Diffusion Models for Image Generation and Inverse Problems

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Motivation

In real-world applications, images are often corrupted by noise, blur, or missing regions, making many imaging tasks inherently ill-posed. Diffusion models have emerged as a powerful class of generative models capable of addressing these challenges.

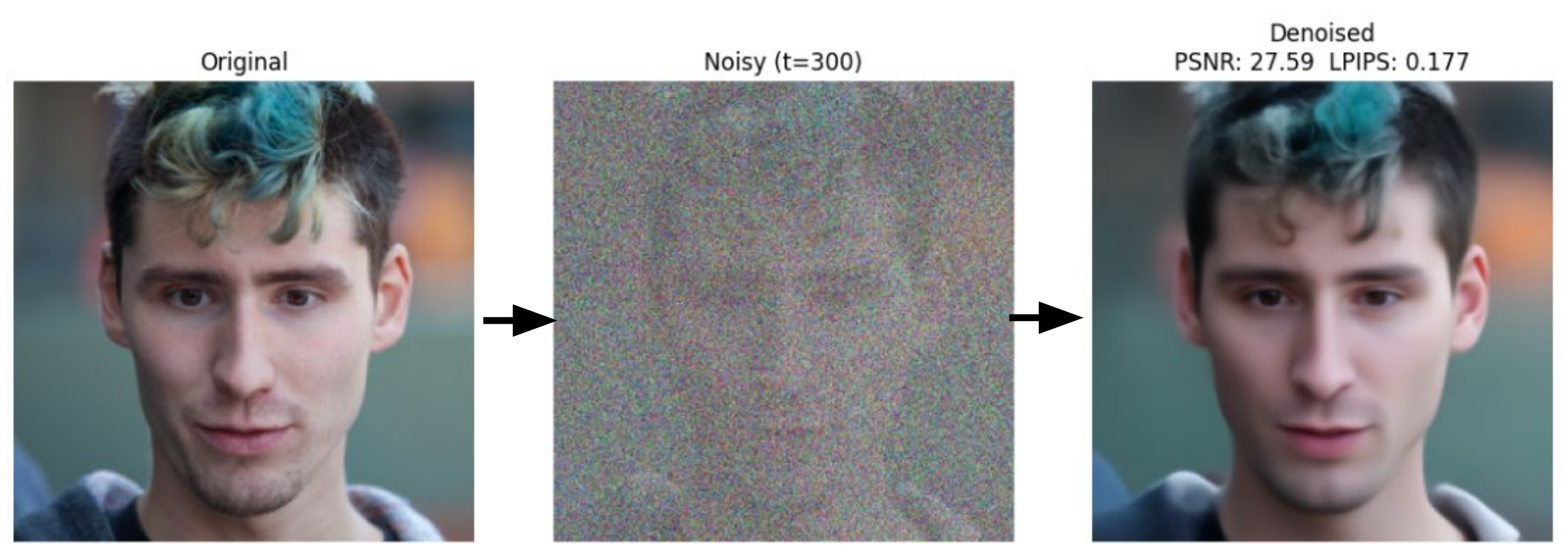
Related Work

Denoising Diffusion Probabilistic Models (DDPM): A diffusion-based generative model that iteratively denoises Gaussian noise to generate realistic images [2].

One-step Forward Diffusion: $x_t = \sqrt{\bar{\alpha}_t}\, x_0 + \sqrt{1-\bar{\alpha}_t}\epsilon$

Training a Noise Estimation Network: $\nabla_\theta \left\| \boldsymbol{\epsilon} - \boldsymbol{\epsilon}_\theta(\sqrt{\bar{\alpha}_t}\mathbf{x}_0 + \sqrt{1-\bar{\alpha}_t}\boldsymbol{\epsilon}, t) \right\|^2$

Reverse diffusion: $x_{t-1} = \frac{1}{\sqrt{\alpha_t}}\left(x_t - \frac{1-\alpha_t}{\sqrt{1-\bar{\alpha}_t}}\epsilon_\theta(x_t, t) \right) + \sqrt{\beta_t}\,\epsilon_t$



References

[1] Chung, H., Kim, J., Mccann, M. T., Klasky, M. L., and Ye, J. C. (2023). Diffusion posterior sampling for general noisy inverse problems. In ICLR.
[2] Ho, J., Jain, A., and Abbeel, P. (2020). Denoising diffusion probabilistic models. In NeurIPS.
[3] Jalal, A., Arvinte, M., Daras, G., Price, E., Dimakis, A. G., and Tamir, J. (2021). Robust compressed sensing mri with deep generative priors.
[4] Meng, C., He, Y., Song, Y., Song, J., Wu, J., Zhu, J.-Y., and Ermon, S. (2022). Sdedit: Guided image synthesis and editing with stochastic differential equations.

Method & Experimental Results

Unconditional Image Generation:

Conditioned Diffusion:

$\boldsymbol{y} \xrightarrow{\text{Add noise}} \boldsymbol{x}_t = \sqrt{\bar{\alpha}_t}\boldsymbol{y} + \sqrt{1-\bar{\alpha}_t}\boldsymbol{z} \xrightarrow{\text{Denoise}} \boldsymbol{x}_0$

Measurement; t is a hyperparameter

Approach:

$\nabla_x \log p(\mathbf{b}|\mathbf{x_0}) \approx \nabla_x \log p_t(\mathbf{b}|\mathbf{x}_t)$

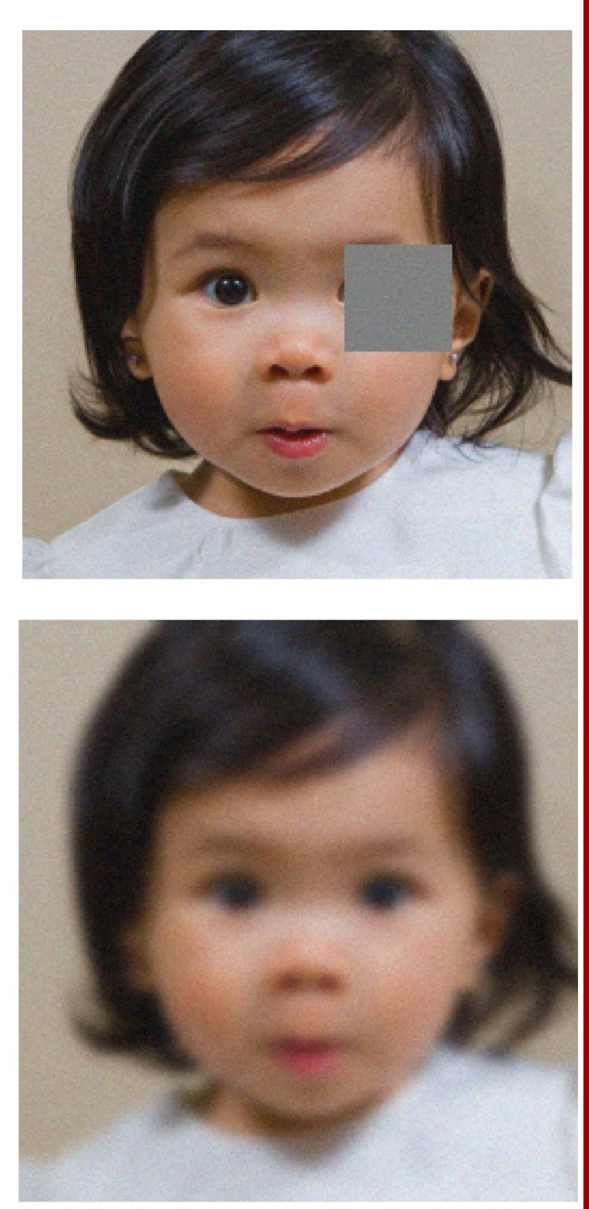
Problem:

$\nabla_x \log p(\mathbf{b}|\mathbf{x_0}) \neq \nabla_x \log p_t(\mathbf{b}|\mathbf{x}_t)$

Input | SDEdit | ScoreALD | DPS

Inpainting

Deconvolution



ScoreALD: $\boldsymbol{x}_{t-1} = \boldsymbol{x}_{t-1} - \frac{1}{2(\sigma^2 + \gamma_t^2)} \nabla_{\boldsymbol{x}_t} \left\| \mathscr{A}(\boldsymbol{x}_t) - \boldsymbol{y} \right\|$

DPS: $\boldsymbol{x}_{t-1} = \boldsymbol{x}'_{t-1} - \zeta_t \nabla_{\boldsymbol{x}_t} \left\| \mathscr{A}(\hat{\boldsymbol{x}}_0) - \boldsymbol{y} \right\|^2 \quad \zeta_t = \frac{\zeta}{\left\| \nabla_{\boldsymbol{x}_t} \left\| \mathscr{A}(\hat{\boldsymbol{x}}_0) - \boldsymbol{y} \right\|^2 \right\|}$