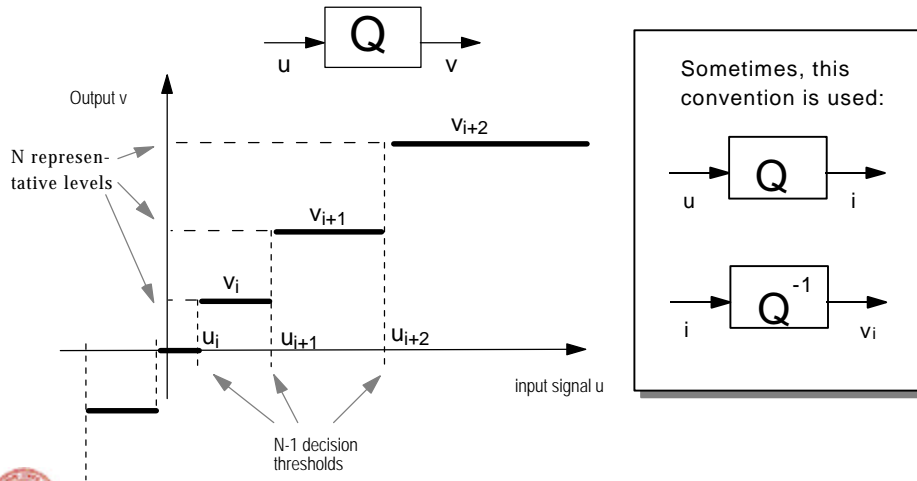
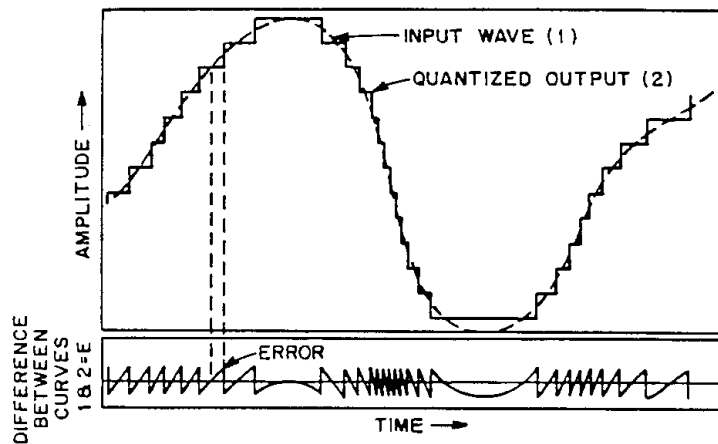


Quantization

Input-output characteristic of a scalar quantizer



Example of a quantized waveform



Lloyd-Max quantization

- **Problem** : For a signal u with given pdf $p_u(u)$ find a quantizer with N representative levels such that

variance of quantization error \longrightarrow min

- **Solution** : Lloyd-Max quantizer (Lloyd, 1957; Max, 1960)

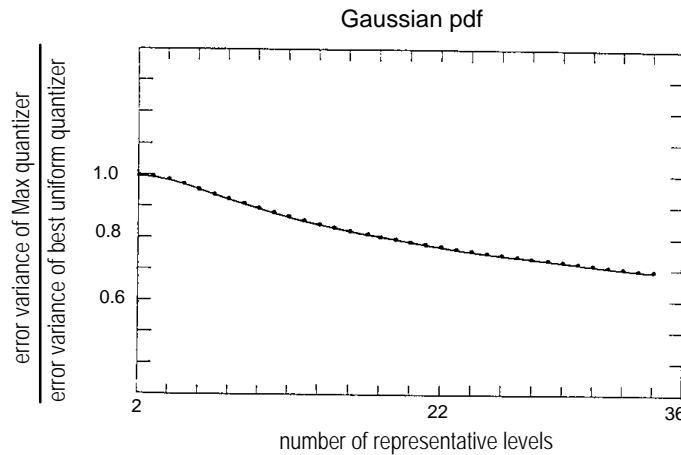
- $N-1$ decision thresholds exactly half-way between representative levels.
- N representative levels in the centroid of the pdf between two successive decision thresholds.

$$u_i = \frac{1}{2}(v_i + v_{i-1})$$

$$v_i = \frac{\int_{u_i}^{u_{i+1}} up_U(u)du}{\int_{u_i}^{u_{i+1}} p_U(u)du}$$



Lloyd-Max quantizer vs. best uniform quantizer



Panter and Dite approximation

- Approximate solution of the "Max quantization problem" by Panter & Dite (1949) assuming smooth pdf:

$$\Delta u(u) = \text{const} \frac{1}{\sqrt[3]{p_u(u)}}$$

Distance between two successive quantizer representative levels
Probability density function of u

- Approximation for the quantization error variance:

$$s_e^2 \approx \frac{1}{12N^2} \left[\int_u \sqrt[3]{p_U(u)} du \right]^3$$

Number of representative levels



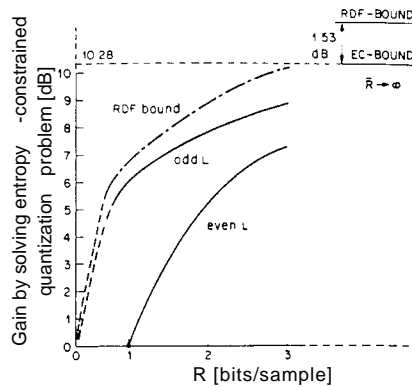
Entropy-constrained quantization

- If entropy coding is used after quantization:

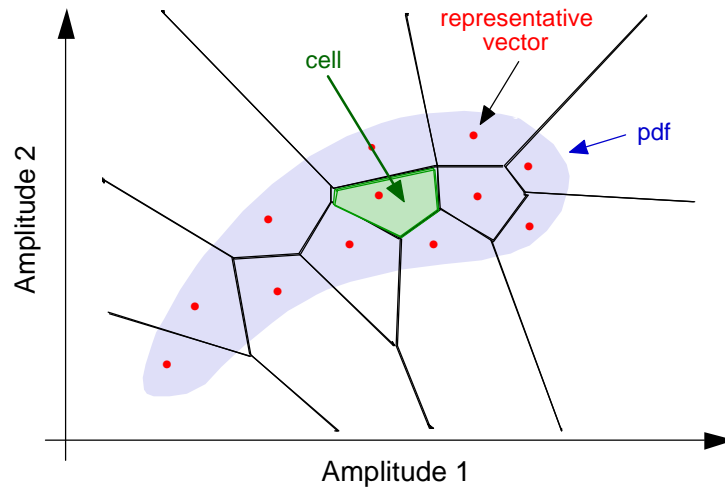
minimize distortion for given entropy!

- For sufficiently fine quantization, **uniform** quantizer yields minimum entropy.

- Example: maximum gain in SNR over Max-quantization for entropy-constraint quantization of a signal with Gamma pdf (from Granzow & Noll, 1983).

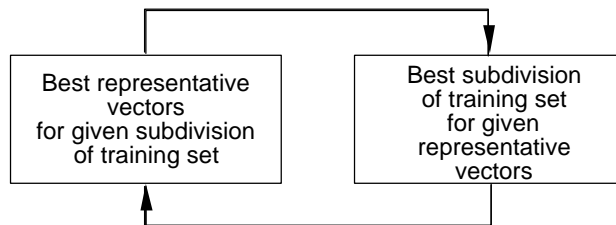


Vector quantization



LBG algorithm

- Linde, Buzo, Gray, 1980: Lloyd algorithm generalized for VQ



- Assumption: fixed code word length
- Code book unstructured: full search



Design of vector quantizers for variable length code words

- Chou, Lookabaugh, Gray, 1989: extended LBG algorithm for entropy-coded VQ
- Lagrangian cost function: solve unconstrained problem rather than constrained problem

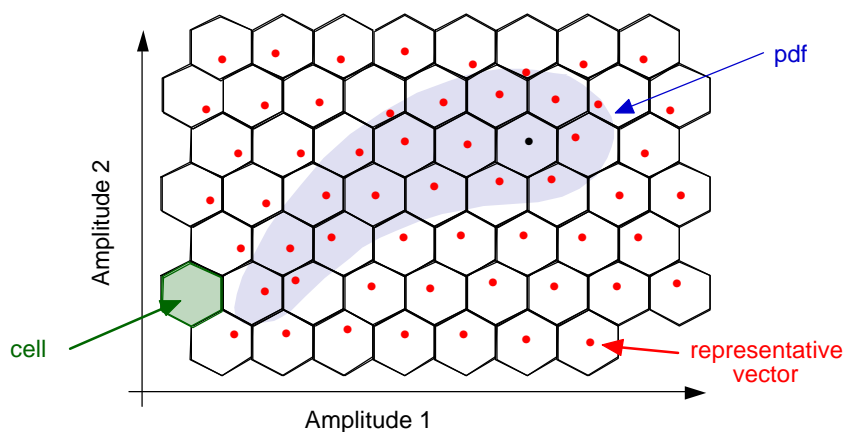
$$\min\{D + \lambda R\}$$

- Unstructured code book: full search for $\min\{D + \lambda R\}$

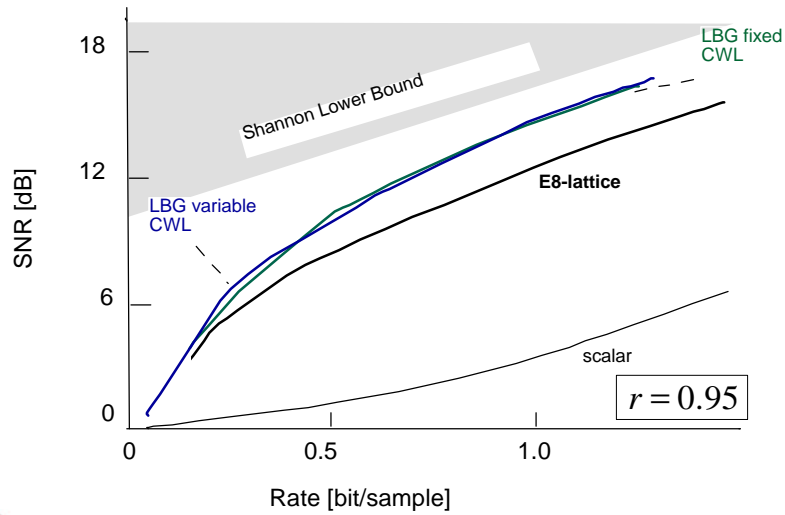
The most general coder structure:
Any source coder can be interpreted as VQ with VLC!



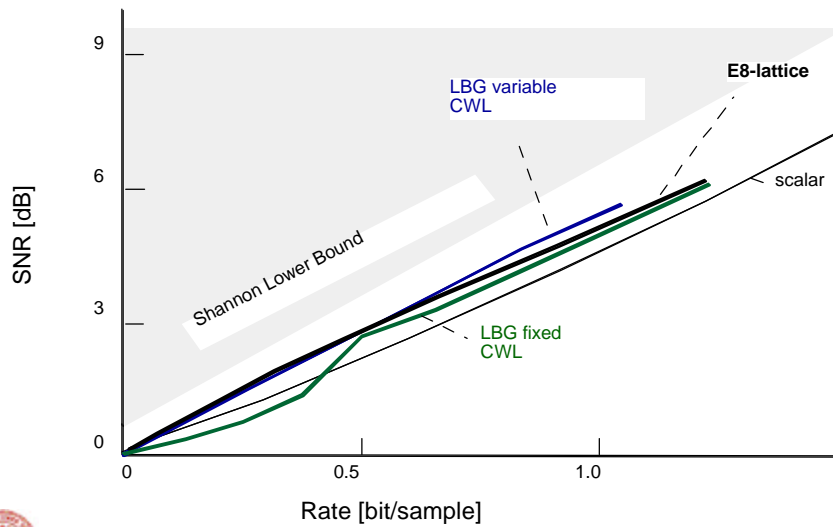
Lattice vector quantization



8D VQ of a Gauss-Markov source



8D VQ of memoryless Laplacian source



Summary: quantization

- Lloyd-Max quantizer: minimum MSE distortion for given number of representative levels
- For variable length coding, additional gains by entropy-constrained quantization
- Minimum mean squared error for given entropy: uniform quantizer (for fine quantization!)
- Vector quantization: quantize and encode several signal samples jointly
- Design optimum unconstrained VQ with LGB algorithm
- Entropy-constrained VQ minimizes Lagrangian cost function $D + \lambda R$
- Lattice VQ: uniform quantizer in multiple dimensions

