Branch and Bound Examples

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Example from control theory

- function minimized is complicated, non-convex function that arises in control theory (stability degree)

- lower and upper bound evaluated using sophisticated methods (convex optimization, control theory)
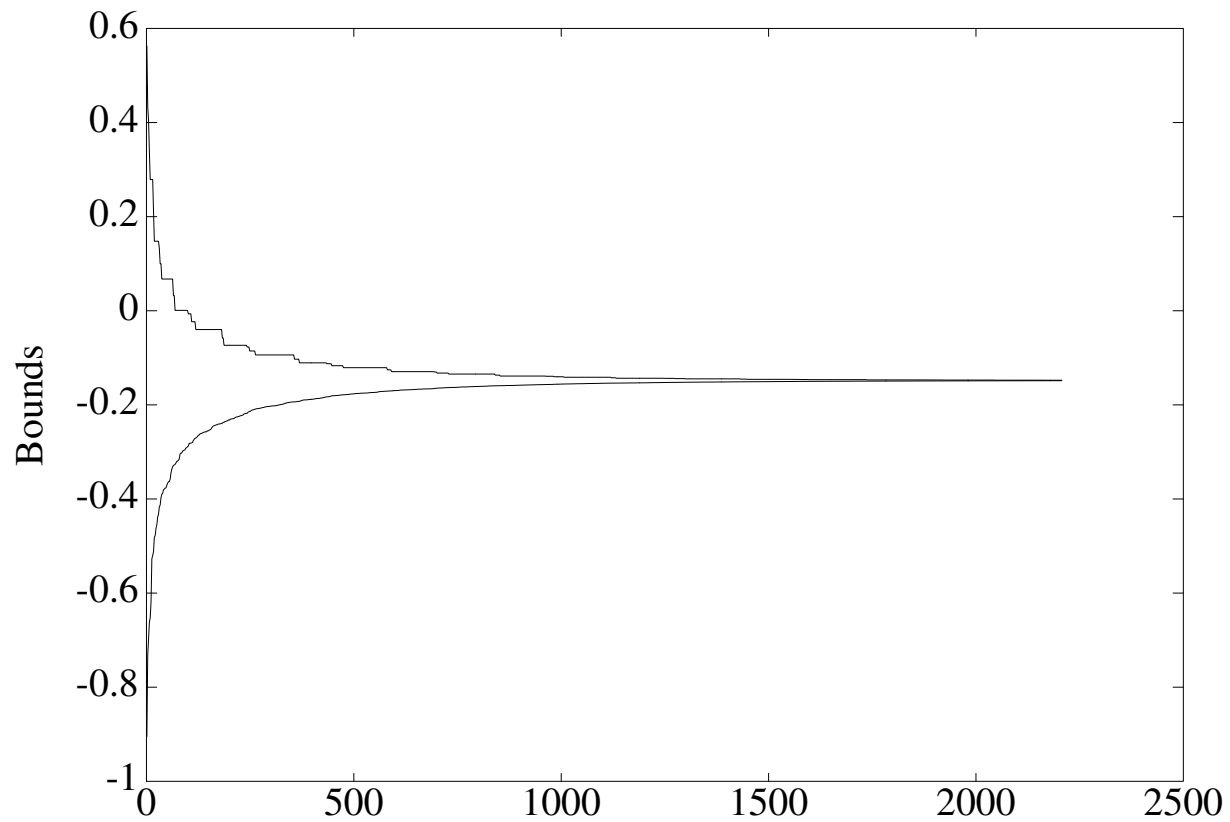
- problem instance has 5 variables

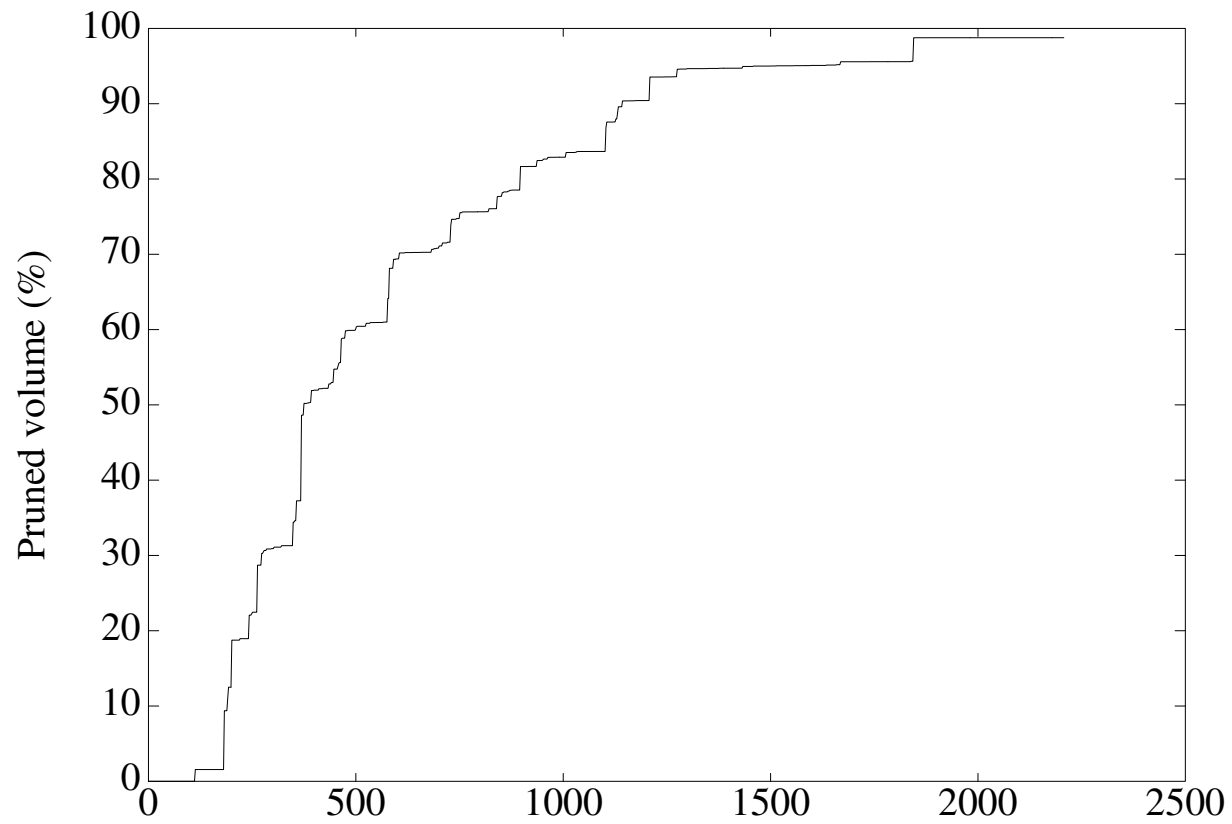**Figure 1:** Upper and lower bound versus number of iterations.
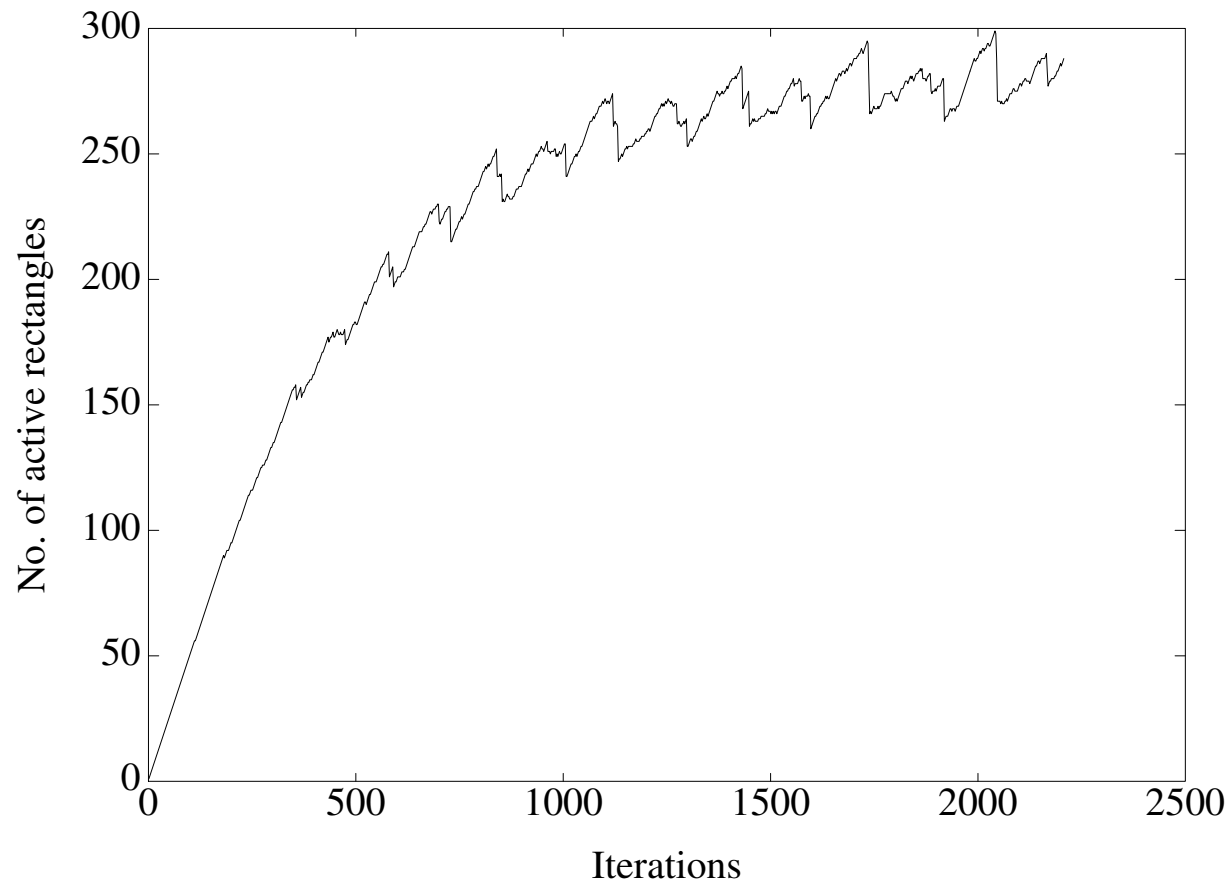
Figure 2: Pruned volume versus number of iterations.

Figure 3: Active rectangles versus number of iterations.

- approximately 1000 iterations to prune 82% of the volume.

- approximately 2000 iterations to find global solution within 0.001

Example of mixed Boolean-convex problem

We consider the problem

$$\begin{array}{ll} \text{minimize} & c^T z \\ \text{subject to} & d^T z \le -1, \\ & z_j \in \{0, 1\}, \quad j = 1, \dots, n, \end{array}$$

where

$$c = (1.1, -2.2, 3.4, -3.7, 6), \qquad d = (-1.1, -2.2, -3.4, 3.2, 5)$$

(easily solved; this is just to illustrate branch and bound)

- split node with lowest lower bound

- fix variable closest to either 0 or 1 in the relaxation, or with largest dual variable among those equal to 0 or 1

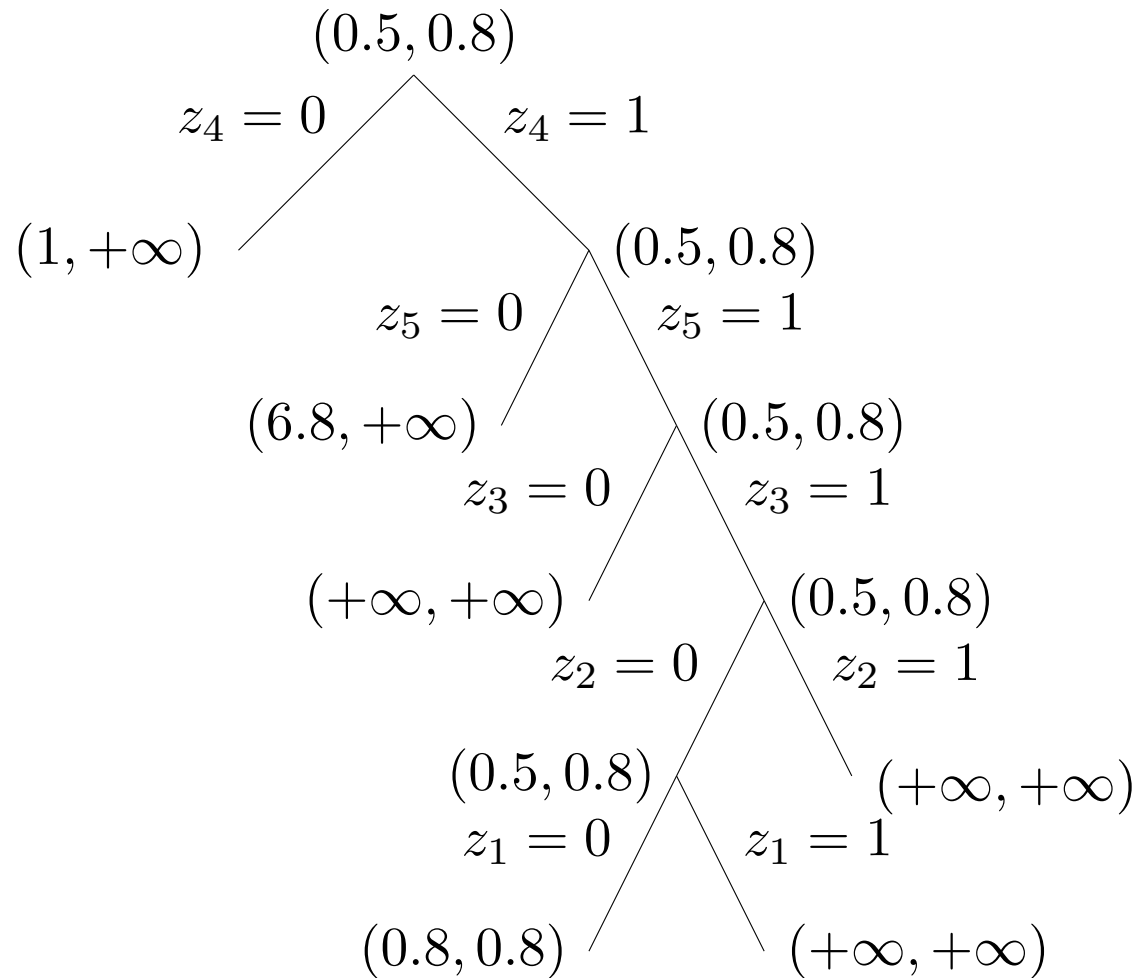- algorithm terminates in 10 steps (cf. 32 for exhaustive search)

Figure 4: Branch and bound algorithm for Boolean problem.