EE367 / CS448I Final Project Proposal

Refraction-Aware ADMM for Optical Projection Tomography Reconstruction of Multimaterial 3D Print Filaments

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Motivation

From 3D printing batteries to artificial hearts, multimaterial additive manufacturing (MMAM) harnesses the incredible ability to rapidly innovate with designs that can be highly customizable, complex, and multimaterial. [1] However, progress in AM innovation is constrained by our inability to fully visualize and understand the fabrication process in 3D in real time. Without this capability, we are unable to immediately correct the small errors that lead to catastrophic failures. [2] This highlights a need for a 4D (3D volume + time) vision system with real-time, in operando guidance, where errors can be rectified before materials, time, and research potential are wasted.

Current state-of-the-art imaging systems for MMAM, such as optical coherence tomography [3] and X-ray computed tomography [4], do not provide 4D monitoring capabilities. They require interrupting the build process or inspecting after the build is completed—when correcting a defect is no longer possible after discovery. Optical projection tomography (OPT) has proven to be a compact, non-destructive imaging system that can be cheaply employed as a multi-camera system; it is thus an excellent candidate for an accessible, high-speed solution to image AM processes in real time. [5, 6]

High-speed imaging alone, however, is not enough to provide rapid responses to make decisions in time—it must be paired with rapid volume reconstruction. When reconstructing 3D volumes from OPT images in real time, several key factors must be addressed: high-speed reconstruction for real-time capability; few observations due to limited camera angles; and nonlinear modeling due to refractive deflections in light ray paths. Current image reconstruction techniques do not fully address these challenges combined. Traditional tomographic methods require dense data and cannot handle nonlinear models [7]; iterative techniques are slow and computationally expensive [8]; and deep learning models are application-specific and demand extensive training data [9, 10]. A promising technique includes using ADMM, which has been effective in tomographic reconstruction for limited-angle views and can incorporate physics priors and nonlinear models [11].

Proposal

To address the challenge of achieving 4D quality assessment capabilities for AM, I aim to develop a high-speed, refraction-aware ADMM-based algorithm for reconstructing semi-transparent materials from optical projection tomography. This will involve developing a nonlinear forward model and combining it into an ADMM-based algorithm. The inputs would be OPT images and the known refractive index field, and the desired output would be a voxel grid where each voxel is assigned a material value that would allow us to understand the 3D structure of each material inside the filament.

Note: The end goal is to learn both the color/absorption values μ AND the refractive index values n of the unknown voxel grid, but for the scope of this project, I will aim to only learn the attenuation values (black/white, noncolored) and provide a given refractive field as a proof of concept for further development of the algorithm.

Refraction-aware forward model. I will implement a nonlinear, refraction-aware forward model in Pytorch using torchdiffeq to account for refractive deflections in OPT. The forward model maps a 3D voxel volume parameterized by spatially varying attenuation $\mu(\mathbf{r})$ and refractive index $n(\mathbf{r})$ to predicted projection images at arbitrary view angles. For a given projection angle θ, each sensor pixel corresponds to a ray launched into the volume. Ray paths would be obtained by numerically integrating the eikonal ray ODEs. The ray evolution is as follows in Eqs. (1) and (2):

$$\frac{dr}{ds} = t \tag{1}$$

$$\frac{dt}{ds} = \nabla \ln(r) - (t \cdot \nabla \ln(r))t, \nabla \ln n(r) = \frac{\nabla n(r)}{n(r)} \tag{2}$$

where $\mathbf{r}(s)$ is the ray position and $\mathbf{t}(s)$ is the unit direction, both parameterized by s.
With the ray trajectory $\mathbf{r}(s)$, the predicted detected intensity can be computed via the Beer-Lambert law:

$$I_\theta(p) = I_0 \exp(-\int \mu(r_\theta(s;\, p))ds) \tag{3}$$

where $\mathbf{p}$ indexes a detector pixel and $\mathbf{r}_\theta(\mathrm{s};\, \mathbf{p})$ is the refracted ray path associated with that pixel at angle θ. The integral can be approximated by sampling and accumulating μ along the discrete ODE trajectory. With this computation done for all sensor pixels, the forward model would produce synthetic projection images that can be directly compared against measured OPT images, explicitly accounting for nonlinearity introduced by refraction.

ADMM-based reconstruction with TV regularization. I will embed the refraction-aware forward model into an ADMM framework with total variation (TV) regularization. I choose TV regularization as the regularizer since it promotes piecewise-smooth volumes, well-suited for multimaterial objects with sharp interfaces. Given a known refractive field n and measured projection images I_θ^{meas}, the algorithm will aim to solve the following minimization problem to find the attenuation field μ:

$$\min_{\mu, z} \sum_{\theta \in \Theta} ||F_\theta(\mu;\, n) - I_\theta^{meas}||_2^{\,2} + \lambda||\nabla\mu||_1 \tag{4}$$

where $F_\theta(\mu;\, n)$ is the nonlinear forward model as defined by Eqs. (1)-(3) in the previous section.
The ADMM iteration is the following:

1. μ-update

$$\mu^{k+1} = \arg\min_{\mu} \sum_{\theta} ||F_\theta(\mu;\, n) - I_\theta^{meas}||^2 + \frac{\rho}{2}||D\mu - z^k + u^k||^2 \tag{5}$$

2. z-update

$$z^{k+1} = \arg\min_{z} \lambda||z||_p + \frac{\rho}{2}||D\mu^k - z + u^k||^2 \tag{6}$$

3. dual update

$$u^{k+1} = u^k + D\mu^{k+1} - z^{k+1} \tag{7}$$

Algorithm evaluation. Upon implementing the algorithm, I will conduct studies varying different parameters such as λ and ρ, the number of projection image inputs to test sparse view, and different ODE solvers and step sizes. I will evaluate the effectiveness of each study using PSNR and SSIM, comparing cross sections with those of the ground truth, and plotting the values against both time and iteration number. Upon completion of this project, I will have a working ADMM-based algorithm coupled with refractive-aware forward model with knowledge of what values of parameters are best suited for MMAM use. This will pave the way for real-time volume reconstruction during 3D prints for high-fidelity quality assurance.

Milestones & Timeline

	Week 1	Week 2	Week 3	Week 4
Refraction-aware nonlinear forward model	■	■		
ADMM + TV with forward model		■		
Model tuning and evaluation			■	■
Final report, poster, and documentation				■

References

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