# Solutions For Homework #3

1. Ozone abundance at different elevations in the atmosphere has very different consequences for life on the surface of the earth. At high altitudes the ozone acts as a screen to filter out hard (high-energy) ultra-violet (UV) radiation emitted from the sun. UV radiation can damage biological cells and is therefor harmful to most life on earth.

   Ozone at low altitudes has the same effect of filtering the UV radiation. However, ozone is also highly reactive, making it a hazard for plants and animals (including humans). It also reacts with certain materials like plastic. Reactions with other pollutants in the lower atmosphere can produce even more damaging pollutants.

   A measurement of ozone concentrations as a function of elevations is necessary to distinguish between this "good ozone" at high altitudes and the "harmful ozone" at lower altitudes. Also, such a measurement still provides the total amount of ozone over the entire height of the atmosphere, if the amounts are integrated over all elevations.

2. (a) The image contains two main features: The dark reservoir and the bright surrounding area. There are few pixels of intermediate brightness in the image. The pixels making up the reservoir correspond to the left bump in the histogram, while the pixels in the bright surroundings make up the right bump.

   (b) The pixels in the first bump make up the reservoir area. Since all the pixels make up $100\%$ of the imaged area ($A = 10\,\mathrm{km} \cdot 10\,\mathrm{km} = 100\,\mathrm{km}^2$), and all the pixels have the same size, $20\%$ of them make up $20\,\mathrm{km}^2$. This is then approximately the area of the reservoir.

3. (a) All pixels values smaller than $80$ are mapped to $0$, all values greater than $130$ are mapped to $255$.

(b) They will not be detectable at all after the stretch. Both values, $60$ and $20$ are mapped to $0$. Notice that the stretch actually reduces the information content in the image, although it may look more pleasing to the eye.

(c) In this case the feature will be more detectable, because the values are less similar after the stretch. This can be expressed with the ratio of brightnesses of target to background before and after the stretch. The linear stretch is performed with the mapping

$$x_{stretched} = (x - 80) \cdot \frac{255 - 0}{130 - 80}$$

Using this equation,

$$x = 115 \longrightarrow x_{stretched} = 178.5$$
$$x = 100 \longrightarrow x_{stretched} = 102$$

Before stretch: $\frac{115}{100} = 1.15$ After stretch: $\frac{178.5}{102} = 1.75$

The greater ratio means that it would be easier to see the feature against the background.

4. Consider the geometry of a radar system (figure 1). The range resolution is measured along the line of sight, while the azimuth resolution is measure along the direction of flight (figure 1).

(a) Referring to figure 1b, the ground range resolution at $\Theta = 30°$ can be calculated as

$$\delta_g = \frac{\delta_s}{\sin(30°)} = \frac{10m}{0.5} = 20m$$

Referring to figure 1b, the ground range resolution at $\Theta = 60°$ can be calculated as

$$\delta_g = \frac{\delta_s}{\sin(60°)} = \frac{10m}{\sqrt{3}/2} = 11.55m$$

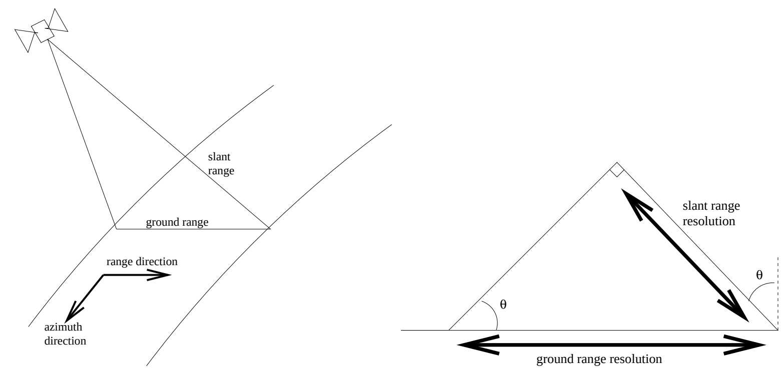


Figure 1: Geometry of radar acquisition. Note the difference between slant range and ground range: $\sin(\Theta) = r_{slant}/r_{ground}$.

(b) To obtain square pixels on the ground, the ground range resolution must be equal to the azimuth resolution, i.e. $\delta_g = 15m$. Rearranging the equation above we get

$$\sin(\Theta) = \frac{\delta_s}{\delta_g} = \frac{10}{15} \Longrightarrow \Theta = 41.8^\circ$$

(c) At $\Theta = 30^\circ$ incidence angle, the ground resolution is $\delta_g = 20m$. The number of pixels per azimuth line can be calculated as

$$n_l = \frac{100\,\mathrm{km}}{20m} = 5000$$

At a velocity of $v = 7500m/s$, and an azimuth resolution of $15m$, the number of lines per second is

$$\mathrm{lps} = \frac{7500\frac{m}{s}}{15m} = 500s^{-1}$$

Note that this is only approximate as we should really use the effective satellite velocity (the velocity of the nadir or sub-satellite point), which is by a factor of $R_{earth}/(R_{earth} + h_{sat})$ smaller.

From the numbers calculated above we can calculate the data rate:

$$\text{data rate} = n_l \cdot \mathrm{lps} \cdot 1\frac{\text{byte}}{\text{pixel}} = 2.5\frac{\mathrm{Mb}}{s} = 20 \cdot 10^6 \frac{\text{bits}}{s}$$

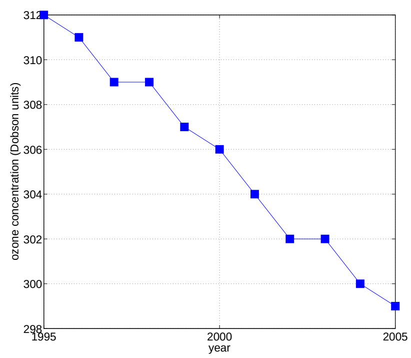


Figure 2: Plot of the ozone concentration versus time.

(d) We need

$$\frac{20 \cdot 10^6}{6 \cdot 10^6} = 3.3 \quad \text{TV channels}$$

to transmit the radar picture.

5. (a) From the plot of the Dobson units versus the intensity ratio we find the relation for the ozone concentration $c$ and the intensity ratio $r$:

$$c = 100r$$

We can use this to convert the given values to ozone concentrations and get the graph in figure 2.

(b) At the current levels of ozone concentration in the atmosphere, a $1\%$ change in ozone concentration approximately causes a $2\%$ change in UV penetration. The measured decrease from $312$DU to $299$DU corresponds to a $4.2\%$ decrease in ozone concentration. This leads to an $8.4\%$ increase in UV radiation.

(c) There are several ways to use this experiment to measure the ozone concentration as a function of height. The easiest way is to pulse the lasers and measure the reflected intensities as a function time. Because the speed of light is known, the measured time delay can be converted to distance, i.e. height.

**f.** (a) The MATLAB code for loading in and displaying an image, say *hw3prob6a.1024* is as follows:

```
f=fopen('hw3prob6a.1024.txt');
a=fread(f,[1024 inf],'uint8');
a = a';
imagesc(a); colormap gray; axis image;
print -djpeg90 red_channel
```

This code reads in a binary file consisting of 8-bit elements into a 2D array of dimensions $1024 \times 1024$. The second last line tells MATLAB to display the image in gray scale and to make the columns and rows have square pixels on the screen. We use the MATLAB code above to read in the Red, Green and Blue channel images and combine them to form a multichannel image as follows

```
picture = zeros(1024,1024,3,'uint8');
picture(:,:,1) = a;   % a = 1024x1024 RED channel
picture(:,:,2) = c;   % c = 1024x1024 GREEN channel
picture(:,:,3) = b;   % b = 1024x1024 BLUE channel
image(picture);axis image
```

The multichannel image formed is shown in Figure 3.

We observe that the image has low contrast. In particular, the Red and Blue channels are more bright than the Green, leading to the observed purple background in the picture. We, therefore, need to stretch the distribution of pixel intensities in each channel to occupy the full dynamic range of 255 levels.

(b) We recall that the linear stretch of an image $x$ is as follows

$$x_{stretched} = \frac{H}{x_{high} - x_{low}}(x - x_{low}) \tag{1}$$

See Figure 4 for the definitions of $x_{low}$, $x_{high}$ and $H$. We need to plot the histogram of pixel values to identify these quantities in order to perform a linear stretch of $x$. Using the MATLAB command (or you can use Scion Image)

```
hist(a(:),255)
```

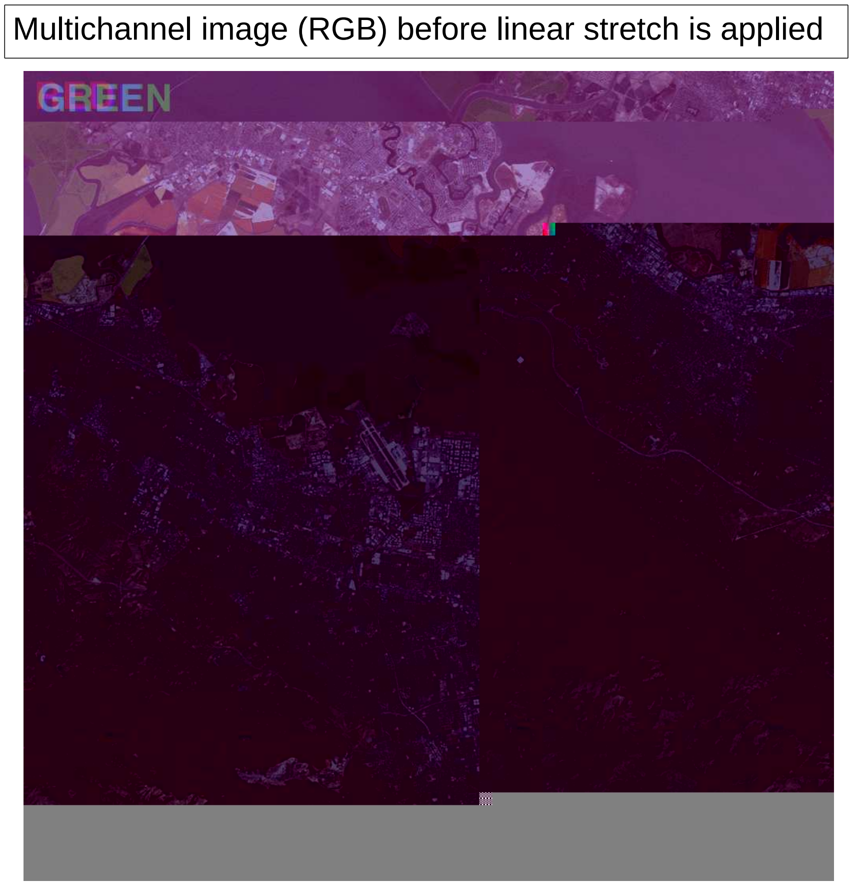


Figure 3: Multichannel (RGB) image before linear stretch is applied. Note the low contrast ratio

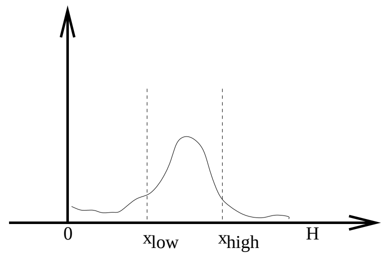


Figure 4: histogram of pixel values in an image, with definitions of $x_{low}$, $x_{high}$ and $H$

we can find the number of pixels in the image $a$ that are at levels between 0 and 255, i.e. the histogram of $a$. Figure 5 displayes the intensity images of the Red, Green and Blue channels along with the corresponding histogram of pixel values. Inspecting the pixel value histograms in Figure 5 , we find that

$$\begin{aligned} \text{red channel}: &\quad x_{low} = 82 \quad x_{high} = 154 \\ \text{green channel}: &\quad x_{low} = 0 \quad x_{high} = 122 \\ \text{blue channel}: &\quad x_{low} = 63 \quad x_{high} = 182 \end{aligned}$$

and $H = 255$. After performing the linear stretch on the Red, Green and Blue intensity images following Equation 1 , we form a new multichannel image shown in Figure 6

(c) We use the IR channel intensity image plus the stretched versions of Red and Green channels as in part (b) to form the false color picture . We examine the histogram of pixel values for the IR channel to obtain values for $x_{low}$ and $x_{high}$. The histogram of IR channel intensities is shown in Figure 7.

We find that

$$\text{IR channel:} x_{low} = 192 \;\; x_{high} = 255$$

Applying the linear stretch equation 1 to the IR image, and then combining the result with the stretched versions of the Red and Green channels from part (b), we obtain the false color image of the Bay Area shown in Figure 8.

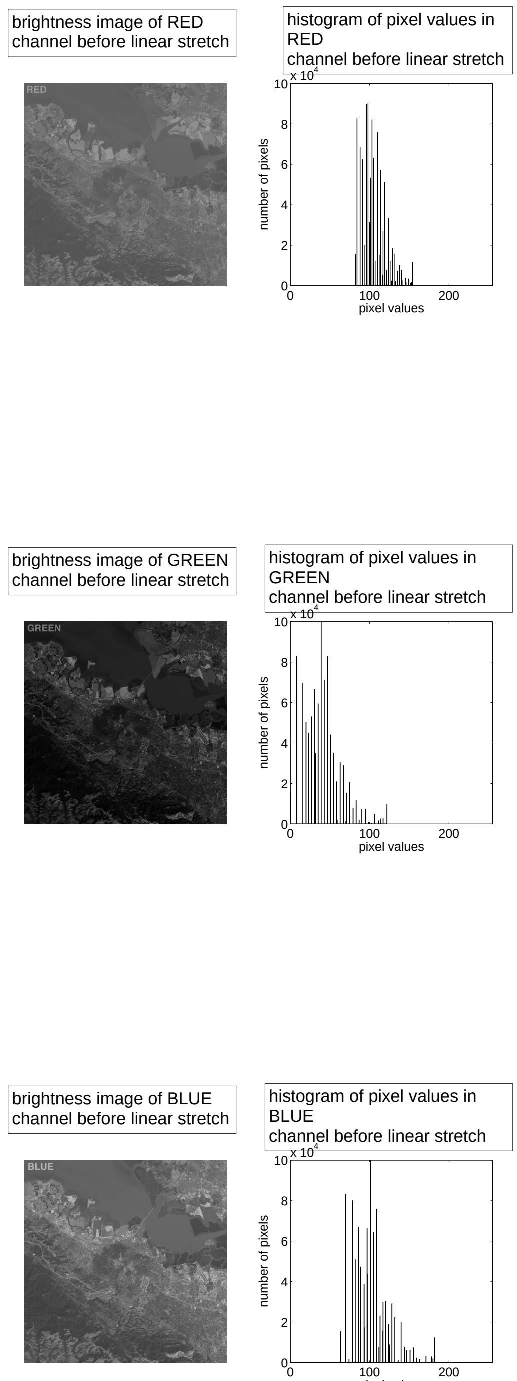




Figure 5: Intensity images and histogram of pixel values for Red, Green and Blue channels

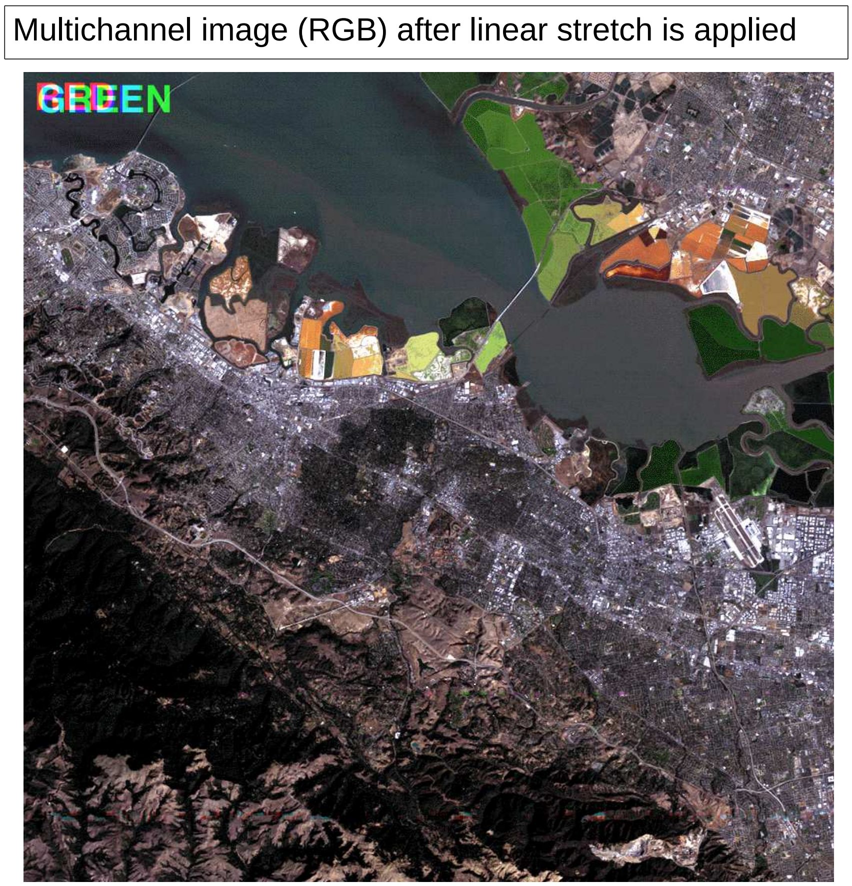


Figure 6: Multichannel (RGB) image after linear stretch is applied. Note that the contrast has improved

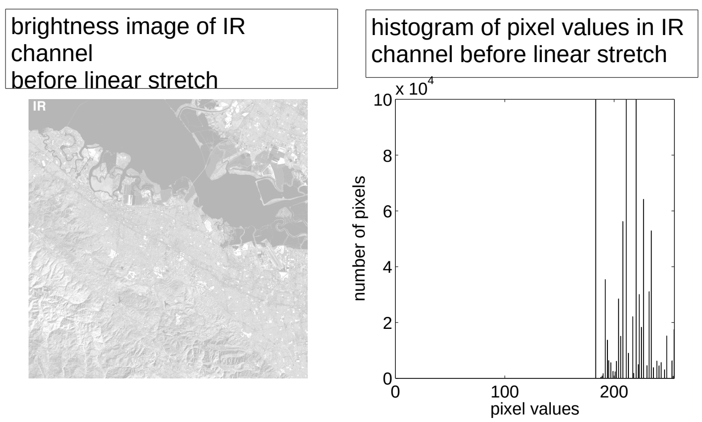


Figure 7: Intensity image and histogram of pixel values for the IR channel

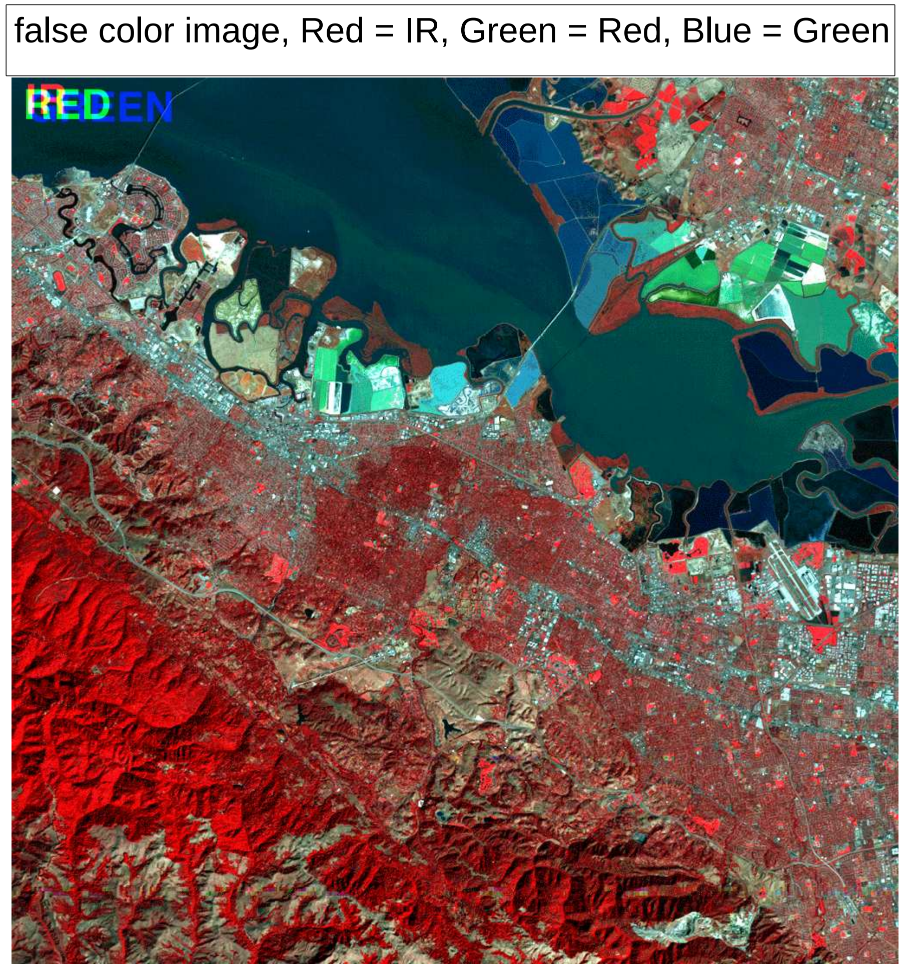


Figure 8: Red, Green and IR channels in false color. IR is mapped to Red, Red is mapped to Green and Green is mapped to Blue in the above

.