EE365: Epidemic Example
Monte Carlo simulation

to approximate

\[ e = \mathbb{E} f(x_0, \ldots, x_T) = \sum_{s_0, \ldots, s_T \in \mathcal{X}} f(x_0, \ldots, x_T) d_{s_0} P_{s_0 s_1} \cdots P_{s_{T-1} s_T} \]

(a sum with \( n^{T+1} \) terms)

- simulate \( N \) trajectories \( x_t^{(i)} \), and let

\[ \hat{e} = \frac{1}{N} \sum_{i=1}^{N} f(x_0^{(i)}, \ldots, x_T^{(i)}) \]

- \( \hat{e} \) is an unbiased estimate of \( e \)

- works for \textit{any} function \( f \)
Example: Epidemic models

- undirected graph
- *k* vertices, each represents an individual
- each may spread infection to neighbors
- each individual is either susceptible, infected, or removed
- called the *S, I, R* model
- transition probabilities for each individual depends on infection state of neighbors

\[ \mathcal{X} = \{S, I, R\}^k, \text{ so } |\mathcal{X}| = 3^k \]

- for modest *k*, we cannot store a distribution on *X*
- one approach is to use Monte Carlo methods to estimate useful quantities
Example: Epidemic models

- graph is $30 \times 30$ grid

- transition probabilities for an individual, with states ordered $(S, I, R)$

\[
\begin{bmatrix}
0.6 & 0.4 & 0 \\
0.2 & 0.6 & 0.2 \\
0 & 0 & 1
\end{bmatrix}
\]
with an infected neighbor

\[
\begin{bmatrix}
1 & 0 & 0 \\
0.2 & 0.6 & 0.2 \\
0 & 0 & 1
\end{bmatrix}
\]
with no infected neighbors

- simulate Markov chain to see spread of infection
Example: Epidemic models

$t = 0$

- Susceptible
- Infected
- Removed
Example: Epidemic models

\[ t = 1 \]

- Susceptible
- Infected
- Removed
Example: Epidemic models

\[ t = 2 \]
Example: Epidemic models

\[ t = 3 \]
Example: Epidemic models

\[ t = 4 \]
Example: Epidemic models

\[ t = 5 \]
Example: Epidemic models

\[ t = 6 \]
Example: Epidemic models

\[ t = 7 \]
Example: Epidemic models

\[ t = 8 \]
Example: Epidemic models

\[ t = 9 \]
Example: Epidemic models

\[ t = 10 \]
Example: Epidemic models

\[ t = 15 \]
Example: Epidemic models

\[ t = 20 \]
Example: Epidemic models

\[ t = 25 \]

- Susceptible
- Infected
- Removed
Example: Epidemic models

\[ t = 30 \]
Example: Epidemic models

\[ t = 35 \]
Example: Epidemic models

\[ t = 40 \]
Example: Epidemic models

\[ t = 45 \]
Example: Epidemic models

\[ t = 50 \]
Example: Epidemic models

\[ t = 55 \]

- Susceptible
- Infected
- Removed
Example: Epidemic models

\[ t = 60 \]
Example: Epidemic models

\[ t = 65 \]
Example: Epidemic models

\[ t = 70 \]
Example: Epidemic models

\[ t = 75 \]
Example: Epidemic models

\[ t = 80 \]
Example: Epidemic models

disease spread over time
Example: Epidemic models

- histogram of fraction of population that is removed after 100 time steps
- in 4% of runs, less than 5% is removed