8.1 (4 points) Trajectory optimization with avoidance constraints. In this problem, you must choose $N$ trajectories (say, of some vehicles) in $\mathbb{R}^n$, which are denoted by $p_i(t) \in \mathbb{R}^n$, $t = 1, \ldots, T$, $i = 1, \ldots, N$. The objective is to minimize

$$J = \sum_{i=1}^{N} \sum_{t=1}^{T-1} \|p_i(t+1) - p_i(t)\|_2^2,$$

subject to fixed starting and final positions,

$$p_i(1) = p_i^{\text{start}}, \quad p_i(T) = p_i^{\text{final}}, \quad i = 1, \ldots, N.$$

The solution to the problem stated so far is simple: Each trajectory follows a straight line from the starting position to the final position, at uniform speed. Here is the wrinkle: We have avoidance constraints, of the form

$$\|p_i(t) - p_j(t)\|_2 \geq D, \quad i \neq j, \quad t = 2, \ldots, T - 1.$$

(Thus, the vehicles must maintain a given distance $D$ from each other at all times.) These last constraints are obviously not convex.

(a) Explain how to use the convex-concave procedure to (approximately, locally) solve the problem by replacing concave functions with their affine approximations.

(b) Implement the method for the problem with data given in traj_avoid_data.m, which involves three vehicles moving in $\mathbb{R}^2$. Executing this file runs a movie showing the vehicle trajectories when they move in straight lines at uniform speed, with a circle around each vehicle of diameter $D$. You can use this code to visualize the trajectories you obtain as your algorithm runs.

You should start the convex-concave procedure from several different initial trajectories for which $p_i(t) \neq p_j(t)$, $t = 1, \ldots, T$. For example, you can simply take $p_i(t) \sim \mathcal{N}(0, I)$. Plot the objective $J$ versus iteration number for a few different initial trajectories (on the same plot). Verify that the avoidance constraints are satisfied after the first iteration.

For the best set of final trajectories you find, plot the minimum inter-vehicle distance, $\min_{i \neq j} \|p_i(t) - p_j(t)\|_2$, versus time. Plot the trajectories in $\mathbb{R}^2$. And of course, for your own amusement, view the movie.

Hint. Don’t try to write elegant code that handles the case of general $N$. We’ve chosen $N = 3$ so you can just name the trajectories ($2 \times T$ matrices) $p_1$, $p_2$, and $p_3$, and explicitly write out the three (i.e., $N(N-1)/2$) avoidance constraints.
8.2 (6 points) *Final Report Peer Review.* Final project report is due on May 28 (Friday), **11:59pm.** You will update your peer-reviews for the submitted midterm reports and provide a detailed final evaluation. You can submit your reviews through the “Final Report” assignment on Canvas, which will be available after all the reports are submitted.

- Please read the reports in-depth. Summarize the entire report in a short paragraph.
- Please try to give constructive feedback. Point out the strong and weak points. The following are possible points you can comment on: writing style, clarity, technical soundness and experimental evaluation (when applicable).
- Check whether your questions and comments on the midterm report are addressed in the final report.
- Finally, you will assign a tentative score to each final report based on two criteria: clarity/organization and technical content. Please see the grading rubric in the Canvas folder 'Files/Project/Grading', and assign a score from 1 to 5 for each criteria (i.e., two scores per report) with 1 being the lowest and 5 the highest. Provide the justification in your review.

The poster presentations will be on June 3 (virtually via Gather.town) from 10:30am to 12pm (in class).