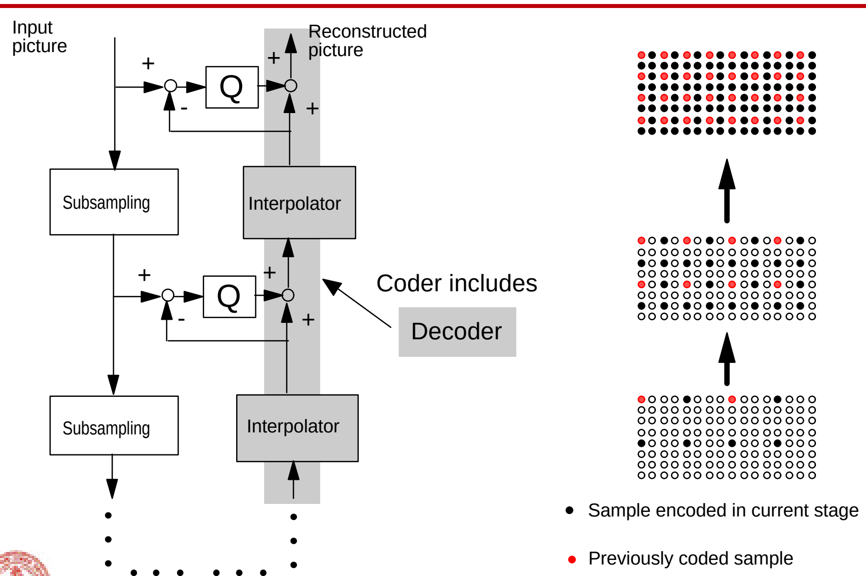
Multiresolution and subband coding

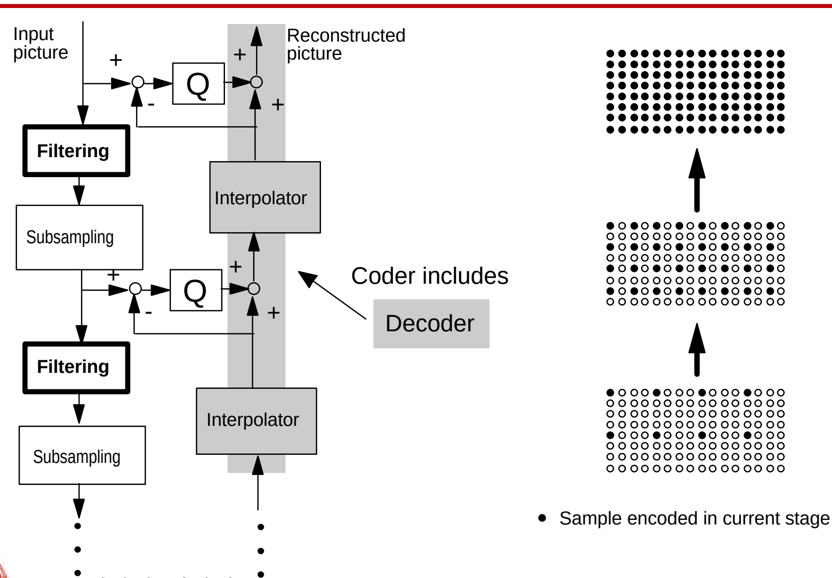
- Predictive (closed-loop) pyramids
- Open-loop ("Laplacian") pyramids
- Subband coding
- Perfect reconstruction filter banks
- Quadrature mirror filter banks
- Discrete Wavelet Transform (DWT)
- Embedded zerotree wavelet (EZW) coding
- Transform coding as a special case of subband coding

Interpolation error coding, I



Interpolation error coding, II

Predictive pyramid, I



Predictive pyramid, II

Number of samples to be encoded =

$$\left(1+\frac{1}{N}+\frac{1}{N^2}+\ldots\right)=\frac{N}{N-1} \text{ x number of original image samples}$$

subsampling factor

Predictive pyramid, III

signals to be encoded

Comparison: interpolation error coding vs. pyramid

- Resolution layer #0, interpolated to original size for display

Interpolation Error Coding

Pyramid

Comparison: interpolation error coding vs. pyramid

- Resolution layer #1, interpolated to original size for display

Interpolation Error Coding

Pyramid

Comparison: interpolation error coding vs. pyramid

- Resolution layer #2, interpolated to original size for display

Interpolation Error Coding

Pyramid

Comparison: interpolation error coding vs. pyramid

- Resolution layer #3

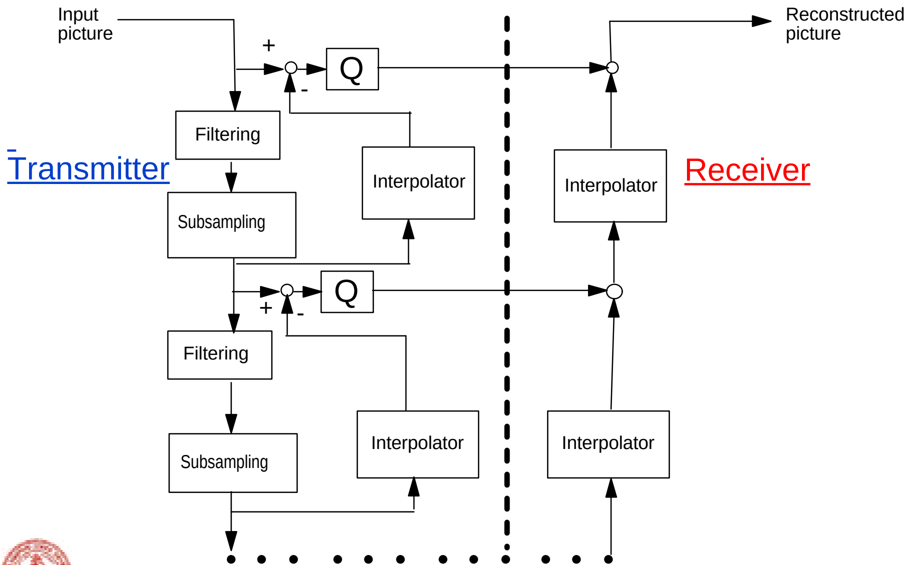
Interpolation Error Coding

=

(original)

Pyramid

Open-loop pyramid (Laplacian pyramid)

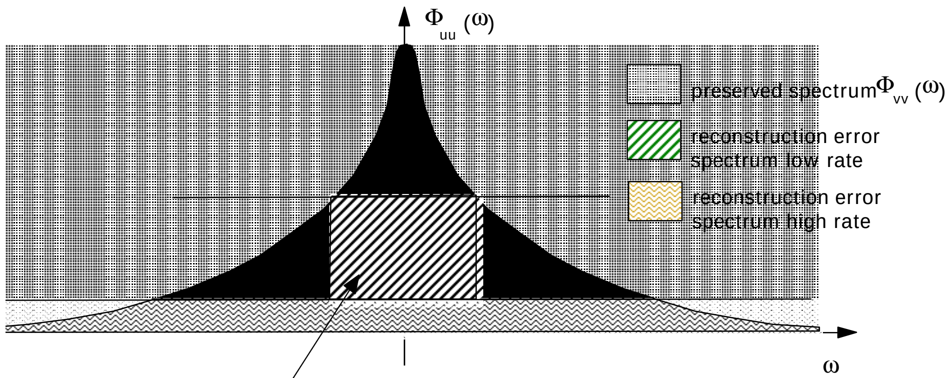


When multiresolution coding was a new idea . . .

This manuscript is okay if compared to some of the weaker papers.
[. . .] however, I doubt that anyone will ever use this algorithm again.

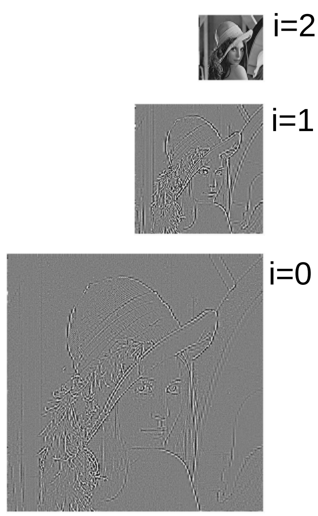
Anonymous reviewer of Burt and Adelson's original paper, ca. 1982

Embedded multiresolution coding



Low frequency components have to be refined for optimal resolution/noise balance

Optimum bit allocation for open-loop pyramid



Assume layer m.s.e distortion-rate function:

$$d_i(r_i) = s_i^2 \, 2^{-2r_i}$$

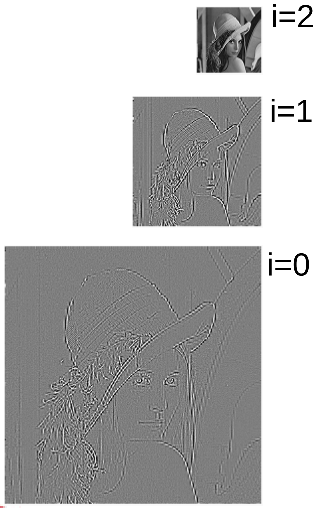
variance

depends on bit-rate R, but not not layer i

$$r_i = \frac{1}{2}\log_2 \frac{s_i^2}{N_i} + f(R) \quad bits/sample$$

number of samples

Optimum bit allocation for closed-loop pyramid



power transfer factor — variance — number of samples

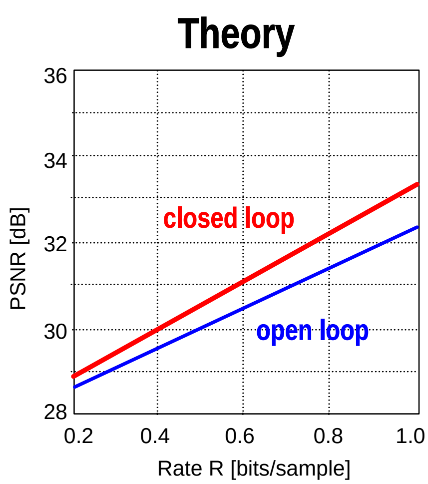
$$r_i = \frac{1}{2}\log_2 \frac{\alpha\,\sigma_i^2 (N_{i-1} - N_i)}{\sigma_{i-1}^2 (N_i - N_{i+1})} \; bits/sample$$

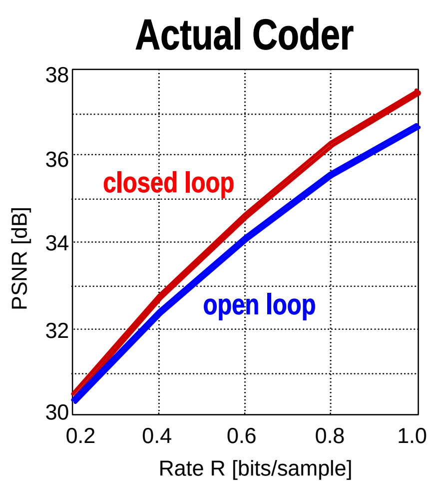
does not depend on R !!

$$r_0 = R - \frac{\sum_{i=1}^{L-1} N_i \, r_i}{\sum_{i=0}^{L-1} N_i} \; bits/sample$$

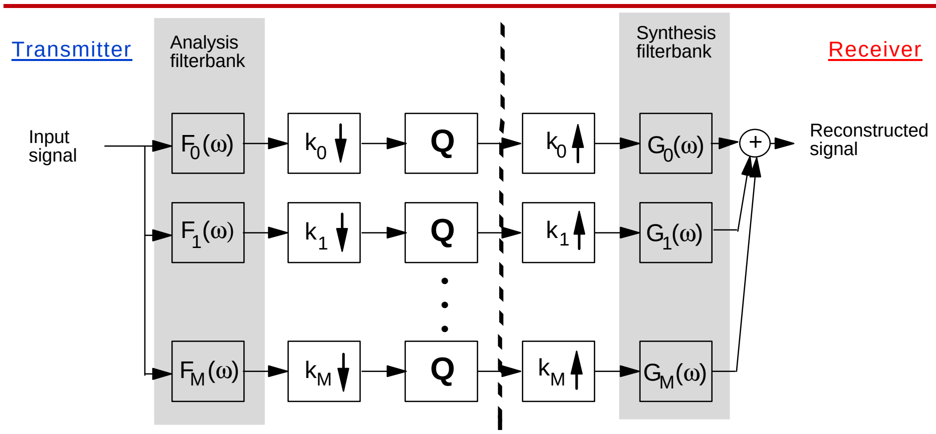
remaining bits

Two-layer open- vs. closed-loop pyramid



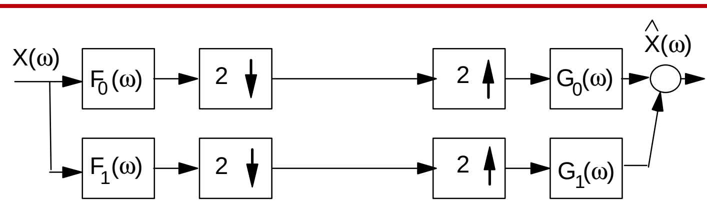


Subband coding



- Number of degrees of freedom is preserved: $\frac{1}{k_0}+\frac{1}{k_1}+\ldots+\frac{1}{k_M}=1$
  "Critically sampled filter bank"
- Perfect reconstruction filter bank required

Two-channel filterbank



$$\hat{X}(w)=\frac{1}{2}\left[F_0(w)G_0(w)+F_1(w)G_1(w)\right]X(w)$$
$$+\frac{1}{2}\left[F_0(w+p)G_0(w)+F_1(w+p)G_1(w)\right]X(w+p)$$

Aliasing

- Aliasing cancellation if : $G_0(w)=F_1(w+p)$, $-G_1(w)=F_0(w+p)$

Example: two-channel filter bank with perfect reconstruction

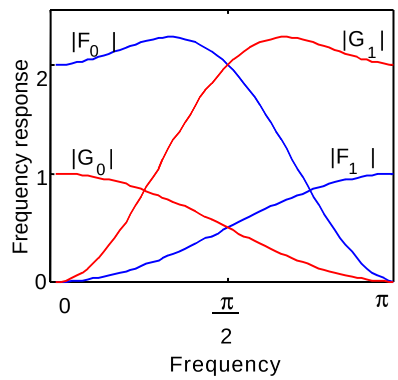
- Analysis filter impulse responses:
  - Lowpass band:

    $\frac{1}{4}(-1,+2,+6,+2,-1)$
  - Highpass band:

    $\frac{1}{4}(+1,-2,+1)$
- Synthesis filter impulse responses:
  - Lowpass band:

    $\frac{1}{4}(+1,+2,+1)$
  - Highpass band:

    $\frac{1}{4}(+1,+2,-6,+2,+1)$
- Frequency responses:



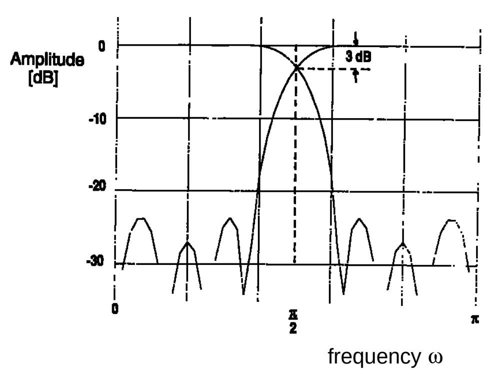
Quadrature mirror filters (QMF)

- QMFs achieve aliasing cancellation by choosing

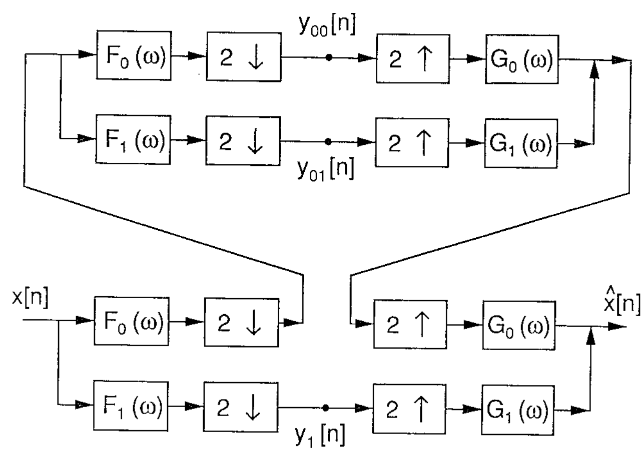
$$F_1(\omega) = F_0(\omega + \pi) = -G_1(\omega) = G_0(\omega + \pi)$$

- Highpass band is the mirror image of the lowpass band in the frequency domain

<u>Example:</u>
<u>16-tap QMF filterbank</u>

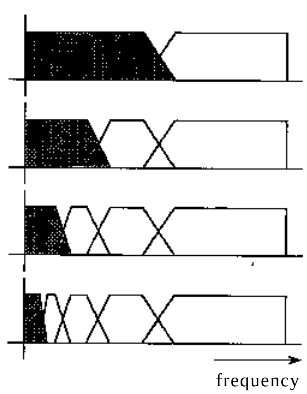


Cascaded analysis / synthesis filterbanks



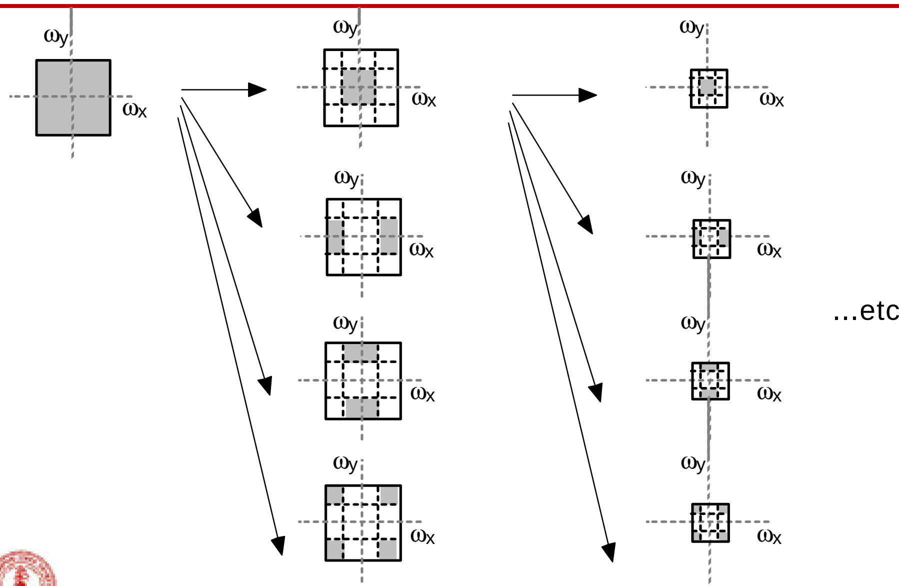
Discrete Wavelet Transform

- Recursive application of a two-band filter bank to the lowpass band of the previous stage yields octave band splitting:

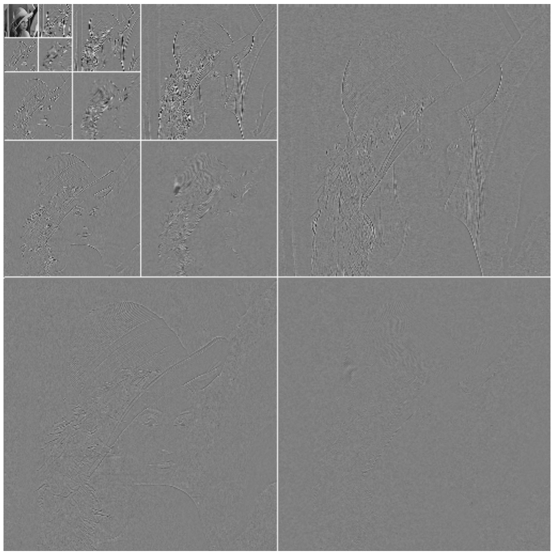


- Same concept can be derived from wavelet theory: Discrete Wavelet Transform (DWT)

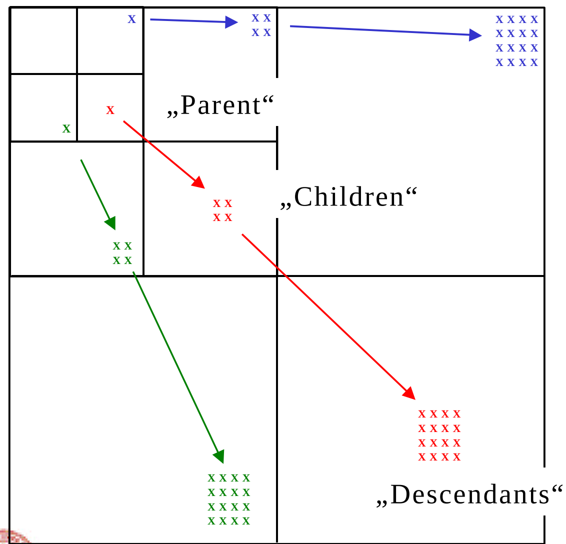
2-d Discrete Wavelet Transform



2-d Discrete Wavelet Transform example



Embedded zero-tree wavelet algorithm



- Idea: Conditional coding of all descendants (incl. children)
- Coefficient magnitude > threshold: significant coefficients
- Four cases
  - ZTR: zero-tree, coefficient and all descendants are **not** significant
  - IZ: coefficient is not significant, but some descendants are significant
  - POS: positive significant
  - NEG: negative significant

Embedded zero-tree wavelet algorithm (cont.)

- For the highest bands, ZTR and IZ symbols are merged into one symbol Z
- Successive approximation quantization and encoding
  - Initial „dominant“ pass
    - Set initial threshold T, determine significant coefficients
    - Arithmetic coding of symbols ZTR, IZ, POS, NEG
  - Subordinate pass
    - Refine magnitude of coefficients **found significant so far** by one bit (subdivide magnitude bin by two)
    - Arithmetic coding of sequence of zeros and ones.
  - Repeat dominant pass
    - Set previously found significant coefficients to zero
    - Decrease threshold by factor of 2, determine new significant coefficients
    - Arithmetic coding of symbols ZTR, IZ, POS, NEG
  - Repeat subordinate and dominate passes, until bit budget is exhausted.

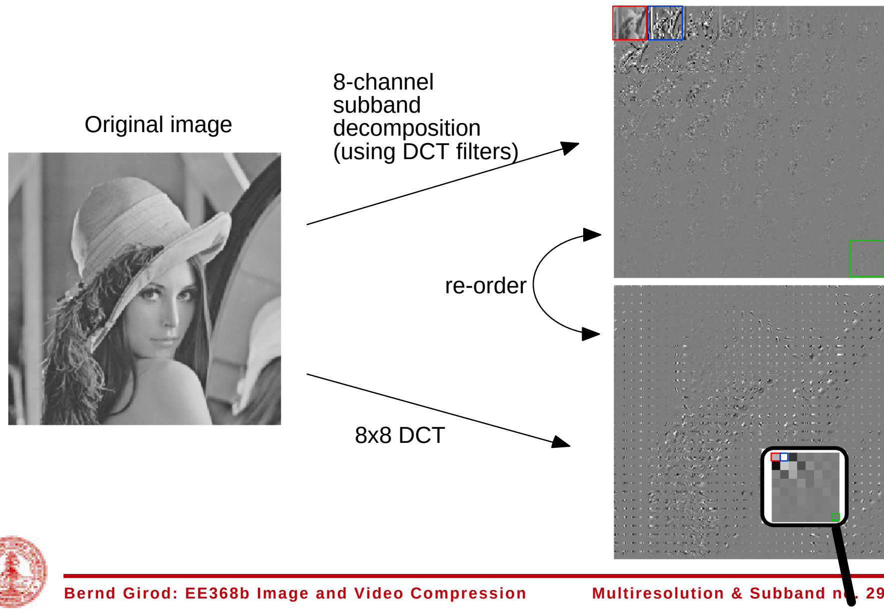
Embedded zero-tree wavelet algorithm (cont.)

- Decoding: bitstream can be truncated to yield a coarser approximation: „embedded“ representation
- Further details: *J. M. Shapiro, „Embedded image coding using zerotrees of wavelet coefficients,“ IEEE Transactions on Signal Processing, vol. 41, no. 12, pp. 3445-3462, December 1993.*
- Enhancement SPIHT coder: *A. Said, A., W. A. Pearlman, „A new, fast, and efficient image codec based on set partitioning in hierarchical trees,“ IEEE Transactions on Circuits and Systems for Video Technology, vol. 63 , pp. 243-250, June 1996.*
- JPEG-2000 standard similar to SPIHT

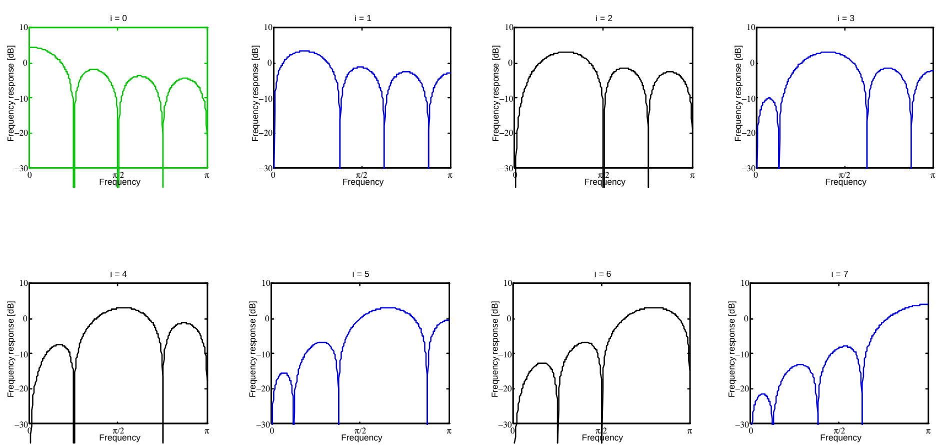
Subband coding vs. transform coding, I

- Transform coding is a special case of subband coding with:
  - Number of bands = order of transform N
  - Subsampling factor $K = N$
  - Length of impulse responses of analysis/synthesis filters N
- Filters used in subband coders are **<u>not</u>** in general orthogonal.

Subband coding vs. transform coding, II



Frequency response of a DCT of order *N*=8



Summary: multiresolution and subband coding

- Resolution pyramids with subsampling 2:1 horizontally and vertically
- Predictive pyramids: quantization error feedback („closed loop")
- Transform pyramids: no quantization error feedback („open loop")
- Pyramids: overcomplete representation of the image
- Critically sampled subband decomposition: number of samples not increased
- Quadrature mirror filters: aliasing cancellation
- Discrete Wavelet Transform = cascaded 2:1 subband splits
- Exploit statistical dependencies across subbands by zero-trees
- Transform coding is a special case of subband coding