Near-optimal control of office park HVAC systems via convex optimization

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1 Abstract

In this project we will examine near-optimal control and operation of multiple heating, ventilation, and cooling (HVAC) systems in an office park via convex optimization techniques in order to minimize total cost of electricity. In this simulation, multiple units on a multiple-floor office building each have their own separate HVAC units, all operated by the same controller. Each HVAC unit has its own cooling rate, energy costs, etc., and each office unit has its own demanded temperature range for each 15 minute time step. We will also experiment with minimizing a multiobjective function that combines electricity cost with the sums of squared deviations from demand that trades off discomfort for cost. We will use model predictive control to plan for a 24-hour planning horizon and re-evaluate every 15 minutes based on actual temperatures and electricity prices.

2 Problem formulation

We start with a network of heat transfer planes; each of \( m \) office units has \( k \) surfaces that transfer heat with either another office unit, the outdoors, or the ground. This is governed by the heat transfer function:

\[
h_{ij} = a_{ij} u_{ij} (y_i - y_j),
\]

where \( a_{ij} \) is the area of the surface between units \( i \) and \( j \) in \( \text{m}^2 \), \( u_{ij} \) is the heat transfer constant of that surface in \( \text{W/m}^2 \text{°K} \), and \( y_i \) is the temperature of each unit \( i \) in \( \text{°K} \) [1]. This gives us heat across surface \( ij \) in Watts, and multiplying by the 15-minute timestep gives us the energy transferred across each surface in each timestep. In order to recover the new difference in temperature, we divide by \( q_i \), which is a thermal resistance constant for each office unit \( i \) based on the number of moles of air in the unit and the heat capacity of air under constant pressure [4].

Our cost minimization problem is dependent on the energy used by each office HVAC unit, \( p_i \). This is linear in the percentage of maximum load, \( \frac{x_i}{x_{i,\text{max}}} \), and parameterized by
\( \alpha_i \) and \( \beta_i \), which take into consideration the typical commercial design load intensity of 10 W/ft\(^2\) [5], as well as the square footage of each unit. Our power is converted into kWh by multiplying by the 15-minute timestep and a conversion factor, so \( c^T p \) represents the dollar cost of running the building’s \( m \) HVAC units for \( T \) timesteps.

We solve the problem:

\[
\begin{align*}
\text{minimize} & \quad c^T p \\
\text{subject to} & \quad y_i^{\min} \leq y_i \leq y_i^{\max}, \\
& \quad x_i^{\min} \leq x_i \leq x_i^{\max}, \\
& \quad h_{ij} = a_{ij} u_{ij} (y_i - y_j), \\
& \quad y_i(t + 1) = y_i(t) + x_i(t) - q_i \sum_{j=1}^{k} h_{ij}(t) \\
& \quad p_i \geq \alpha_i |x_i^{\max}| + \beta_i, \\
& \quad y_i \in \mathbb{R}^T, \\
& \quad x_i \in \mathbb{R}^T, \\
& \quad h_{ij} \in \mathbb{R}^T, \\
& \quad p_i \in \mathbb{R}^T,
\end{align*}
\]

where \( x \in \mathbb{R}^{m \times T} \) is our decision variable - the temperature each HVAC unit lowers or raises its office unit’s temperature by each timestep. This makes \( c \) and \( p \in \mathbb{R}^{mT} \), where \( c = [c(1) c(2) \ldots c(T)] \), and \( c(1) \) is the current price of electricity in \$/kWh repeated \( m \) times, and each subsequent \( c(t) \) is the forecasted price at time \( t \) (repeated \( m \) times). Note that \( p_i \) is an inequality, but the nature of cost minimization (and the fact that all \( c \geq 0 \)) and the convexity of the constraint means that this will always be a strict equality.

We also have constraints on the temperatures of each office unit. During the night time, these constraints are relaxed, but during working hours temperatures are expected to be within a certain range, different for each office unit. These constraints also govern the temperature of the outdoors and the ground: \( y_{k-1}^{\max} = y_{k-1}^{\min} = y_{\text{outdoor}} \) and \( y_k^{\max} = y_k^{\min} = y_{\text{ground}} \), which are forecasted outdoor and ground temperatures, except for the first entry in each vector (the current measured outdoor and ground temperature). We also have time-independent constraints on each HVAC unit, \( x_i^{\max} \), which determine the maximum temperature impact that office unit \( i \)'s HVAC system may have.

We will solve this problem based on our forecasted prices and temperatures, then recalculate every 15 minutes once real-time electricity prices, outdoor temperatures, and indoor temperatures are measured. At each time step, we will reduce \( T \) by one, so that we are still only planning until the end of our original 24-hour time period. Also, we will experiment with a multiobjective function to minimize, which trades off cost for discomfort:

\[
\begin{align*}
\text{minimize} & \quad c^T p + \lambda \| u \|_2^2 \\
\text{subject to} & \quad u \geq (y - y_{\max})_+ + (y_{\min} - y)_+ , \quad u \in \mathbb{R}^{mT},
\end{align*}
\]

with all other constraints the same, except the strict lower and upper bounds on \( y \) are removed in favor of the discomfort function \( u \). Again, \( u \geq 0 \) and the nature of the objective function ensures that the equality holds.

### 3 Methods

The problem as described above is coded in MATLAB using CVX to minimize the cost function. Several different numerical examples have been constructed, with varying results.
Our baseline system is optimal, assuming perfect knowledge of temperature and electricity price fluctuations. After that, we use two different temperature functions, one varying over a range of 14 °C and one varying over 8 °C, in conjunction with our initial optimization problem (1) and our multiobjective problem (2).

3.1 Example system

Currently, the problem is being modeled with toy data, in a model of an office park that is 100m x 100m x 7m, with two stories and five office units. Each office unit has its own temperature requirements and HVAC system limits. Modeled in 15-minute time chunks over a 24-hour period, we have a baseline optimum for when we have perfect information. Introducing uncertainty will showcase the flexibility of the model predictive control system compared to our baseline.

In the toy dataset, each office unit has a minimum temperature of 63 + \(i\) °F, where \(i\) is the index of the office park unit, and a maximum temperature of 81.5 – 1.5\(i\) °F, between the hours of 7 a.m. and 8 p.m. In addition, each office unit’s HVAC system has max cooling/heating power of 5 + \(i\) °F per 15-minute time step. The surface area matrix \(A\) is:

\[
\begin{pmatrix}
0 & 350 & 3250 & 0 & 1750 & 700 & 5000 \\
350 & 0 & 0 & 3250 & 1750 & 700 & 5000 \\
3250 & 0 & 0 & 227.5 & 175 & 3652.5 & 0 \\
0 & 3250 & 227.5 & 0 & 175 & 3652.5 & 0 \\
1750 & 1750 & 175 & 175 & 0 & 4095 & 0 \\
700 & 700 & 3652.5 & 3652.5 & 4095 & 0 & 0 \\
5000 & 5000 & 0 & 0 & 0 & 0 & 0
\end{pmatrix}
\text{m}^2,
\]

with index 6 and 7 representing the outdoors and the ground respectively. The heat transfer constant \(u_{ij}\) is 8.9 W/m\(^2\) °K when surface \(ij\) is between two office units, and 5.9 W/m\(^2\) °K when either \(i\) or \(j\) is the outdoors or the ground \([1]\). The thermal resistance constant vector \(q_i\) is:

\[
\begin{pmatrix}
.4040 \\
.4040 \\
.6215 \\
.6215 \\
.5771
\end{pmatrix}
\times 10^{-4}\ \text{°K/W},
\]

and power consumption vectors:

\[
\alpha = \begin{pmatrix}
107.6391 \\
107.6391 \\
69.9654 \\
69.9654 \\
75.3474
\end{pmatrix}\ \text{kWh}, \quad \beta = \begin{pmatrix}
26.9098 \\
26.9098 \\
17.4914 \\
17.4914 \\
18.8368
\end{pmatrix}\ \text{kWh}.
\]
We model the forecasted cost of power as  

$$c = \frac{-0.0068326 t^3 + 0.229743 t^2 - 1.57789 t + 9}{100} \$/kWh,$$

and at each 15-minute timestep, \(c_{\text{current}}\) varies uniformly by ±1 cent, centered on the forecasted value of \(c\) [2]. Our example temperature forecast is either  

\[
y_{\text{out}}^{(1)} = \frac{5}{9}(-0.0173611 t^3 + 0.520833 t^2 - 2.5 t + 30.777777) \text{ (from 15.5 to 29.5 } ^\circ \text{C)} \text{ or}
\]

\[
y_{\text{out}}^{(2)} = 4 \sin\left(\frac{\pi}{12}(t - 10)\right) + 22 \text{ (from 18 to 26 } ^\circ \text{C)},
\]

with current outdoor temperature calculated by uniformly varying around the forecasted temperature in the range ±1 °C [3].

### 3.2 Optimal control

With the setup described in §3.1, assuming full knowledge of the upcoming outdoor temperatures and power costs, it costs $1694.80 to keep the building temperatures within each office unit’s desired range under the first set of outdoor temperatures, and $1125.60 under the second set of outdoor temperatures. However, it should be noted that $1091.70 of the total electricity cost is from the sum of the \(\beta_i\) values, which describe power consumption based on the HVAC units being on, but not actively impacting office temperature. That means that in the second case, the outdoor temperatures fluctuate so little that only 3% of the total cost is due to temperature modulation, compared to 35.6% in the first case.

If we look at the first set of temperatures with our multiobjective function, setting \(\lambda = 10\), we see how much money the building owner saves if they pay each tenant $10 times the squared residual of the actual temperature minus their desired temperature each timestep. In this case, the building owner pays $1546.10, a savings of $148.70. Now the electricity bill is just $1484.90, and the owner pays $61.20 in “discomfort tax.”

In the provided Figure 1, office unit temperature is plotted against time, as is the activity of each office’s HVAC unit. There is a small amount of heating around 6-7 a.m. to prepare the office at the minimum temperature, then cooling for most of the day, as office
units 3-5 have stricter demanded temperature ranges. Note that the temperature in units 3 and 5 naturally soars higher than that of office unit 1, since units 3 and 5 are on the top floor, and therefore they have more surfaces in contact with the outdoors.

3.3 Model predictive control

The next step builds uncertainty into the model. Instead of complete knowledge of temperature and pricing, we instead only know the current time’s temperature and electricity price, relying on our previous forecasts in order to plan for each 15-minute timestep. This is much more realistic, but also unfortunately suboptimal. We begin with our initial temperature set to the outdoor temperature, and then solve the complete convex optimization problem (1) based on our knowledge of the current temperature and price, as well as our forecasts for the rest of the day. After determining the HVAC use values $x(t)$, we set the current building temperature to our calculated $y(t+1)$ and then solve the problem again, with the new outdoor temperature and electricity price. We do this successively, shortening the planning horizon by one timestep each time, until we have a complete HVAC usage pattern $x \in \mathbb{R}^{m \times T}$.

When using the multiobjective function with $\lambda = $ $10$, we reach a total cost of $1645.80$, with $1571.40$ allocated to the electricity bill, and $74.40$ allocated to paying the “discomfort tax” described earlier. This is $23.40$ less than the optimal cost of problem (1), and $99.70$ more than the optimal solution using the multiobjective function (2). See Figure 2 for graphs of temperatures under this objective function. When subtracting out the fixed cost we discussed in §3.2, model predictive control solving problem (2) costs 22% more than our optimal variable cost of $454.50$. However, if we deal with our total objective cost values, model predictive control only costs 6.5% more than the optimum.

When trying to solve problem (1) using model predictive control, it turns out that the uncertainty introduced by the random variations in temperature actually makes the problem infeasible at certain points in the middle of the day. If we are already cooling office unit 5 at

Figure 2: Graphs of temp. in units 1, 3, and 5 under model predictive control. Office temp. in blue, temp. bounds in black, outside temp. in red, and HVAC in green.
and the temperature jumps higher than previously anticipated, our lack of planning for this uncertainty in the past means that we are unable to bring the temperature within the required bounds. However, the multiobjective function in problem (2) does not have strict temperature constraints, just a penalty function, so it is able to come close to the optimal HVAC schedule we determined in §3.2. Note in particular the similarity between HVAC usage in office unit 5 in Fig 1 and Fig 2. In reality, tenants are unlikely to notice a temperature change of less than one degree, and no deviation was more than half a degree.

4 Conclusion

As seen in §3.3, model predictive control provides a schedule for HVAC usage that, while suboptimal, can be made to handle the unpredictable nature of the real world quite well. In the future, it would be interesting to experiment with a model that allows HVAC systems to shut on and off, saving a lot of cost in times when the systems are on but not actually affecting the temperature (in stand-by mode). There would necessarily be a large start-up cost, but this could definitely reduce electricity costs on more temperate days, such as with our secondary temperature function. Also, it would be worth experimenting with more sudden changes in temperature or electricity price - to examine the robustness of the system. It’s highly likely that the initial model (1) would become infeasible during a sudden temperature change, due to the $x^{\text{max}}$ limits, as it is already infeasible in our example with an unpredictable temperature over a 14 °C range. But with the multiobjective function (2), we should be able to handle a more gradual return to the desired office unit comfort zones. It would also be worth experimenting with the dollar value of $\lambda$, as well as the different penalty function $\lambda\|u\|_1$, which would incentivize fewer deviations from the desired temperature range.

References


