EE 360 - Multiuser Wireless Systems and Networks - Winter 2014
Homework 2: Due Mar. 10, 5pm.
Up to three people can collaborate on this homework.
(Total: 200 pts.)

February 20, 2014

1. **Relays and power efficiency:** [**20 pts**] Problem 16-1 in the textbook.

2. **Power control:** [**25 pts**] Problem 16-2 in the textbook.

3. **Distributed power control:** [**20 pts**] Problem 16-3 in the textbook.

4. **Number of nodes and connectivity:** [**20 pts**] Problem 16-5 in the textbook.

5. **Delay/throughput in multipath routing:** [**25 pts**] Problem 16-6 in the textbook.

6. **Delay metrics:** [**20 pts**] Problem 16-9 in the textbook.

7. **Gains from cross-layer design:** [**25 pts**] Problem 16-10 in the textbook.

8. **Null space learning in cognitive radios with an interference beacon:** [**25 pts**] Consider the system in Fig. 1. The SU-Tx wants to transmit in the same frequency and time slots with PU-Tx to a secondary receiver without interfering with the transmissions of the primary system. Denote as $\mathbf{H}_{12}$ the interference channel from SU-Tx to PU-Rx, and as $\mathbf{H}_{11}$ the channel from PU-Tx to PU-Rx. The channel at the PU-Rx is thus

$$\mathbf{y}(k) = \mathbf{H}_{11}\mathbf{x}_1(k) + \mathbf{H}_{12}\mathbf{x}_2(k) + \mathbf{v}(k), \tag{1}$$

where $\mathbf{v}$ is additive stationary noise. Consider the case of an SU-Tx with N_T antennas and a PU-Rx with N_R antennas where $N_T > N_R$.
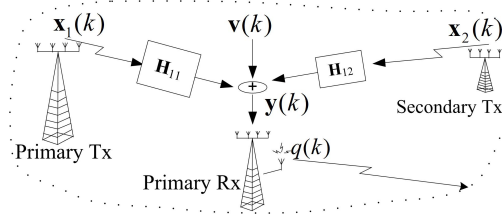

Figure 1: System Model

We divide the learning timeline into Transmission Cycles (TCs), where at TC k, SU-Tx transmits a learning signal $\tilde{\mathbf{x}}(k) \in \mathbb{C}^{N_T}$. By the end of each TC, PU-Rx calculates and broadcasts the following information

$$q(k) \triangleq \alpha\|\mathbf{H}_{12}\tilde{\mathbf{x}}(k)\|^2 + c, \tag{2}$$

where α, c, are constants unknown to the SU-Tx.

The objective of SU-Tx is to estimate the null space of $\mathbf{H}_{12}$, denoted as $\mathcal{N}(\mathbf{H}_{12})$, from $\{\tilde{\mathbf{x}}(k), q(k)\}_{k=1}^N$. Define the TC Complexity $TC(N_T, N_R)$ of a null space learning algorithm to be the minimum number N of TCs required to obtain $\mathcal{N}(\mathbf{H}_{12})$.

Questions:
Consider a random matrix $\mathbf{H}_{12} \in \mathbb{C}^{N_R \times N_T}$, $N_R < N_T$, with N_R rank, where $[H_{1,2}]_{l,m} \sim \mathcal{CN}(0,1)$. We assume that $\mathbf{H}_{12}$ is constant during the learning process.

(a) Show how a perfect knowledge of $\mathcal{N}(\mathbf{H}_{12})$ can be used by the SU-Tx to null the interference to the PU-Rx. How many spatial degrees of freedom are left for SU-Tx to SU-Rx transmission?

(b) Provide a scheme to learn $\mathbf{G} = \alpha \mathbf{H}_{12}^* \mathbf{H}_{12}$ using the feedback scheme specified in (2). What is the TC complexity of your scheme?
   Hint: Use the fact that $\mathbf{G}$ is hermitian and positive definite. The diagonals can be learnt by sending $\tilde{\mathbf{x}}$ vectors with only 1 non zero entry. Each off diagonal can be learnt by sending two $\tilde{\mathbf{x}}s$ with two non zero entries each.

(c) Express the the null space of $\mathbf{H}_{12}$ in terms of the vectors/values in the singular value decomposition of $\mathbf{G}$.

9. **Energy efficiency and multihop routing: [20 pts]** This problem investigates the energy efficiency of multihop routing protocols and the implications of considering circuit energy together with transmit energy. Consider the follow topology of equally spaced nodes, distance 1 unit apart:

The lifetime of the network is defined as the time till the first node runs out of energy. Our goal is to come up with a routing protocol to maximize lifetime. Assume the following:

i. The routing protocol is completely specified by $f_{ij} > 0$ which represents the fraction of time node i transmits to node j. Please also assume that only one transmission is active at a single time (due to the fact that every node can hear each other's transmission).

ii. The *power* spent in transmission from node i to node j is due to both transmit power and receive power: Thus net energy consumed per unit time at node i is

$$\sum_j p_{tx,ij} f_{ij} + \sum_j p_{rx} f_{ji}.$$

Furthermore, the transmit power can be decomposed into

$$p_{tx,ij} = \epsilon_c + \epsilon_t d_{ij}^2,$$

where d_{ij} is the distance between nodes i and j, and the receiver power is

$$p_{rx} = \epsilon_c,$$

for some constants $\epsilon_c, \epsilon_t > 0$.

iii. Nodes 1 to 3 have to send in 0.1 bit per unit time to the AP. That is the net output from each of the non AP nodes is 0.1 bit per unit time.

iv. Nodes 1 to 3 have an individual energy constraint of 50 units each. The AP node has no energy constraint.

Given the above, answer the following:

(a) Formulate the problem of maximizing lifetime as a linear programming problem.
   Hint: Each of the assumptions corresponds to a constraint.

(b) Solve the above problem for

   i. $\epsilon_c = 1, \epsilon_t = 20$,

   ii. $\epsilon_c = 1, \epsilon_t = 0.05$.

   Please submit code used to solve this and the optimal lifetime values that you get.

(c) Based on the above, which routing strategy (singlehop or multihop) would you prefer when the transmit energy dominates? Which strategy would you prefer when the circuit energy dominates? (Please justify.)