Solutions For Homework #2

1. (a) The orbital period can be calculated using the equation

$$T = 2\pi r\sqrt{\frac{r}{gR_e^2}} \qquad \text{where} \quad r = R_e + h$$

where $R_e = 6378\,\mathrm{km}$ is the earth's radius, r is the satellites distance from the earth's center and $h = 205\,\mathrm{km}$ is the satellite's orbital altitude, and $g = 9.81 m/s^2$ is the gravitational acceleration. With these given values the orbital period is

$$T_{orbit} = 5312.5s = 1.4757h$$

(b) To calculate the orbital velocity either of the equations

$$v = \sqrt{\frac{gR_e^2}{r}} \qquad \text{or} \quad T = \frac{2\pi r}{v} \Leftrightarrow v = \frac{2\pi r}{T}$$

can be used. For the given values the result is

$$v \approx 7786 m/s$$

(c) To calculate the *minimum* number of ascending passes needed to cover the entire equator, divide the perimeter of the equator by the swath width. The perimeter of the equator is $p = 2\pi R_e = 40074 km$, so the minimum number of passes is

$$\frac{p}{w} = \frac{40074\,\mathrm{km}}{50\,\mathrm{km}} \approx 802 \text{ passes}$$

The time needed to acquire the calculated coverage can be calculated by

$$\begin{aligned} T_{global} &= (\#of passes) \cdot T_{orbit} \\ &= 802 \cdot 5312.5s = 4260625s = 49.3128 \text{ days} \approx 49.3 \text{ days} \end{aligned}$$

(d) To calculate the *minimum* number of ascending passes needed to cover the entire equator, the perimeter of the equator is divided by the *effective* swath width. The effective swath width is the length along the

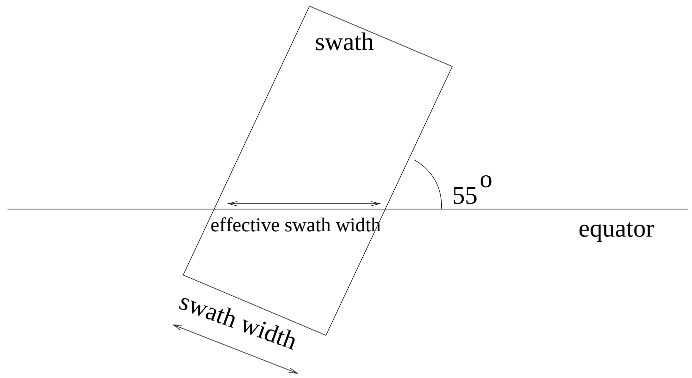

Figure 1: The effective swath width is the length along the equator covered by a single crossing of swath at an angle of 55^o

equator covered by the swath crossing it at a given inclination angle, (Figure 1) For the given swath width $w = 50\,\text{km}$, at an inclination of $\theta = 55°$ the effective swath width is

$$\sin(\theta) = \frac{w}{h} \Leftrightarrow h = \frac{50\,\text{km}}{\sin(55°)} = 61\,\text{km}$$

The perimeter of the equator is $p = 2\pi R_e = 40074 km$, so the minimum number of ascending passes is

$$\frac{p}{h} = \frac{40074\,\text{km}}{61\,\text{km}} \approx 657 \text{ passes}$$

The time needed to acquire the calculated coverage can be calculated by

$$\begin{aligned} T_{global} &= (\#of passes) \cdot T_{orbit} \\ &= 657 \cdot 5312.5 s = 3490294 s = 40.3969 \text{ days} \approx 40.5 \text{ days} \end{aligned}$$

If both ascending and descending passes are used, the satellite swath crosses the equator twice in every orbit, cutting the time in half, i.e. $T_{global} \approx 20$days.

(e) The polar orbit is most useful for studying ice motions in the Arctic Ocean.

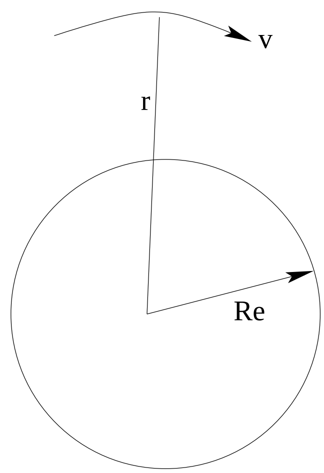

Figure 2: The velocity of the nadir point (point on the earth's surface directly under the satellite) is by the radius ratio (r/R_e) smaller than the orbit velocity. The ground-velocity is what matters for coverage calculations.

2. The rate of coverage is measured in area covered per unit time. Given the formula for the orbital velocity,

$$v = \frac{2\pi r}{T_{orbit}} = \sqrt{\frac{gR_e^2}{r}}$$

a rate of coverage can be calculated by

$$\text{rate of coverage} = v \cdot \frac{R_e}{r} \cdot w$$

where w is the swath width (see also Figure 2). To get equal coverage:

$$v_1 \frac{R_e}{r_1} w_1 = v_2 \frac{R_e}{r_2} w_2$$

$$\Rightarrow w_2 = w_1 \frac{v_1 r_2}{v_2 r_1} = w_1 \frac{\sqrt{\frac{gR_e^2}{r_1}} r_2}{\sqrt{\frac{gR_e^2}{r_2}} r_1} = w_1 (\frac{r_2}{r_1})^{\frac{3}{2}} = 160\,\text{km} \cdot (\frac{6655\,\text{km}}{6948\,\text{km}})^{\frac{3}{2}} \approx 150\,\text{km}$$

Alternatively, this result can be arrived at by calculating the two orbital periods and the coverage per day for each satellite:

$$T_1 = 5760.4s (15 \text{ orbits/day}) \text{ and } T_2 = 5400s (16 \text{ orbits/day})$$

The coverage per orbit:

$$c_1 = 2\pi R_e w_1 = 6.4113 \cdot 10^6 \,\mathrm{km}^2$$
$$c_2 = 2\pi R_e w_2$$

Thus, satellite 1 acquires per day:

$$15 \frac{\text{orbits}}{\text{day}} \cdot 6.4113 \cdot 10^6 \frac{\mathrm{km}^2}{\text{orbit}} = 9.618 \cdot 10^7 \frac{\mathrm{km}^2}{\text{day}}$$

Satellite 2 should have the same coverage, therefore:

$$9.618 \cdot 10^7 \frac{\mathrm{km}^2}{\text{day}} = 16 \frac{\text{orbits}}{\text{day}} \cdot 2\pi R_e w_2 \Rightarrow w_2 = 150 \,\mathrm{km}$$

To compute the data rate of each satellite we first find the number of lines per second, then multiply by the number of samples per line. Next we multiply by 8-bit per sample and by 3 channels. So,

(i) Satellite 1 acquires

$$\text{ground-velocity} = v_1 \times \frac{R_e}{r_1} = 6953 \text{ m/s}$$

$$\frac{6953 \, m/s}{30 \, m} = 232 \text{ lines/s}$$

$$232 \text{ lines/s} \times \frac{160,000 \text{ m}}{30 \text{ m}} = 1,236,000 \text{ samples/s}$$

$$1,236,000 \text{ samples/s} \times 3 \text{ channels} \times 8 \text{ bits/sample} = 29.7 \text{ Mbits/s}$$

(ii) Satellite 2 acquires

$$\text{ground-velocity} = v_2 \times \frac{R_e}{r_2} = 7418 \text{ m/s}$$

$$\frac{7418 \, m/s}{30 \, m} = 247 \text{ lines/s}$$

$$247 \text{ lines/s} \times \frac{150,000 \text{ m}}{30 \text{ m}} = 1,236,000 \text{ samples/s}$$

$$1,236,000 \text{ samples/s} \times 3 \text{ channels} \times 8 \text{ bits/sample} = 29.7 \text{ Mbits/s}$$

3. The normalized vegetation index (NDVI) is an indicator for vegetation density, which uses the ratio between the difference of the reflectance in the near infrared and in the visible red part of the spectrum and the sum of these two reflectances:

$$\text{NDVI} = \frac{R_{ir} - R_v}{R_{ir} + R_v}$$

To make a similar measurement with the Thematic mapper instrument we can use those bands which correspond to the near infrared and visible red wavelengths. Bands $4 - 6$ fall within the infrared and band 3 lies in the red part. One possible TMVI equation would be

$$\text{TMVI} = \frac{R_4 - R_3}{R_4 + R_3}$$

4. Since there are H_2O absorption bands in the spectrum on each side of the wavelengths covered by band 5 there is little to no energy coming from those regions. A spectral band within these absorption bands would not be useful to study surface properties.

If band 5 were chosen to look at a slightly different part of the spectrum, it would be less sensitive to minerals which have characteristic signatures in the areas just above and below $1.55\,\mu\text{m} - 1.75\,\mu\text{m}$. These include the water bearing minerals, such as gypsum, montmorillonite and quartz, as well as the hydroxyl (OH) bearing minerals, such as Muscovite, Kaolinite and Actinolite.

Although the regions just above or below the band 5 wavelengths are strongly absorbed by the atmosphere, they may be useful in applications where the thematic mapper instrument is mounted on an airborne platform, rather than a satellite. If the amount of atmosphere between the scanner and the ground is reduced, enough energy may reach the scanner to be able to use these parts of the spectrum.

5. (a) The purpose of this problem is to show that increasing levels of atmospheric CO_2 concentration results in more power being absorbed

than emitted by the Earth. Recall from lecture that if more radiation is absorbed than is emitted by the Earth, then the Earth warms up. We need to compute the *net* power per square meter on Earth to find the corresponding increase in temperature.

The incident solar radiation is given as 1000 W/m^2, of which 25% is reflected by clouds. This gives a $1000 \times (1 - 0.25) = 750$ W/m^2 incident on Earth. Assuming no absorption by atmospheric CO_2, this incident radiation is absorbed by the Earth and re-radiated resulting in a net power density balance of zero. However, in the presence of atmospheric CO_2 absorption, the Earth emits only $\tau \times 750$ W/m^2 of radiation because some of Earth's emitted blackbody radiation is absorbed again by atmospheric CO_2. Here,

$$\tau = 1 - \frac{Q_{CO2} - 270}{20000} \tag{1}$$

is the model of atmospheric CO_2 transmission, as described in the problem statement. Consequently, we find a net power balance, P_{net} of

$$\begin{aligned} P_{net} &= (1 - \tau) \times 750 \\ &= \left(\frac{Q_{CO2} - 270}{20000} \right) 750 \;\; \text{W/m}^2 \end{aligned}$$

Now, referring to Handout 7 of the lecture notes, we find a plot of atmospheric CO_2 concentrations (in parts per million) from the year 1700 A.D to the present. Taking the levels of atmospheric CO_2 concentration in the present to be 360 ppm, we find that

$$P_{net} = 3.375 \text{ W/m}^2 \tag{2}$$

We are given that the temperature on Earth increases by 0.25 ^{o}K for every additional Watt per square meter of *net* power. Thus, the temperature increase compared to pre-industrial times (i.e. the year 1700 A.D when the atmospheric CO_2 concentration was at 270 ppm, implying $\tau = 1$) is

$$\text{temperature increase} = 0.25^o\text{Km}^2/\text{W} \times P_{net} = 0.84^o\text{K} \tag{3}$$

(b) The present levels of atmospheric CO_2 concentration, according to Handout 7 of the lecture notes, is about 360 ppm. If this level were to double in 100 years, then the net power density on Earth a hundred years into the future would be

$$P_{net} = \frac{2 \times 360 \text{ ppm} - 270 \text{ ppm}}{20000} \times 750 = 16.88 \text{ W/m}^2$$

subtract the power level of today

$$16.88 \text{ W/m}^2 - 3.375 \text{ W/m}^2 = 13.5 \text{ W/m}^2$$

Thus, the temperature will rise an amount of $13.5 \text{ W/m}^2 \times 0.25 \text{ Km}^2/\text{W} = 3.375 \text{ K}$ over the next century