MATH 215B, WINTER 2012

The course: Math 215b is a graduate course in algebraic topology. We meet on Tuesdays and Thursdays from 12:50 to 2:05pm in room 380-380F. Algebraic topology is the study of homotopy invariants, where a homotopy equivalence describes an equivalence between two topological spaces that is weaker than homeomorphism, one that allows for richer set of algebraic invariants. This course focuses on the computation of homotopy invariants of topological spaces, in particular the fundamental group, the homology groups and the cohomology ring. Some consequences of these computations are studied as well.

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Prerequisites: Formal prerequisites are Math 113, 120 and 171. Apart from formal prerequisites, I will assume that you are intimately familiar with point-set topology, homological algebra and modern algebra. In particular, these are things you should know really well

in algebra: equivalence relations and quotient sets, groups, quotient groups, rings, homomorphisms, modules, exact sequences, categories and functors. These are things you should know really well in point-set topology: constructions with spaces such as cartesian products, quotient topology, connectedness and path-connectedness, compactness and consequences of these properties.

Math 215a is not a prerequisite for Math 215b. Neither is Math 148.

Textbook: Algebraic Topology, by Allen Hatcher will be the main textbook. This book can be downloaded for free at http://www.math.cornell.edu/~hatcher/AT/ATpage.html and it is also on reserve in the library. You can also find it in the bookstore and it is not expensive. The book Topology and Geometry, by Glen E. Bredon, is a good alternative reference.

Course grading. There will be a take-home midterm, an open-book final exam (March 22nd, 3:30-6:30pm) and six homework sets. Late homework will not be accepted. You are encouraged to discuss the problems with each other, but you must try the problems first on your own, and also work on your own when you write them down. You can not discuss the problems of the midterm or the final with each other. Homework problems will be posted on the course website, more than a week before they are due. Homework assignments count for 25 % of your final grade, the midterm for 35 %, and the final exam for 40 %.

Course learning objectives. By the end of this course, you should be able to:

- Define and illustrate the concepts of homotopy type, homotopy invariant, CW complex.
- Construct homotopy equivalences between certain spaces.
- Compute the fundamental group of graphs.
- Compute the fundamental group of some spaces using covering spaces.
- Compute the fundamental group of some spaces using Van Kampen's theorem.

- Justify that the fundamental group, the homology groups and the cohomology ring are homotopy invariants.
- Compute the homology of a chain complex.
- Compute the homology and cohomology of some spaces using their axiomatic characterization.
- Use the fundamental group, homology or cohomology to compare the homotopy types of two spaces.
- Prove some classical results in topology and algebra using the fundamental group and the homology groups.
- Apply Poincare duality to obtain computations of homology/cohomology or to prove properties of manifolds.