HOMEWORK 5 DUE MARCH 6TH BY 5PM

You may use any results in the book up to and including section 2.B.

1. Exercise 12, page 156.

2. Let $f : S^n \to S^n$ be a continuous map such that $||f(p) - p|| < 1$ for all $p \in S^n$, where $||\ ||$ denotes the canonical norm in $\mathbb{R}^{n+1}$. Show that f is surjective.

3. A continuous map $f : X \to Y$ between CW complexes is called cellular if $f(X^n) \subset Y^n$ for all $n \geq 0$, where X^n, Y^n denote the n-skeleton of X and Y, respectively.

(a) Show that a cellular map f induces a map between the cellular chain complexes of X and Y. Deduce from this fact that f induces maps in cellular homology, $f_*^{CW} : H_k^{CW}(X) \to H_k^{CW}(Y)$ for all $k \geq 0$.

(b) Show that these maps are natural with respect to the isomorphism of singular and cellular homology, that is, if $\Phi_X : H_k(X) \to H_k^{CW}(X)$ is the isomorphism given in the proof of theorem 2.35 in page 139 of Hatcher's book (or in class), then $f_*^{CW}\Phi_X = \Phi_Y f_*$.

(c) Using parts (a) and (b), and a suitable CW-structure on S^n, show that a reflection $r : S^n \to S^n$ across a hyperplane in $\mathbb{R}^{n+1}$ that contains the origin has degree -1.

4. Exercise 2, page 176.

5. Let $U(2)$ denote the group of all unitary 2×2 matrices over $\mathbb{C}$ and $SU(2)$ the subgroup of $U(2)$ of unitary matrices with determinant one. We endow both sets with the topologies induced by the inclusions $SU(2) \subset U(2) \subset \mathbb{C}^4 \subset \mathbb{R}^8$.

(a) Show that $SU(2)$ is homeomorphic to S^3.

(b) Compute $H_*(U(2))$.