Image Encoding and Reconstruction with Unconventional MRI Hardware

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1 Introduction

Magnetic Resonance Imaging (MRI) is typically performed in the presence of an extremely strong and uniform magnetic field. Such fields are necessary to strongly polarize spins (tiny magnetic fields produced by hydrogen atoms) to produce the sufficient SNR required to image within reasonable time constraints (less than an hour). Image encoding is performed with spatially linear varying magnetic fields produced by gradient coils. By modulating the strength of these gradient coils over the three coordinate axes, one can acquire all necessary points in the spatial frequency domain (k-space), and reconstruct using an Inverse Fourier Transform. In addition to linear spatial encoding, multiple MRI RF receiver coils are often placed throughout the surface of the subject to create additional image encoding with coil-sensitivity based spatial weighting maps.

Having powerful magnetic fields (1 to 7 Tesla) are great for achieving a high SNR however they are the main reason why MRIs are expensive, uncomfortable, and have a reduced accessibility globally. In order to address these pitfalls, there have been many proposed low field systems (less than 1 Tesla) that operate at a reduced field strength [1], [2]. Such systems instead use opt for cheaper permanent magnets to generate the strong main field, and keep all other encoding hardware relatively unchanged. While this does indeed reduce the cost, the primary challenge is the reduced SNR at lower fields.

2 Motivation

The most straightforward way to gain the lost SNR is to simply image for a longer time (i.e. averaging). However, most subjects are unable to sit still for such a long period of time (more than 30 minutes). Thus, many have applied deep learning techniques to low field images in order to denoise and improve image quality to that of high field systems. While this does help, the use of deep learning for denoising can only go so far before hallucinations become apparent – which is very dangerous in medical imaging applications. Motivated by the fundamental SNR wall at low field, we turn to a slightly different strategy:

Can we instead build low field system that is comfortable enough to wear while you sleep, and is insensitive to motion?

Assuming that such a system did exist, and was indeed comfortable enough to wear, then the SNR lost due to using a low field system can be bought back by imaging over the course of the human sleep cycle (6-8 hours). Such systems have not yet been built, but can be simulated with some fairly simple wearable MRI coil designs. Placing the imaging coils on the subject's head will result in spatially non-linear phase encoding, in contrast to spatially linear phase encoding (i.e. sampling in the spatial frequency domain). **This presents an opportunity to create a non-Fourier based image reconstruction algorithm for MRI with unconventional hardware, and pair it with strong image priors to help with denoising.**

3 Related Work

Unconventional encoding techniques typically show up in the literature to either accelerate existing hardware at high field, or to enable lower cost hardware. We list a few of these ideas below:

- The authors of [1] construct a low field where the main magnetic field has a fixed pattern using a halbach array of permanent magnets, and this field is *mechanically* rotated in order to produce sufficient spatial encoding. Our idea is heavily inspired from this – we are instead opting for an *electrically* rotating magnetic field that is instead comfortably worn on the subject to justify scanning overnight.
- The authors of [3] replace conventional MRI gradient coils (which produce linearly varying fields along coordinate axes) with a set of coils that produce a non-linear field variation over space. They place the coils around the head of a subject, giving excellent spatial encoding around the extremities of the head, but very poor encoding in the center. In this work, a large uniform main field is used for spin polarization. In our case, we would be extending the encoding coils to also provide the main magnetic field needed for polarization, enabling low field applications.
- The authors of [4] allow for a variable main magnetic field, which is controlled by changing the applied current to such a field. The authors decided to target for spatially linearly varying magnetic fields, and perform additional image encoding using an additional set of RF coils. We extent this by exploring wearable non-linearly varying magnetic fields.

4 Overview

4.1 MRI Physics

In our imaging setup, we will have a set of C receive coils to receive RF signals, and a set of K encoding coils to generate magnetic fields. The encoding magnetic field $\mathbf{B}_0(\mathbf{r})$ is generated by modulating the currents $I_1, \cdots, I_K$ applied to each of the K coils:

$$\mathbf{B}_0(\mathbf{r}) = \sum_k I_k \mathbf{B}_0^{(k)}(\mathbf{r}) \tag{1}$$

This encoding field will cause tiny magnetic moment vectors $\mathbf{M}_0(\mathbf{r})$ to rotate about the field vector $\mathbf{B}_0(\mathbf{r})$ with a rotational velocity given by $\gamma||\mathbf{B}_0(\mathbf{r})||_2$, where γ is a constant known as the gyromagnetic ratio. As this magnetic moment rotates, it produces an RF signal which can be received by each of the C receive coils. $\mathbf{M}_0(\mathbf{r})$ turns out to depend on many tissue properties, hence yielding excellent medically relevant contrast if estimated properly.

Using some MRI physics and Faraday's law of induction, the signal received by the c^{th} receive coil is then given by

$$s_c(t) = -\frac{\partial}{\partial t} \int_{\mathbf{r}} \mathbf{B}_1^{(c)}(\mathbf{r}) \cdot \Big(\mathbf{R}_{\hat{\mathbf{B}_0}} \big(\gamma ||\mathbf{B}_0(\mathbf{r})||_2 t\big) \mathbf{M}_0(\mathbf{r}) \Big) d\mathbf{r} \tag{2}$$

where $\mathbf{B}_1^{(c)}(\mathbf{r})$ is the known receive sensitivity of the c^{th} coil and $\mathbf{R}_{\mathbf{v}}(\theta)$ described a rotation operator about $\mathbf{v}$ by angle θ. **The details of this equations are not too important, only that the received signal is in fact linear in $\mathbf{M}_0(\mathbf{r})$.**

4.2 Linear System Formulation

Upon discretization of the multi-coil received temporal signals $s_c(t) \to \mathbf{b}_c$ and the desired magnetization vectors $\mathbf{M}_0(\mathbf{r}) \to \mathbf{m}$, we can combine (1) and (2) to arrive at a much simpler linear system, where the forward model depends on the applied currents $I_1, \cdots, I_K$:

$$\mathbf{A}(I_1, \cdots, I_K)\mathbf{m} + \mathbf{n} = \mathbf{b}_c \tag{3}$$

where $\mathbf{n} \sim \mathcal{N}(0, \sigma^2 I)$ is the Gaussian thermal noise.

In order to have sufficient encoding to invert (3), and battle the relatively large levels of Gaussian noise at low field, we can repeat our imaging experiment with a set of N currents $\{(I_1^{(i)}, \cdots, I_K^{(i)})\}_{i=1}^N$, and attempt to solve the following regularized inverse problem:

$$\min_{\mathbf{m}} || \underbrace{\begin{bmatrix} \mathbf{A}(I_1^{(1)}, \cdots, I_K^{(1)}) \\ \vdots \\ \mathbf{A}(I_1^{(N)}, \cdots, I_K^{(N)}) \end{bmatrix}}_{\mathbf{A}} \mathbf{m} - \underbrace{\begin{bmatrix} \mathbf{b}_c^{(1)} \\ \vdots \\ \mathbf{b}_c^{(N)} \end{bmatrix}}_{\mathbf{b}} ||_2^2 + \mathcal{R}(\mathbf{m}) \tag{4}$$

5 Goals

With equation (4) in mind, we will break up the goals of this project into four parts:

1. **Craft a physics simulator capable of generating the forward model A and received data b for some realistic currents** $I_1, \cdots, I_K$:

   Most of the required physics simulation tools are already accessible in our lab. We won't be putting too much emphasis in making this simulator very accurate, rather we will build just enough to allow us to perform reconstructions down the line. Expected time is a few days.

2. **Estimate m without any regularization or noise**:

   We will attempt to get reasonable reconstructions without any noise, using GD and/or Conjugate Gradient on the *unregularized* ($\mathcal{R}(m) = \mathbf{0}$) version of (4). Expected time is a few days.

3. **Estimate m using a variety of different regularization choices over various different noise levels**:

   We will be using ADMM, as described in lecture, to implement the regularized reconstructions with priors such as Tikonov, TV, ℓ_1 Wavelet, and deep learning denoisers. Expected time is a few days.

4. **Explore techniques to optimize the imaging currents** $I_1, \cdots, I_K$**.** :

   This goal has the highest potential for failure, and hence is mostly viewed as a soft goal. This will consist of first just trying out various different heuristics and strategies for choosing optimal currents, without any formal optimization. If time permits, attaching gradients to the currents and running SGD will be the next logical step. Expected time is one week.

References

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