

WINTER 2012 MATH 215B FINAL EXAM

1. (12 marks) Let X and Y be path-connected and locally contractible spaces such that $H^1(X, \mathbb{Q}) \neq 0$ and $H^1(Y, \mathbb{Q}) \neq 0$. Show that $X \vee Y$ is not a retract of $X \times Y$.
2. (12 marks) The surface M_g of genus g , embedded in \mathbb{R}^3 in the standard way, bounds a compact region R . Two copies of R , glued together by the identity map between their boundary surfaces M_g , form a space X . Compute the homology groups of X and the relative homology groups of (R, M_g) .
3. (8 marks) Show that the spaces $S^2 \times \mathbb{R}P^4$ and $S^4 \times \mathbb{R}P^2$ are not homotopy equivalent.
4. (12 marks) Let $f : \mathbb{R}P^{2n} \rightarrow Y$ be a covering map, where Y is a path-connected CW-complex. Prove that f must be a homeomorphism. (Careful: When is the composition of covering maps a covering map?)
5. Let X be the space obtained by attaching two 2-cells to S^1 , one via the map $z \mapsto z^3$ and the other via $z \mapsto z^5$, where z^n denotes the n th power of a complex number and we consider the standard inclusion $S^1 \subseteq \mathbb{C}$.
 - (a) (6 marks) Compute the fundamental group and the cohomology ring of X .
 - (b) (6 marks) Show that X is not homeomorphic to S^2 .