Flutter Shutter Deconvolution under Gaussian and Poisson Noise

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**Techniques and Algorithms**

The Gaussian noise term follows a zero mean i.i.d. Gaussian distribution, \( \eta = \mathcal{N}(0, \sigma^2) \), where \( \sigma^2 \) is the variance of the noise. The Poisson term follows a Poisson distribution, \( \text{Pois}(\lambda) \), where \( \lambda \) is both the mean and the variance. The mixed noise model approximates the Poisson noise term by a signal dependent gaussian distribution \( \mathcal{N}(\mathbf{Ax}, \mathbf{Ax}) \) (Foi et al., 2009), which can now be considered additive noise such that the total noise term is \( \eta = \mathcal{N}(\mathbf{Ax,Ax}) + \mathcal{N}(0,\sigma^2) = \mathcal{N}(\mathbf{Ax,Ax} + \sigma^2) \). Before deconvolution, noise following these distributions is added to the simulated flutter shutter image.

Two iterative ADMM algorithms for deconvolution of the fluttered image are implemented. The general formulation for image reconstruction is:

\[
\min_{x} \frac{1}{2} \Vert Ax - b \Vert^2 + \Gamma(x)
\]

Subject to \( Dx - z = 0 \)

Where C is the convolution kernel, b is the blurred image, \( \Gamma(x) \) is the regularization term. In this project, two regularization terms are utilized: A weighted \( l_1 \) norm and the weighted difference of \( l_1 \) and \( l_2 \) norms:

\[
\Gamma(z) = \lambda \Vert Dx - z \Vert^2 + \frac{\rho}{2} \Vert v - u \Vert^2
\]

For ADMM in general, the Augmented Lagrangian is formed as:

\[
L_j(x,v) = f(x) + g(z) + \frac{\rho}{2} \Vert Ax - v \Vert^2 + \frac{\lambda}{2} \Vert z - u \Vert^2
\]

Now the update rules for both regularization schemes can be derived. For the isotropic case, the update rules derived by Boyd et al. defines 2 auxiliary variables, \( z \) and \( u \):

\[
x \leftarrow \text{prox}_{g}(v) = \arg\min_{x} f(x) + \frac{\rho}{2} \Vert Ax - v \Vert^2 + \frac{\lambda}{2} \Vert z - u \Vert^2
\]

\[
z \leftarrow \text{prox}_{g}(u) = \arg\min_{z} g(z) + \frac{\lambda}{2} \Vert z - u \Vert^2 + \frac{\rho}{2} \Vert v - Ax \Vert^2
\]

For the isotropic case, we follow the method utilized by Lou et al., based on the difference of convex algorithm. Another auxiliary variable, q, is introduced to linearize the isotropic term. This only changes the z and q updates:

\[
z \leftarrow \text{prox}_{g}(u) = \arg\min_{z} g(z) + \frac{\lambda}{2} \Vert z - u \Vert^2 + \frac{\rho}{2} \Vert v - Ax \Vert^2
\]

\[
q \leftarrow Dq + z
\]

**Results**

To the left: Stationary image, flutter shutter blurred image, and motion blurred image for the car test.

Results of ADMM using \( \Gamma = \text{regularizer for the car}. \) Outer images: Top row: gaussian noise with \( \sigma = 0.001, 0.01, .1, .15 \); Middle row: poisson noise with \( \lambda = 1000,500,50 \); Bottom row: Mixed gaussian-poisson noise, \( \lambda = 1000, \sigma = .001; \lambda = 500 \).

PSNR: 19.89
PSNR: 19.05
PSNR: 18.99
PSNR: 17.86
PSNR: 16.89
PSNR: 15.49
PSNR: 11.83
PSNR: 11.21
PSNR: 10.53
PSNR: 7.86

To the right: Stationary image, flutter shutter blurred image, and motion blurred image for the cars test.

Results of ADMM using \( \Gamma = \text{regularizer for the car}. \) Outer images: Top row: gaussian noise with \( \sigma = 0.001, 0.01, .1, .15 \); Middle row: poisson noise with \( \lambda = 1000,500,50 \); Bottom row: Mixed gaussian-poisson noise, \( \lambda = 1000, \sigma = .001; \lambda = 500 \).

PSNR: 25.58
PSNR: 25.54
PSNR: 25.16
PSNR: 23.49
PSNR: 23.69
PSNR: 25.56
PSNR: 25.44
PSNR: 25.34
PSNR: 21.46
PSNR: 18.21
PSNR: 17.91
PSNR: 16.90
PSNR: 15.70
PSNR: 15.49
PSNR: 12.15
PSNR: 11.82
PSNR: 11.21
PSNR: 10.50
PSNR: 9.05
PSNR: 7.86

**References**