Refraction-Aware ADMM Reconstruction for Optical Projection Tomography of Multimaterial 3D-Print Filaments

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Motivation

- **Multimaterial additive manufacturing (MMAM)** enables rapid innovation and development of multifunctional materials systems.[1-2]

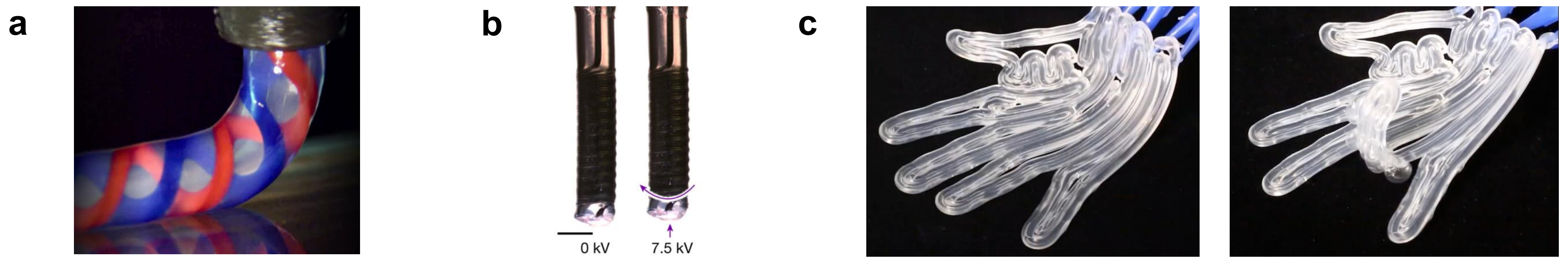


Figure 1. Recent MMAM innovations. **a,** Rotational subvoxel control. **b,** Electronic muscles. **c,** Pneumatic-actuated soft-robotics.

- However, MMAM innovation is constrained by the inability to **visualize, verify, and correct internal structures** *during* the printing process.
- This highlights a need for a **3D vision system** coupled with an algorithm capable of **rapid, refraction-aware,** and **sparse-view** reconstructions.

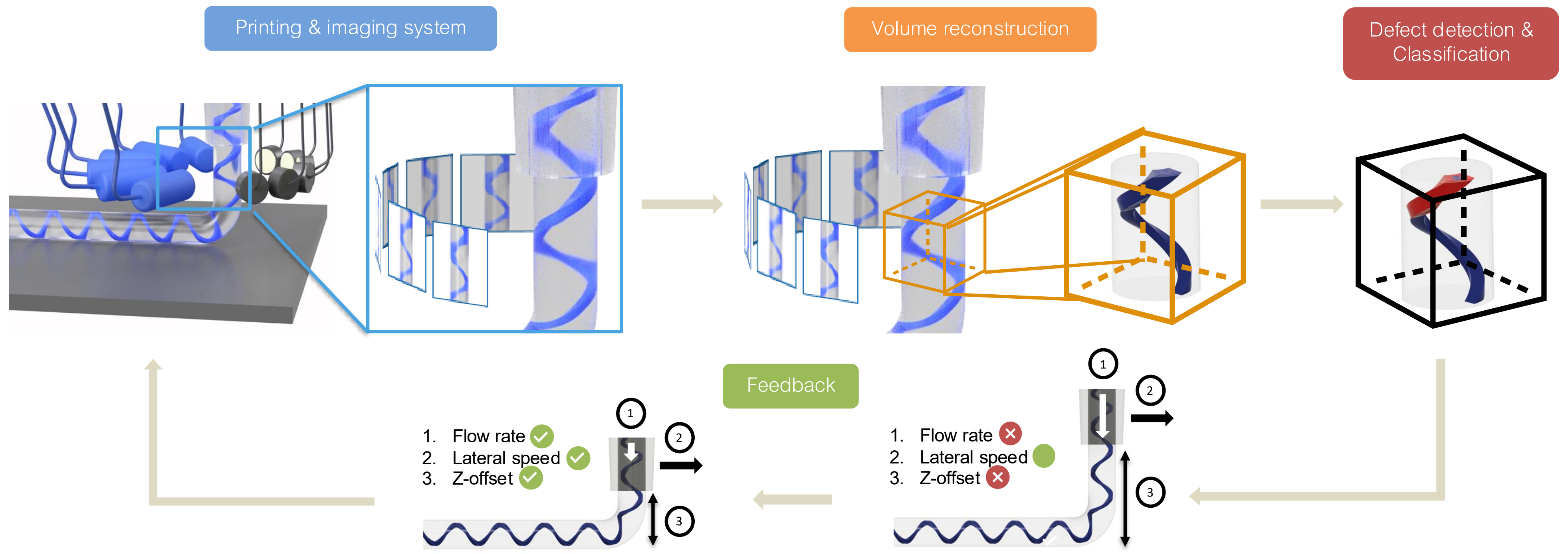


Figure 2. 4D vision system pipeline for real-time operando guidance of extrusion-based MMAM.

Related Work

- **Traditional tomographic reconstruction (e.g., FBP)** for OPT needs an adequate number of projections, and the sample must be RI matched.[3]
- **Neural methods (e.g., SinNet[4], Tomo-NeRF[5])** are application-specific and demand extensive training data.
- **ADMM-based methods (e.g., PFITRE[6])** are effective for sparse view reconstruction and can incorporate physics priors and nonlinear models.[7]

References

[1] Larson *et* al., *Nature*, 2023 **[2]** J. K. Wilt *et al.*, *arXiv*, 2025 **[3]** R. Clackdoyle and M. Defrise, *IEEE Signal Process. Mag.*, 2010 **[4]** J. Sun *et al., Biomed Opt Express*, 2023 [**5**] Y. Zheng and K. B. Hatzell, *ACS Appl. Mater. Interfaces*, 2023 **[6]** C. Zhao, M. Ge, X. Yang, *et al.*, *npj Comput Mater*, 2025 **[7]** J. Tao, Y. Wei, B. Bo, *ICCGIV*, 2024.

New Technique

- We propose an ADMM-based reconstruction algorithm with TV regularization and a material discreteness prior to estimate a volume absorption coefficient field $\mu(x)$ from set of projection images $\{I_{\theta_1}, I_{\theta_2}, \ldots, I_{\theta_n}\}$, given a refractive index field $n(x)$, and K material/absorption levels.

ADMM Update

K-level material discreteness prior $\sum_x \min_{m=1,\ldots,K} (\mu_x - c_m)^2$

$$\min_{\mu,z} \sum_{\theta\in\Theta} ||F_\theta(\mu;\ n) - I_\theta^{meas}||_2^2 + \lambda \underbrace{||z||_1}_{\text{TV regularization}} + \beta R_K(\mu;\ c) \quad \text{s.t. } z = D\mu$$

1. *μ*-update

$$\mu^{k+1} = \arg\min_\mu \sum_{\theta\in\Theta} ||F_\theta(\mu;\ n) - I_\theta^{meas}||^2 + \tfrac{\mu}{2}||D\mu - z^k + u^k||^2 + \beta R_K(\mu;\ c)$$

2. *z*-update

$$z^{k+1} = \arg\min_z \lambda||z|| + \tfrac{\rho}{2}||D\mu - z^k + u^k||^2$$

3. Dual update

$$u^{k+1} = u^k + D\mu^{k+1} - z^{k+1}$$

Forward Model $\hat{I}_{\theta_i} = F(\mu, n, \theta_i)$

Predicted intensities I_{θ_i} accumulated at a detector pixel are computed via the Beer-Lambert Law:

$$\hat{I}_{\theta_i}(p) = I_0 \exp\left(-\int \mu(r_{\theta_i}(s;\ p))ds\right)$$

Ray paths r_{θ_i} are obtained by numerically integrating Eikonal ODEs:

$$\frac{dr}{ds} = t\ ;\ \frac{dt}{ds} = \nabla \ln(r) - (t \cdot \nabla \ln n(r)), \qquad \nabla \ln n(r) = \frac{\nabla n(r)}{n(r)}$$

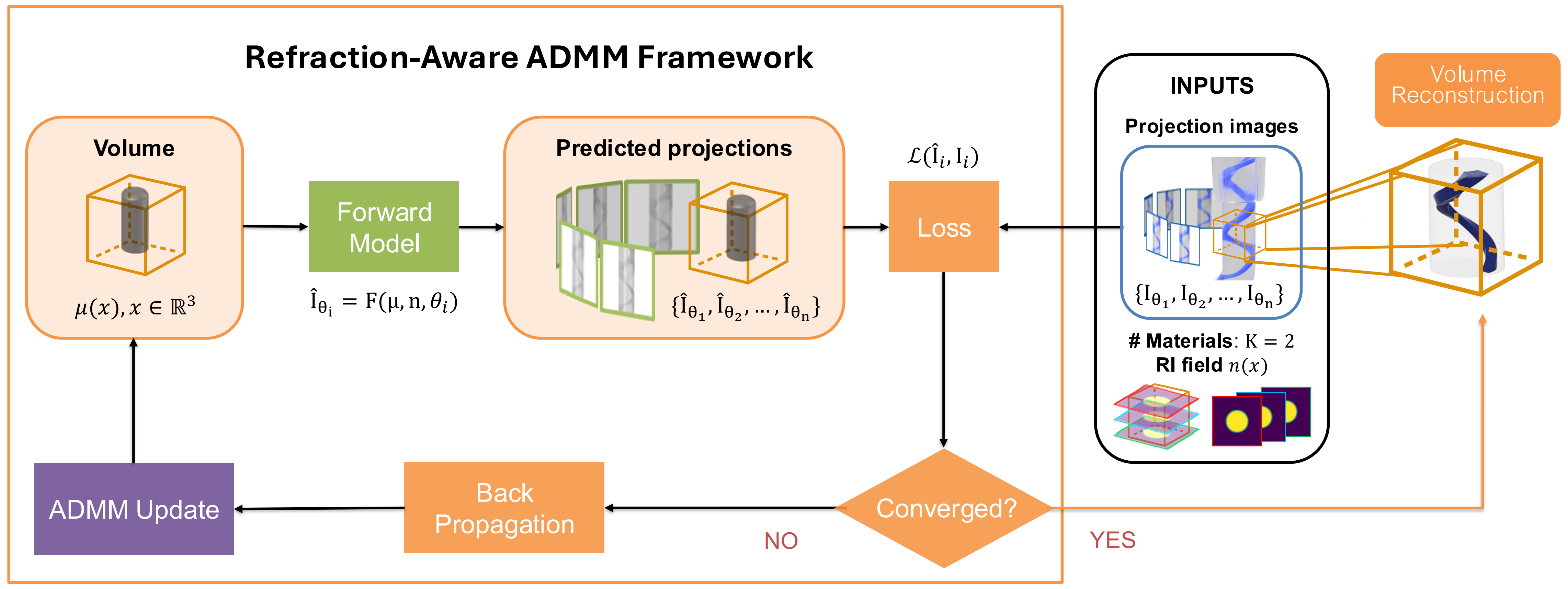
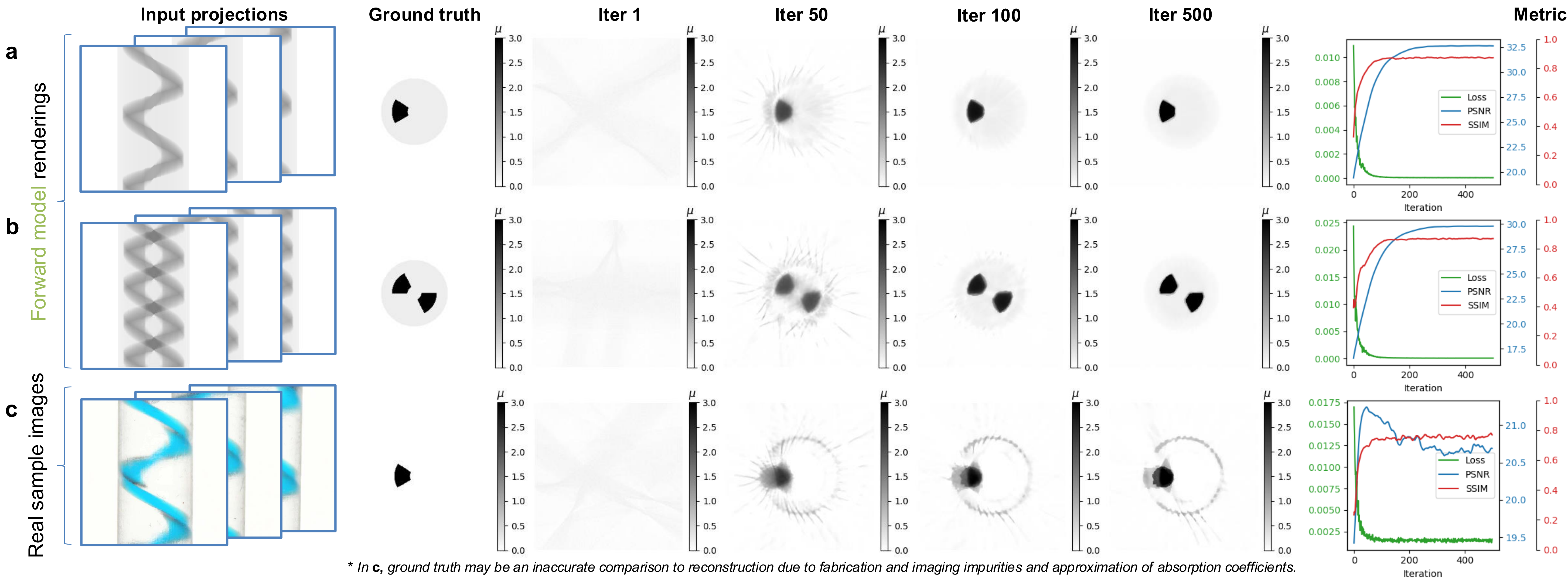


Figure 3. Refraction-aware ADMM framework with TV regularization and material discreteness prior.

Experimental Results



Time at iter 500: 216.692s
Max PSNR: 32.660 @ Iter 448
Max SSIM: 0.874 @ Iter 447

Time at iter 500: 219.760s
Max PSNR: 29.770 @ Iter 336
Max SSIM: 0.873 @ Iter 428

Time at iter 500: 218.302s
Max PSNR: 21.243 @ Iter 46
Max SSIM: 0.779 @ Iter 495

** In **c**, ground truth may be an inaccurate comparison to reconstruction due to fabrication and imaging impurities and approximation of absorption coefficients.*
** Color is not accounted for—only absorption is.*

Figure 4. Experimental results of reconstructions with visuals and metrics based on the *center cross-section* from input projections with 36 evenly spaced angles across 360°. All inputs are the same resolution (256x256) and use the same K=2 and RI field input (known cylindrical volume as RI=1.5 and RI=1.0 elsewhere). **a,** Volumetrically rendered images of single helix within cylinder. **b,** Volumetrically rendered images of double helix within same cylinder. **c,** A *real* set of images of the same structure as (a), fabricated with a *clear* resin-printed cylinder with RI=1.5 and injected with dyed pluronic gel through the helix.