EE365: Hitting Times
Example: Inventory re-ordering

if we start in state $C$, how long before we re-order?

$$\tau_E(x_0, x_1, \ldots) = \min\{t > 0 \mid x_t \in E\}$$

- $\tau_E$ is a random variable, called the first passage time or hitting time to set $E$
- $\tau_E$ is the earliest time when $x_t \in E$
- we set $E = \{0, 1\}$
Computing the distribution of first passage times

replace states in $E$ (in this case 0 and 1) with absorbing states

hitting times to set $E$ are the same for both chains
Computing the distribution of first passage times

let $Q$ be the transition matrix of the new chain

for $j \in E$

$$\text{Prob}(\tau_{\{j\}}(x) = t \mid x_0 = i) = (Q^t)_{ij} - (Q^{t-1})_{ij}$$

i.e., conditioned on $x_0 = i$,

$$\text{Prob}(t \text{ is the first time at which } j \text{ is reached}) = \text{Prob}(j \text{ has been reached by time } t) - \text{Prob}(j \text{ has been reached by time } t - 1)$$
Example: Inventory re-ordering

► how long before we re-order, given that we start fully stocked?

► plot shows $\text{Prob}(\tau_{\{0,1\}} = t \mid x_0 = 6)$ vs. $t$ (mean is 13.1)