Inverse Imaging with Diffusion Model Prior

Sikandar Y. Mashayak
Stanford Online Student

Motivation

The goals of this project are to learn basics of diffusion models and solve inverse imaging problems with a pre-trained diffusion model prior.

Related Work

- To solve inverse imaging problems, we implement and compare three methods for conditional terms:
  1. Score-distillation editing (SDEdit) [1].
  2. ScoreALD [2].
  3. Diffusion posterior sampling (DPS) [3].
- For a diffusion model, we use a pre-trained diffusion model on FFHQ data and DDPM sampling method [4].

References

[1] Meng et al., SDEdit: guided image synthesis and editing with stochastic differential equations, ICLR, 2022.
[2] Jalal et al., Robust compressed sensing MRI with deep generative priors, NeurIPS, 2021.
[3] Chung et al., Diffusion posterior sampling for general noisy inverse problems, ICLR, 2023.
[4] Ho et al., Denoising diffusion probabilistic models, NeurIPS, 2020.

Method

Maximum-a-posterior (MAP) solution to inverse problem

$$x_{\text{MAP}} = \text{arg min}_x - \log\left(p(\mathbf{b}\,|\,\mathbf{x},\sigma)\right) - \log\left(p(\mathbf{x})\right)$$

Sampling with guidance based on measurements

$$p_t(\mathbf{x}\,|\,\mathbf{b}) \propto \mathbf{p_t}(\mathbf{b}\,|\,\mathbf{x})\mathbf{p_t}(\mathbf{x})$$

$$d\mathbf{x} = \left[\mathbf{f}(\mathbf{x},\mathbf{t}) - \mathbf{g}^2(\mathbf{t})\left(\nabla_{\mathbf{x}}\log \mathbf{p_t}(\mathbf{x}) + \nabla_{\mathbf{x}}\log \mathbf{p_t}(\mathbf{b}\,|\,\mathbf{x})\right)\mathbf{dt} + \mathbf{g}(\mathbf{t})\mathbf{d}\tilde{\mathbf{w}}\right]$$

Approximations for conditional score term

$$\nabla_x \log p_t(\mathbf{b}\,|\,\mathbf{x_0}) \neq \nabla_{\mathbf{x}}\log \mathbf{p_t}(\mathbf{b}\,|\,\mathbf{x_t})$$

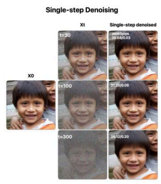
<u>ScoreALD[3]</u>

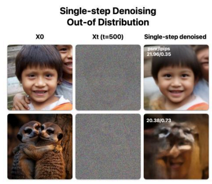
$$\nabla_x \log p_t(\mathbf{b}\,|\,\mathbf{x_0}) \approx \nabla_x \log p_t(\mathbf{b}\,|\,\mathbf{x_t}) \approx -\frac{1}{\sigma^2+\gamma_t^2}\left(\mathbf{A}^{\mathbf{T}}(\mathbf{b}-\mathbf{A}\mathbf{x})\right)$$

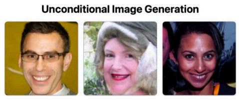
<u>DPS[4]</u>

$$\nabla_x \log p_t(\mathbf{b}\,|\,\mathbf{x_0}) \approx \nabla_{\mathbf{x}}\log \mathbf{p_t}\left(\mathbf{b}\,|\,\mathbf{x_0} = \mathbb{E}\left[\mathbf{x_0}\,|\,\mathbf{x_t}\right]\right)$$

Results: Diffusion Sampling







Results: Inverse Imaging