

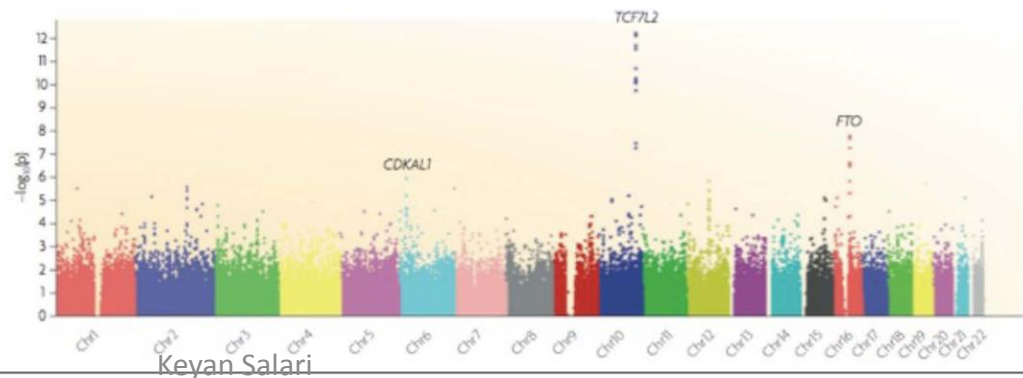


I. Natural variation in the human genome

2. Genetic Association & Linkage Disequilibrium



3. Genome-wide association studies



ORIGINAL INVESTIGATION

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**Sequence polymorphism at the human apolipoprotein AII gene (*APOA2*):
unexpected deficit of variation in an African-American sample**

Sequence *APOA2* in 72 people

Look at patterns of polymorphisms

Chimp		Site no. ^a														
SNP haplotype no.	Sequence haplotype no.	1	2	1	2	1	2	2	2	2	2	2	3	3	3	
		1	2	8	2	6	0	0	1	2	8	8	9	0	0	2
		5	0	7	1	7	3	8	1	3	1	6	9	2	9	0
		5	1	2	8	1	8	5	5	3	8	8	4	7	2	8
		C	G	T	G	?	G	C	G	C	C	C	C	T	A	G

Find polymorphisms at these positions.

Reference sequence is listed.

Chimp		Site no. ^a														Sample				
SNP haplotype no.	Sequence haplotype no.	1	2	1	2	1	2	2	2	2	2	2	3	3	3	2	J	N	R	T
		5	0	7	1	7	3	8	1	3	1	6	9	2	9	0				
		5	1	2	8	1	8	5	5	3	8	8	4	7	2	8				
		C	G	T	G	?	G	C	G	C	C	C	C	T	A	G				
Core re-sequenced samples																				
S9	G		C		20	●		A	●	●	●	●	●	●	●	●	0	0	1	1
S9a	G		C		18	●		A	●	●	●	●	●	●	●	●	0	1	0	1
S2	G		C		19	●		●	●	●	●	●	●	●	●	●	15	10	12	37
S2a	G		C		20	●		●	●	●	●	●	●	●	●	●	0	2	3	5
S2b	G		C		18	●		●	●	●	●	●	●	●	●	●	0	2	1	3
S2c	G		C		21	●		●	●	●	●	●	●	●	●	●	1	0	1	2
S1d	G		●		19	●		●	●	●	●	●	●	●	●	●	5	0	0	5
S1	G		●		16	●		●	●	●	●	●	●	●	●	●	17	19	14	50
S1a	G		●		18	●		●	●	●	●	●	●	●	●	●	5	1	0	6
S1b	G		●		15	●		●	●	●	●	●	●	●	●	●	2	0	0	2
S1c	G		●		17	●		●	●	●	●	●	●	●	●	●	1	0	0	1
S6	●		●		16	●		●	●	●	●	●	●	●	●	●	1	2	0	3
S5	●		●		14	●		●	T	●	A	●	●	●	●	●	1	4	2	7
S3	●		●		14	●		●	T	●	A	●	C	G	A	●	0	3	6	9
S7	●		●		13	C		●	●	T	●	●	●	●	●	●	0	2	0	2
S8	●		●		13	C		●	●	T	●	●	C	G	●	●	0	1	1	2
S4	●		●		13	C		●	●	T	●	T	C	G	●	●	0	1	6	7
S4a	?		●		14	C		●	●	T	●	T	C	G	●	●	0	0	1	1

Chimp		Site no. ^a														Sample				
SNP haplotype no.	Sequence haplotype no.	1	2	1	1	2	2	2	2	2	2	2	3	3	3	2	J	N	R	T
		5	0	7	1	7	3	8	1	3	8	6	9	2	9	0				
		5	1	2	8	1	8	5	5	3	8	8	4	7	2	8				
		C	G	T	G	?	G	C	G	C	C	C	C	T	A	G				
Core re-sequenced samples																				
S9	G		C			20	●		A	●	●	●	●	●	●		0	0	1	1
S9a	G		C			18	●		A	●	●	●	●	●	●		0	1	0	1
S2	G		C			19	●		●	●	●	●	●	●	●		15	10	12	37
S2a	G		C			20	●		●	●	●	●	●	●	●		0	2	3	5
S2b	G		C			18	●		●	●	●	●	●	●	●		0	2	1	3
S2c	G		C			21	●		●	●	●	●	●	●	●		1	0	1	2
S1d	G		●			19	●		●	●	●	●	●	●	●		5	0	0	5
S1	G		●			16	●		●	●	●	●	●	●	●		17	19	14	50
S1a	G		●			18	●		●	●	●	●	●	●	●		5	1	0	6
S1b	G		●			15	●		●	●	●	●	●	●	●		2	0	0	2
S1c	G		●			17	●		●	●	●	●	●	●	●		1	0	0	1
S6	●		●			16	●		●	●	●	●	●	●	●		1	2	0	3
S5	●		●			14	●		●	T	●	A	●	●	●		1	4	2	7
S3	●		●			14	●		●	T	●	A	●	C	G	A	0	3	6	9
S7	●		●			13	C		●	●	T	●	●	●	●		0	2	0	2
S8	●		●			13	C		●	●	T	●	●	C	G	●	0	1	1	2
S4	●		●			13	C		●	●	T	●	T	C	G	●	0	1	6	7
S4a	?		●			14	C		●	●	T	●	T	C	G	●	0	0	1	1

Commonly Used Descriptors

- Haplotype Frequencies
 - The frequency of each type of chromosome
 - Contain all the information provided by other summary measures
- Commonly used summaries
 - D
 - D'
 - r^2 or Δ^2

Haplotype Frequencies

	<u>Locus B</u>		Totals
	<i>B</i>	<i>b</i>	
<u>Locus A</u>	<i>A</i>	p_{AB} p_{Ab}	p_A
	<i>a</i>	p_{aB} p_{ab}	p_a
Totals		p_B p_b	1.0

Fill out this table.

X11 is number of times that haplotype is seen.

	2818 C	2818 T	
3027 T	X11	X21	# 3027 T alleles
3027 C	X12	x22	#3027 C alleles
	# 2818 C Allele	# 2818 T allele	

	2818 C	2818 T	
3027 T	125/146	2/146	127/146 T alleles
3027 C	9/146	10/146	19/146 C alleles
	134/146 C Allele	12/146 T allele	

Convert to fractions

	2818 C	2818 T	
3027 T	.86	.013	.87 T alleles
3027 C	.061	.068	.13 C alleles
	.92 C Allele	.08 T allele	

Linkage Equilibrium Expected for Distant Loci

$$p_{AB} = p_A p_B$$

$$p_{Ab} = p_A p_b = p_A (1 - p_B)$$

$$p_{aB} = p_a p_B = (1 - p_A) p_B$$

$$p_{ab} = p_a p_b = (1 - p_A)(1 - p_B)$$

Linkage Disequilibrium Expected for Nearby Loci

$$p_{AB} \neq p_A p_B$$

$$p_{Ab} \neq p_A p_b = p_A(1 - p_B)$$

$$p_{aB} \neq p_a p_B = (1 - p_A)p_B$$

$$p_{ab} \neq p_a p_b = (1 - p_A)(1 - p_B)$$

Disequilibrium Coefficient D_{AB}

$$D_{AB} = p_{AB} - p_A p_B$$

$$p_{AB} = p_A p_B + D_{AB}$$

$$p_{Ab} = p_A p_b - D_{AB}$$

$$p_{aB} = p_a p_B - D_{AB}$$

$$p_{ab} = p_a p_b + D_{AB}$$

Calculate D_{AB}

$$\begin{aligned} D_{AB} &= P_{AB} - P_A P_B \\ &= .86 - (.87)(.92) \\ &= .86 - /80 \\ &= .06 \end{aligned}$$

D_{AB} is hard to interpret

- Sign is arbitrary ...
 - A common convention is to set A, B to be the common allele and a, b to be the rare allele
- Range depends on allele frequencies
 - Hard to compare between markers

D' – A scaled version of D

$$D'_{AB} = \begin{cases} \frac{D_{AB}}{\min(p_A p_B, p_a p_b)} & D_{AB} < 0 \\ \frac{D_{AB}}{\min(p_A p_b, p_a p_B)} & D_{AB} > 0 \end{cases}$$

- Ranges between -1 and $+1$
 - More likely to take extreme values when allele frequencies are small
 - ± 1 implies at least one of the observed haplotypes was not observed

Calculate D'

If $D_{AB} > 0$

$$D' = D_{AB} / \min(P_A P_b, P_a P_B)$$

$$= .06 / \min[(.87 * .08), (.13 * .92)]$$

$$= .06 / \min (.069 , .12)$$

$$= .06 / .069 = .87$$

If $D_{AB} < 0$

$$D' = D_{AB} / \min (P_A P_B, P_a P_b)$$

More on D'

- **Pluses:**
 - $D' = 1$ or $D' = -1$ means no evidence for recombination between the markers
 - If allele frequencies are similar, high D' means the markers are good surrogates for each other
- **Minuses:**
 - D' estimates inflated in small samples
 - D' estimates inflated when one allele is rare

Δ^2 (also called r^2)

$$\Delta^2 = \frac{D_{AB}^2}{p_A(1-p_A)p_B(1-p_B)}$$
$$= \frac{\chi^2}{2n}$$

- Ranges between 0 and 1
 - 1 when the two markers provide identical information
 - 0 when they are in perfect equilibrium
- Expected value is $1/2n$

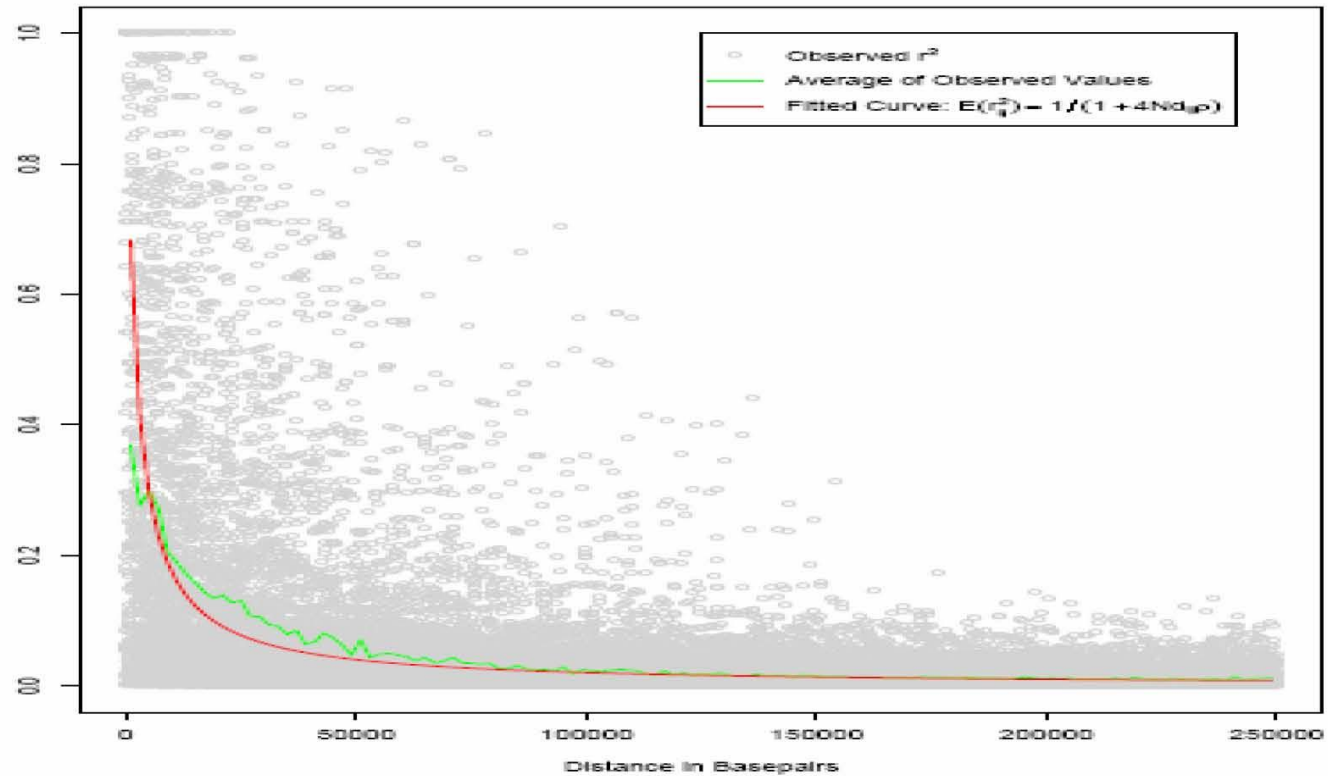
Calculate R

$$\begin{aligned} R &= D_{AB} / \text{SQR}(P_A P_a P_B P_b) \\ &= .06 / \text{SQR}(.87 * .13 * .92 * .08) \\ &= .06 / \text{SQR}(7.2 \times 10^{-3}) \\ &= .06 / .085 = .706 \\ R^2 &= .497 \end{aligned}$$

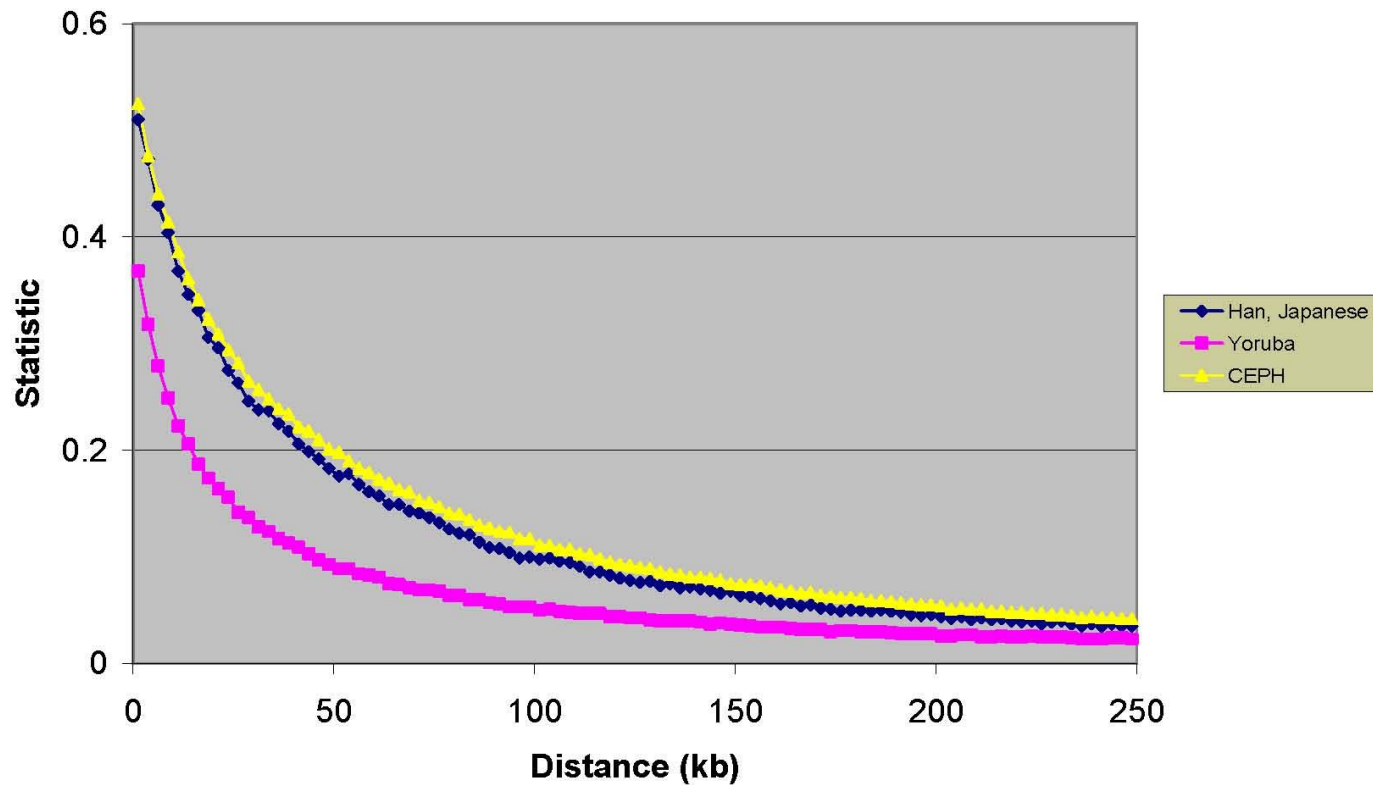
More on r^2

- $r^2 = 1$ implies the markers provide exactly the same information
- The measure preferred by population geneticists
- Measures loss in efficiency when marker A is replaced with marker B in an association study
 - With some simplifying assumptions (e.g. see Pritchard and Przeworski, 2001)

Summarizing Disequilibrium



Comparing Populations ...



LD extends further in CEPH and the Han/Japanese than in the Yoruba