Markov Random Fields and Gibbs Sampling for Image De-noising
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Motivation
- High-order MRFs are more expressive.
- Filters like Gaussian blur are deterministic. Gradient ascent needs careful initialization and converges to suboptimal. Gibbs Sampling provides exploration, and does not require sophisticated initialization.
- There might be some patterns that pixels within a window tend to show.

New Technique
- Tested different neighbor sizes, found that using 9 neighbors (3×3) is generally better than using 5 (pairwise) or 25 (5×5).
- Define energy as loss + prior + score.
- $E(Y_{ij}) = \sum_{x \in N(Y_{ij})} \| Y_{ij} - X \|_1^n + \lambda \sum_{x \in EN(Y_{ij})} X_{ij} \| Y_{ij} - Z \|_2^n + \psi(Y_{ij})$.
- For loss, $L_1$ norm has higher PSNR, but $L_2$ norm has better visualization. Lorentzian is robust to outliers, and can provide smoothing.
- Applied sparse gradient and close-to-mean-of-neighbors priors.
- With the assumption that pixels within a window tend to have some patterns, defined ‘score’ as the dot product of patterns (filters) and pixels in window.
- By using first $n$ principal components as filters and weighting dot products by variance explained, found that ‘close to center’ prior gives improvements.

Related Work
- MRF: $p(Y_{ij}) = \frac{1}{Z} \exp \left( -E(Y_{ij}) \right)$.
- Metropolis-Hastings for Markov Chain Monte Carlo. $A(x' | x) = \min \left( 1, \frac{p(x')Q(x | x')}{p(x)Q(x' | x)} \right)$.
- Ising, Potts, hierarchical MRF models.
- Roth & Black: Fields of experts for learning image priors.
- McAuley et al: High-order MRF priors of color image.
- Barbu: Active Random Fields.

Experimental Results

<table>
<thead>
<tr>
<th>Filters</th>
<th>Noisy</th>
<th>Gaussian</th>
<th>$L_1$ norm</th>
<th>$L_2$ norm</th>
<th>Lorentzian</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma=50$</td>
<td><img src="image1.jpg" alt="Image" /></td>
<td><img src="image2.jpg" alt="Image" /></td>
<td><img src="image3.jpg" alt="Image" /></td>
<td><img src="image4.jpg" alt="Image" /></td>
<td><img src="image5.jpg" alt="Image" /></td>
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<tr>
<td></td>
<td>14.78 dB</td>
<td>23.53 dB</td>
<td>25.94 dB</td>
<td>25.50 dB</td>
<td>25.76 dB</td>
</tr>
<tr>
<td>$\sigma=100$</td>
<td><img src="image7.jpg" alt="Image" /></td>
<td><img src="image8.jpg" alt="Image" /></td>
<td><img src="image9.jpg" alt="Image" /></td>
<td><img src="image10.jpg" alt="Image" /></td>
<td><img src="image11.jpg" alt="Image" /></td>
</tr>
<tr>
<td></td>
<td>10.26 dB</td>
<td>18.55 dB</td>
<td>23.34 dB</td>
<td>21.08 dB</td>
<td>23.85 dB</td>
</tr>
</tbody>
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