Diffusion Posterior Sampling for Fourier Compressed Sensing

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Motivation

- Fourier Compressed sensing in MRI is crucial for reducing scan time.
- **Goal1 : Solve Fourier Compressed Sensing using the Diffusion Posterior Sampling (DPS) method.**
- **Goal2 : Compare reconstruction performance across undersampling masks.**

Background

- MRI forward operator & MRI reconstruction problem

$$b = PFS(x) + n$$

$$\hat{x} = \arg\min_{x} \|PFS(x) - b\|_2^2 + \mathcal{R}(x)$$

- The prior $p(x)$ can be modeled using diffusion models [1]
- Latent Diffusion Models (LDM) [2]
  - Employ an Encoder(E) – Decoder(D) structure
  - The prior $p(z)$ is modeled in the latent space
- Diffusion Posterior Sampling [3]

$$\nabla_{x_t} \log p(x_t|y) = \nabla_{x_t} \log p(x_t) + \nabla_{x_t} \log p(y|x_t)$$

- Data consistency in latent space

$$\nabla_{z_t} \log p(y|z_t) = \nabla_{z_t} \log p(y|\mathcal{D}(E([z_0|z_t])))$$

References

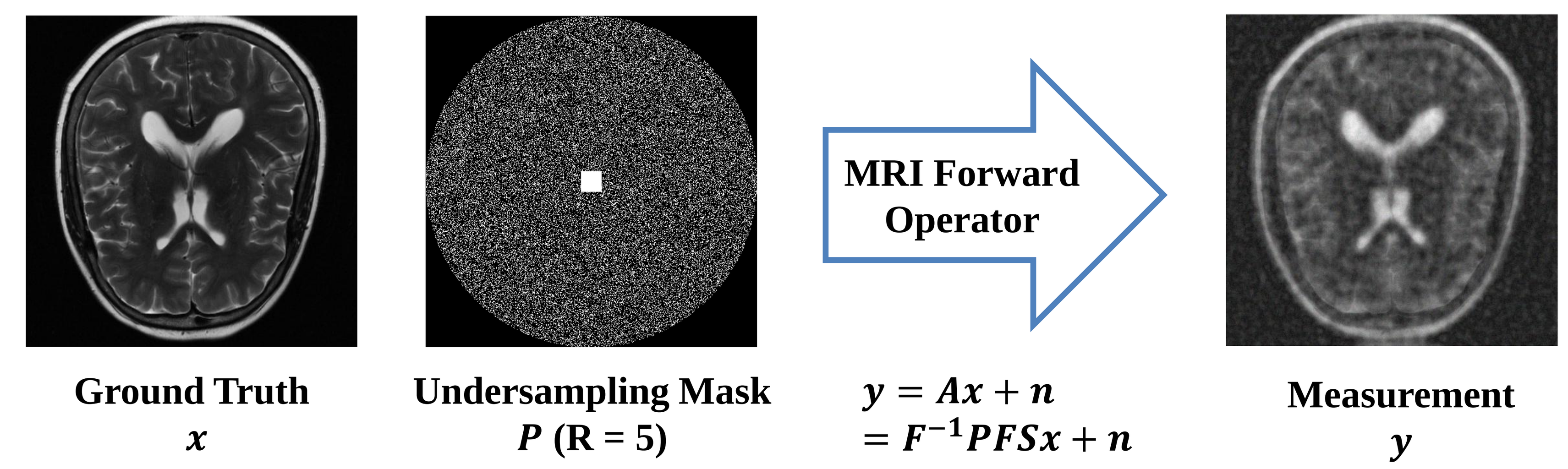
[1] A. Jalal, M. Arvinte, G. Daras, E. Price, A. Dimakis, J. Tamir, "Robust Compressed Sensing MRI with Deep Generative Priors", NeurIPS 2021 .

[2] R. Rombach, A. Blattmann, D. Lorenz, P. Esser, B. Ommer, "High-resolution image synthesis with latent diffusion models", CVPR 2022.

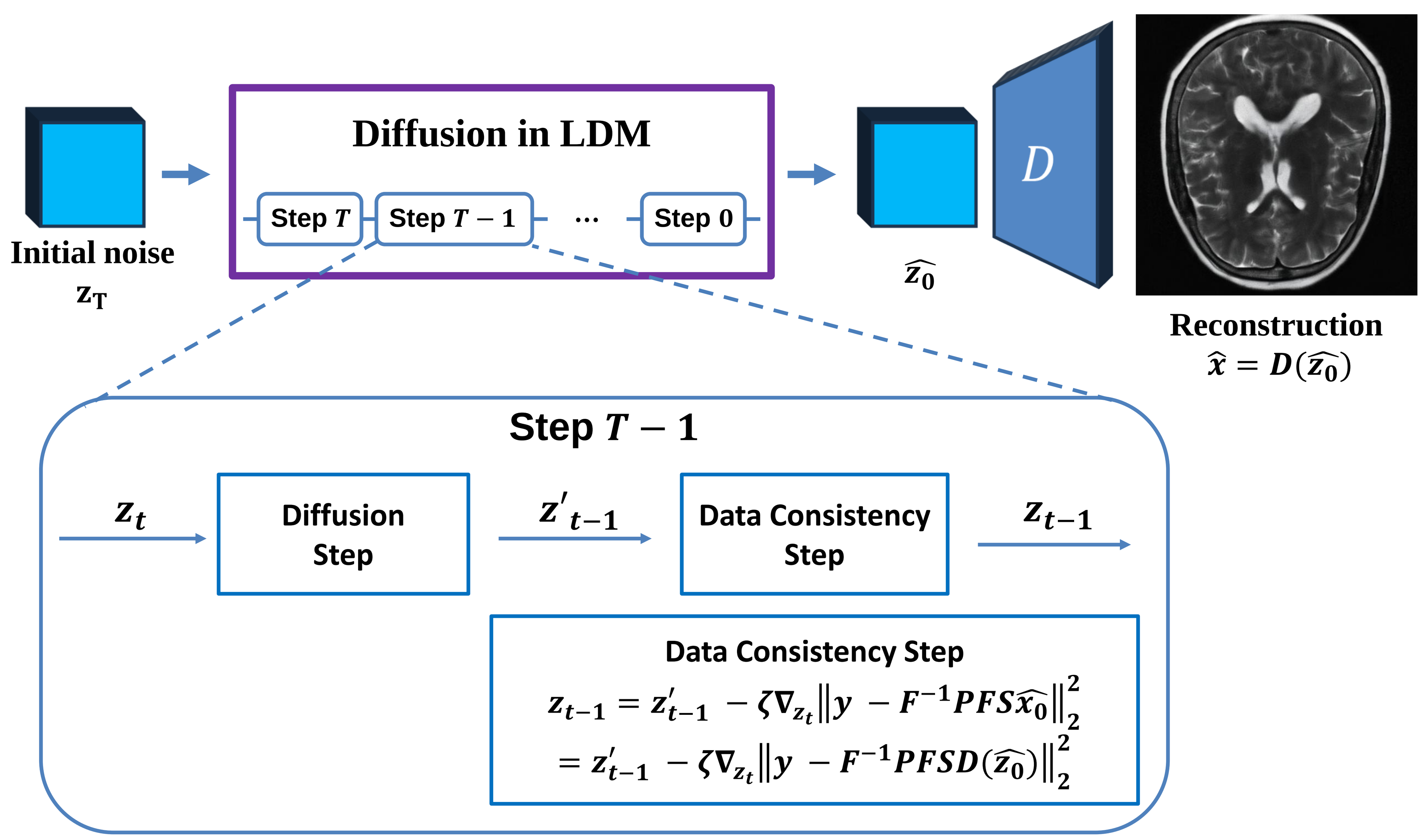
[3] H. Chung, J. Kim, M. T. Mccann, M. L. Klasky, and J. C. Ye, "Diffusion posterior sampling for general noisy inverse problems," ICLR 2023.

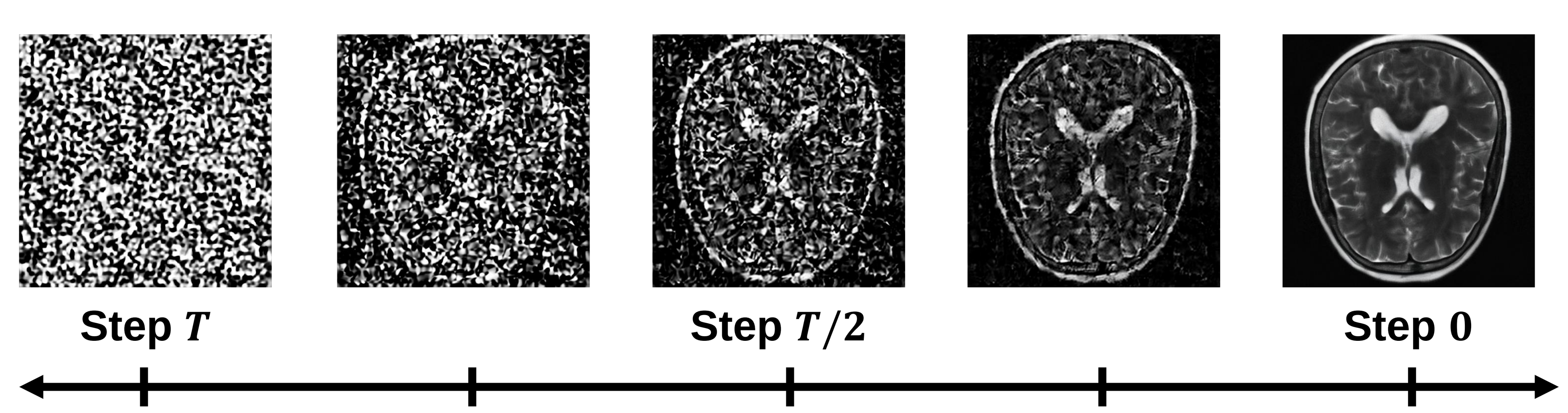
Method

- MRI Forward Operator with a designed undersampling mask



- Utilize a pretrained LDM (Stable Diffusion v2.1) [2]
- Iteratively alternate between denoising diffusion and data consistency (DC) step.
  - DC step : Perform stochastic optimization over the latent space to enforce consistency with the MRI forward model.

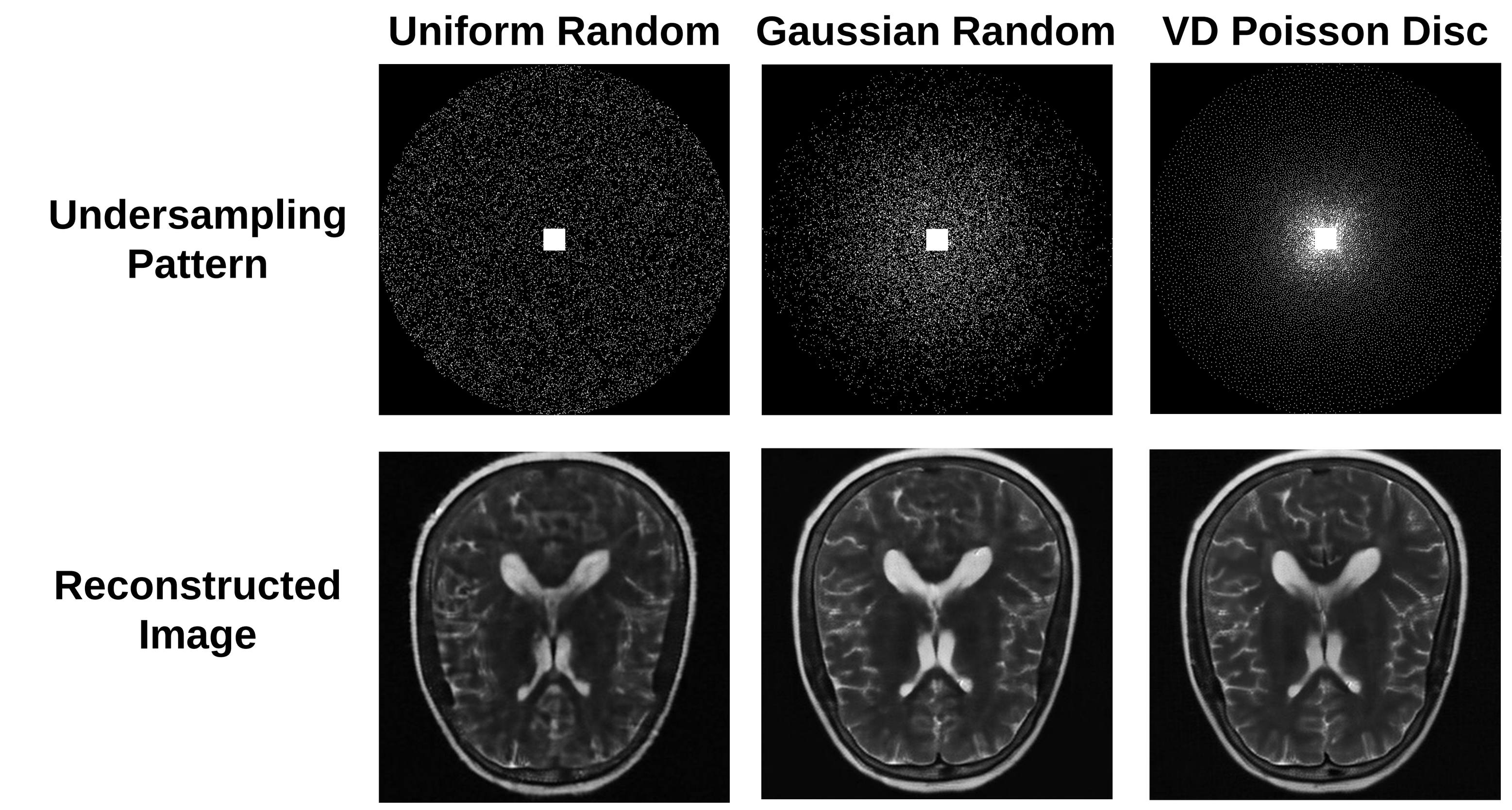




Results

- Experiments with various acceleration factors and undersampling patterns.
  - In most cases, the VD Poisson Disc Pattern achieves the highest reconstruction performance using LDM.

Acceleration (R)	Undersampling Pattern	SSIM	PSNR
5x	Uniform Random	0.8728	29.63
	Gaussian Random	0.9015	29.72
	VD Poisson Disc	**0.9119**	**30.88**
10x	Uniform Random	0.8175	27.68
	Gaussian Random	0.8783	**29.57**
	VD Poisson Disc	**0.8981**	28.88
15x	Uniform Random	0.7749	26.15
	Gaussian Random	0.8648	29.38
	VD Poisson Disc	**0.8852**	**30.31**
20x	Uniform Random	0.7230	24.41
	Gaussian Random	0.8454	28.49
	VD Poisson Disc	**0.8832**	**29.86**



Conclusion

- The Diffusion Posterior Sampling method effectively solves the Fourier compressed sensing problem.
- **Future work** : Optimize the undersampling pattern to achieve the best reconstruction performance, potentially by incorporating an additional trainable layer to learn the optimal pattern.