1) \( a) \quad \ln y = (\sin x) (\ln x) \)

\[
\frac{y'}{y} = (\cos x) \ln x + \frac{\sin x}{x}
\]

\[
y' = \left[ (\cos x) \ln x + \frac{\sin x}{x} \right] x^{\sin x}
\]

\( b) \quad f'(z) = -\sin \left( (\ln z)^2 \right) \left( 2 \ln z \right) \frac{1}{z} \)

\( c) \quad g'(t) = A \cdot \frac{1}{\sqrt{1-(Bt+c)^2}} \cdot B = \frac{AB}{\sqrt{1-(Bt+c)^2}} \)

2) \( a) \lim_{\theta \to 0} \frac{1-\cos \theta}{\theta^2} = \lim_{\theta \to 0} \frac{\sin \theta}{2\theta} = \lim_{\theta \to 0} \frac{\cos \theta}{2} \)

\[
= \frac{\cos(0)}{2} = \frac{1}{2}.
\]

(OR: you can recognize \( \lim_{\theta \to 0} \frac{\sin \theta}{2\theta} = \frac{1}{2} \lim_{\theta \to 0} \frac{\sin \theta}{\theta} = \frac{1}{2} \))

\( b) \quad \ln \left( \lim_{x \to \infty} \left( e^x + x \right)^{\frac{1}{x}} \right) = \lim_{x \to \infty} \frac{\ln(e^x + x)}{x} \)

\[
= \lim_{x \to \infty} \frac{\ln(e^x + x)}{x} = \lim_{x \to \infty} \frac{\frac{e^x + 1}{e^x + x}}{1} = \lim_{x \to \infty} \frac{e^x}{e^x + 1} = \lim_{x \to \infty} \frac{e^x}{e^x} = 1.
\]

So \( \lim_{x \to \infty} (e^x + x)^{\frac{1}{x}} = e^1 = e. \)
3. \( \cos \left( \frac{12\pi}{25} \right) = f(x) \) with \( f(x) = \cos x \),

\( x = \frac{12\pi}{25} \). We use \( a = \frac{\pi}{2} \) so

\( x - a = \frac{12\pi}{25} - \frac{\pi}{2} = -\frac{\pi}{50} \). Then

\[
\begin{align*}
 f(x) \approx L(x) &= f(a) + f'(a) \left( x - a \right) \\
 &= \cos \left( \frac{\pi}{2} \right) + \sin \left( \frac{\pi}{2} \right) \left( -\frac{\pi}{50} \right) \\
 &= 0 - 1 \left( -\frac{\pi}{50} \right) = \frac{\pi}{50}.
\end{align*}
\]

So \( \cos \left( \frac{12\pi}{25} \right) \approx \frac{\pi}{50} \).

4. If \( F(x) = \text{mass (in g)} \) of the first \( x \) cm,

then \( F'(x) = \frac{1}{1+x^2} \). So \( F \) is an antiderivative of \( \frac{1}{1+x^2} \). So \( F(x) = \arctan(x) + C \).

\[
0 = F(0) = \arctan(0) + C = C
\]

\( (\tan 0 = 0 \text{ so } \arctan 0 = 0) \).

So \( F(x) = \arctan x \).

The mass of the first 1 cm is

\[
F(1) = \arctan(1) = \frac{\pi}{4} \text{ g}
\]

\( (\tan \frac{\pi}{4} = 1 \text{ so } \arctan 1 = \frac{\pi}{4}) \).
\( 3x^2 + \tan^{-1} y + \frac{x}{1+y^2} \frac{dy}{dx} = 8e^y \frac{dy}{dx}. \)

At \((2,0): \quad 12 + 0 + 2 \frac{dy}{dx} = 8 \frac{dy}{dx} \Rightarrow 6 \frac{dy}{dx} = 12 \Rightarrow \frac{dy}{dx} = 2. \)

So the tangent line is
\[ y - 0 = 2(x - 2), \quad \text{or} \quad y = 2x - 4. \]

\[ V = x^2 y \quad 4x + y = 108 \quad \Rightarrow \begin{array}{c}
0 \leq y \leq 108 \\
0 \leq x \leq \frac{108}{4} = 27
\end{array} \]

Maximize \( V(x) = x^2 \cdot 4(27-x) \)
\[ = 4(x^2 27x^2 - x^3) \]

for \( 0 \leq x \leq 27. \)

\[ V'(x) = 4(54x - 3x^2) = 12x(18-x) \]

\[ V'(x) = 0 \quad \text{for} \quad x = 0, 18. \]

Candidate points for maxima: \( x = 0, 18, 27. \)
\[ V(0) = V(27) = 0, \quad V(18) = (18)^2 \cdot 4 \cdot (27-18) \]
\[ = 4 \cdot 9 \cdot 18^2 \text{ in}^3. \]

The maximum volume is \( 4 \cdot 9 \cdot 18^2 \text{ in}^3. \)
We start by pretending the mug extends downward like a cone.

These triangles are similar, so the height of the bottom part is also 6.

If we pretend the bottom part is also full of coffee, then nothing changes about the question since it asks about only changes in the height and volume, not about the actual amount of coffee. Just notice that we are interested in the time when \( y = \frac{4}{3} \), not \( y = 6 \).

The total (real + imaginary) volume of coffee is \( V = \frac{\pi}{3} r^2 y \) and \( \frac{y^2}{r} = \frac{12}{4} \), so

\[ r = \frac{y^2}{3} \text{ and } V = \frac{\pi}{27} y^3. \]

(I put things in terms of \( y \), not \( r \), because...
we want to know about $\frac{dy}{dt}$, not $\frac{dr}{dt}$.

Differentiating w.r.t. $t$:

$$\frac{dV}{dt} = \frac{\pi}{9} y^2 \frac{dy}{dt}.$$

We know $\frac{dV}{dt} = 20$ and we're interested in the moment when $y = 9$. So

$$20 = \frac{\pi}{9} \cdot 9^2 \frac{dy}{dt} = 9\pi \frac{dy}{dt}$$

$$\Rightarrow \frac{dy}{dt} = \frac{20}{9\pi} \text{ cm/s}.$$
As I've drawn things, it appears that successive iterations make \( x_n \rightarrow -\infty \).

A slightly different guess of what the tangent line at \( x=3 \) looks like would lead to \( x_1 = 1 \). In that case, the tangent line at the second step is horizontal.

Either way, the method fails to find a root.
\[ f(x) = e^{1/x} \]

a) Horizontal asymptotes:
Using the substitution \( t = \frac{1}{x} \),

\[
\lim_{x \to \infty} e^{\frac{1}{x}} = \lim_{t \to 0^+} e^t = e^0 = 1
\]

and

\[
\lim_{x \to -\infty} e^{\frac{1}{x}} = \lim_{t \to 0^-} e^t = e^0 = 1.
\]

So \( y = 1 \) is the only horizontal asymptote.

Vertical asymptotes: \( f \) is continuous except at \( x = 0 \), so that is the only possible vertical asymptote.

\[
\lim_{t \to \infty} e^{\frac{1}{x}} = \lim_{t \to \infty} e^t = \infty \quad (t = \frac{1}{x} \text{ again})
\]

\[
\lim_{t \to -\infty} e^{\frac{1}{x}} = \lim_{t \to -\infty} e^t = 0.
\]

So \( x = 0 \) is the only vertical asymptote.

b) \[ f'(x) = e^{\frac{1}{x}}(-\frac{1}{x^2}) = -\frac{e^{\frac{1}{x}}}{x^2} \]

\[ f''(x) = -\left[ \frac{x^2 e^{\frac{1}{x}}(-\frac{1}{x^2}) - e^{\frac{1}{x}}(2x)}{x^4} \right] \]

\[ = \frac{e^{\frac{1}{x}}(1+2x)}{x^4} \]
c) $f'(x)$ is never 0, but is undefined at $x = 0$.
   $f'(x) < 0$ for $x \neq 0$ since $e^x > 0$ and $x^2 > 0$ for $x \neq 0$. So $f$ is decreasing on $(-\infty, 0)$ and $(0, \infty)$.

d) $f''$ is undefined at $x = 0$ and $f''(-\frac{1}{2}) = 0$.
   ($(1+2x)$ is the only part of the numerator which can be 0.)
   
   $f''(-1) = e^{-1}(-1) < 0 \quad \checkmark$

   $\implies f$ is concave down on $(-\infty, -\frac{1}{2})$.

   $f''(-\frac{1}{4}) = e^{-\frac{1}{4}} \frac{(1-\frac{1}{2})}{(\frac{1}{2})^2} = 32e^{-\frac{1}{4}} > 0 \quad \checkmark$

   $\implies f$ is concave up on $(-\frac{1}{2}, 0)$

   $f''(1) = 3e > 0$

   $\implies f$ is concave up on $(0, \infty)$.

   Concavity changes at $x = -\frac{1}{2}$, so we calculate $f(-\frac{1}{2}) = e^{-2} = \frac{1}{e^2} < 1$
There are no local extrema.

Inflection point