Non-Blind Deconvolution of Partial Fourier MRI

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INTRODUCTION

Magnetic resonance imaging (MRI) operates by capturing samples from an object's spatial frequency domain. In the most basic case, the frequency space is fully sampled (in the Nyquist sense) so that a human-interpretable image can be reconstructed via inverse Fourier transform of the raw frequency data. The resulting image is generally complex-valued.

Due to hardware and safety limitations, MRI is very limited in terms of the speed with which frequency data can be collected. This translates into long exam times which pose significant challenges in the clinical application of MRI. A common protocol for shortening MRI data acquisition is Partial Fourier (PF) imaging, whereby a contiguous portion of (high)-frequency space is not sampled (inset of right-most panel in Figure 1).

In the image domain, the PF sampling scheme amounts to convolving a 2D image with a 1D phase-modulated sinc kernel (Figure 1). My project will explore a variety of approaches to solve this non-blind deconvolution problem using tools from EE367.

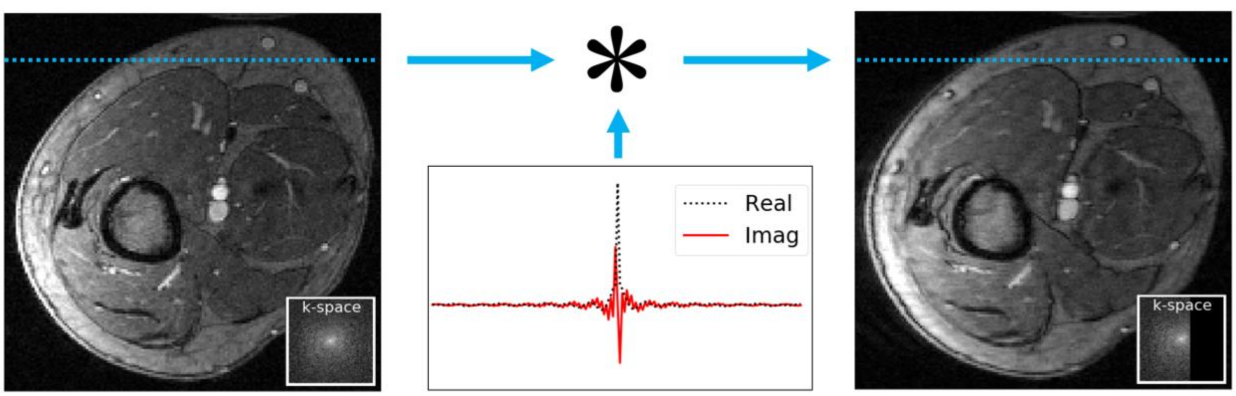

Figure 1: Relationship between Partial Fourier (PF) and fully sampled MRI for a transverse image of a human leg in vivo. The PF image on the right is generated via retrospective PF sampling. Insets show the corresponding frequency space, which is commonly referred to as k-space in the MRI literature.

RELATED WORK

PF sampling has been around for a few decades now, so naturally many schemes have been developed for reconstructing such data (McGibney 1993). Standard methods for PF reconstruction (Noll 1991, Haacke 1991) are very limited in scope because they require the phase of the complex-valued image to be slowly varying, which is often not the case. More recently, a few papers have proposed using CNNs to learn the mapping from undersampled to fully sampled images in MRI (Jin 2017, Wang 2016, Schlemper 2017), but not with the PF sampling scheme.

PROJECT OVERVIEW

I will investigate three novel approaches to the PF reconstruction problem based on: (i) non-local means (Buades 2005) (ii) ADMM (Boyd 2011), and *time permitting* (iii) deep learning (Lecun 2015, Diamond 2017). I have already written code that establishes the baseline performance of standard PF reconstruction methods (Noll 1991, Haacke 1991) for comparison.

Data & Equipment

I will make use of two datasets. For early stage sanity checks, I will use simple, low-resolution natural images from CIFAR-10 (Krizhevsky 2018). Once I establish proper functioning of my code, I will use a database of 53 3D MRI exams of miscellaneous anatomy which I have access to through my research group. Both datasets will be retrospectively undersampled to generate examples of PF images with the original image serving as ground truth. My research group has ample computational infrastructure (many CPUs, GPUs) which I will use for the latter stages of this project which will be quite computationally intensive.

Non-Local Means

With inspiration from (Buades 2005), I posit that a given MRI image contains similar features in disjoint regions. Considering that the PF blur kernel is 1D, I will exploit the existence of similar 1D features which are oriented orthogonal to one another. In other words, the MRI dimensions orthogonal to the PF blurring will provide a sort of dictionary of example features from which to learn. This should work well on images with some rotational symmetry, such as transverse images of limbs (e.g. Figure 1).

ADMM

I will implement ADMM as in HW3, with a few modifications to adapt it for this problem. First, the data consistency term will be modified to enforce fidelity in the frequency domain. Second, I will experiment with different priors for the MRI context, which I expect will form the bulk of the work for this part of the project. So far, I intend to explore an "approximate conjugate symmetry" prior on the frequency space, such as L1(Fx – conj(flip(Fx))) where F is the fourier transform operator. I may also try an NLM prior if the first NLM method goes well.

Deep Learning

I have already trained a basic CNN for the PF task which I will compare to the above methods. Time permitting, I will explore Deep Priors method detailed in this paper (Diamond 2017).

TIMELINE

Feb 18-20: Non-Local Means
Feb 21-28: ADMM
Mar 1-8: Deep Learning
Mar 9-12: Prepare presentation and report

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