• Complete the following problems. In order to receive full credit, please show all of your work and justify your answers. You do not need to simplify your answers unless specifically instructed to do so. You may use any result proved in class or the text, but be sure to clearly state the result before using it, and to verify that all hypotheses are satisfied.

• Please check that your copy of this exam contains 11 numbered pages and is correctly stapled.

• This is a closed-book, closed-notes exam. No electronic devices, including cellphones, headphones, or calculation aids, will be permitted for any reason.

• **You have 2 hours.** Your organizer will signal the times between which you are permitted to be writing, including anything on this cover sheet, and to have the exam booklet open. During these times, the exam and all papers must remain in the testing room. When you are finished, you must hand your exam paper to a member of teaching staff.

• Paper not provided by teaching staff is prohibited. If you need extra room for your answers, use the back side of the question page or other extra space provided at the front of this packet, and clearly indicate that your answer continues there. Do not unstaple or detach pages from this exam.

• It is your responsibility to arrange to pick up your graded exam paper from your section leader in a timely manner. You have only until **Tuesday, October 29**, to resubmit your exam for any regrade considerations; consult your section leader about the exact details of the submission process.

• Please sign the following:

  “On my honor, I have neither given nor received any aid on this examination. I have furthermore abided by all other aspects of the honor code with respect to this examination.”

  **Signature:** __________________________
1. (16 points) Find each of the following limits, with justification. If the limit does not exist, explain why. If there is an infinite limit, then explain whether it is $\infty$ or $-\infty$.

(a) \( \lim_{x \to 1^+} \frac{x^2 \ln(x/2)}{x^2 - 1} \)

(b) \( \lim_{z \to -2} \frac{z^2 - 4}{z^2 + z - 2} \)
(c) \( \lim_{x \to \infty} \frac{e^x + 2^x}{3^x + 1} \)

(d) \( \lim_{t \to 0^+} t \sin(\ln t) \)
2. (8 points)
   (a) Complete the following sentence: the rigorous ("ε, δ") definition of the statement
   \[ \lim_{x \to a} f(x) = L \]
   is that for every positive value of \( \varepsilon \), there is a corresponding positive value of \( \delta \) so that
   whenever \( \varepsilon \)
   , then \( \delta \).

   (b) Use the \( \epsilon, \delta \) definition to prove that \( \lim_{x \to 0} x^{2/3} = 0 \).
3. (14 points) Let \( h(x) = e^{-e^x} - 2. \)

(a) Find the domain and range of \( h. \)

(b) Find the equations of all vertical asymptotes of \( h, \) or explain completely why none exist. (As justification for each asymptote \( x = a, \) calculate both the one-sided limits \( \lim_{x \to a^-} h(x) \) and \( \lim_{x \to a^+} h(x), \) with reasoning.)
For easy reference, $h(x) = e^{(-e^x)} - 2$.

(c) Find the equations of all horizontal asymptotes of $h$, or explain why none exist. Justify using limit computations.

(d) It is a fact that $h$ is one-to-one (which you do not have to prove). Find an expression for $h^{-1}(x)$, the inverse of $h$. 
4. (9 points) The day length $T$ (in minutes) on October 15th varies with latitude $L$ (in degrees North of the Equator) according to the following table:

<table>
<thead>
<tr>
<th>$L$</th>
<th>30</th>
<th>35</th>
<th>40</th>
<th>45</th>
<th>50</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T(L)$</td>
<td>703</td>
<td>699</td>
<td>695</td>
<td>690</td>
<td>684</td>
</tr>
</tbody>
</table>

(a) Give your best estimate for the value of $T'(40)$, showing your reasoning, and make sure to specify the units of this quantity.

(b) What is the practical meaning of the quantity $T'(40)$? Give a brief but complete one- or two-sentence explanation that is understandable to someone who is not familiar with calculus.

(c) The average latitude of the U.S. is 38 degrees North. Apply your value of $T'(40)$ to approximate the day length on October 15th at this latitude, showing your reasoning.
5. (8 points) Let \( f(x) = \sqrt{3x + 2} \). Find \( f'(x) \) using the limit definition of the derivative. Show the steps of your computation.
6. (8 points) Find the derivative, using any method you like. You do not need to simplify your answers.

(a) \( g(x) = \frac{x^4 - 4}{1 + e} + \frac{\sqrt[3]{x^7}}{\sqrt[3]{x^2}} + \pi^2 \)

(b) \( h(x) = x^e e^x + \frac{5 \tan x}{\cos x + 1} \)
7. (5 points) Show that the graph of

\[ f(x) = e^x \cos x + x \]

has a horizontal tangent line at some point \((a, f(a))\), where \(a\) lies in the interval \([0, \pi]\).
8. (8 points) For constants $a$, $b$, and $c$, let $f(x) = \begin{cases} ax^2 + b & \text{if } x < 4 \\ c & \text{if } x = 4 \\ \sqrt{x} & \text{if } x > 4 \end{cases}$

(a) Find, with complete reasoning, necessary conditions on $a$, $b$, $c$ so that $\lim_{x \to 4} f(x)$ exists.

(b) Find, with complete reasoning, necessary conditions on $a$, $b$, $c$ so that $f$ is differentiable at $x = 4$. 
9. (12 points)

(a) On the set of axes below, sketch the graph of a function \( f \) that satisfies all of the following conditions. Be sure to label the scales on your axes.

- \( f \) is continuous on \((-\infty, \infty)\)
- \( f(-2) = 0 \) and \( f(-1) = -1 \)
- \( f'(x) = 1 \) for \( x < -2 \)
- \( f(-x) = -f(x) \) for all \( x \) satisfying \(-1 \leq x \leq 1\)
- \( \lim_{x \to 0} f'(x) = \infty \)
- \( f(x) > 0 \) for \( x > 0 \)
- the only local maximum of \( f(x) \) for \( x > 0 \) is located at \( x = 1 \)
- the only inflection point of \( f(x) \) for \( x > 0 \) is located at \( x = 2 \)

(b) On the axes below, sketch the graph of \( f' \) (the derivative of \( f \)), labeling your axes as appropriate.