EE356 Elementary Plasma Physics H.O. #12

Inan 12 April 2002

Spring 2002

HOMEWORK ASSIGNMENT #2

(due Friday, April 19th)

1. **Short Review Questions:**

   a. *Mobility and conductivity.* An electric field $\mathbf{E}$ causes electrons to drift at a velocity $\mathbf{v} = \mu\mathbf{E}$, while the protons remain stationary. Determine the mobility, current density, and conductivity of the electrons in terms of N_e and q_e, i.e., the electron number density and charge respectively.

   b. *Uniform distribution function.* Bittencourt p. 138, Problem 5.1.

   c. *Constant velocity distribution.* A plasma with density N_0 consists only of particles all of which drift in the $\hat{\mathbf{x}}$ direction with a constant velocity v_x. Write down the distribution function $f(\mathbf{r}, \mathbf{v}, t)$ describing this plasma.

   d. *Magnetic heating of plasma.* A 1-keV proton with $v_\parallel = 0$ in a uniform magnetic field $B = 0.1$ T is accelerated as B is slowly increased to 1 T. The proton subsequently makes an elastic collision with a heavy particle and changes direction so that now $v_\perp = v_\parallel$. The B field is then slowly decreased back to 0.1 T. What is the final energy of the proton?

2. **Plasma distribution function.** Measurements made on a plasma indicate that the electron velocities in the $\hat{\mathbf{x}}$ direction are distributed in a Gaussian manner. Further measurements reveal the same trend for the velocities in the $\hat{\mathbf{y}}$ and $\hat{\mathbf{z}}$ directions, so, armed with EE356 street-smarts, you write down the velocity space distribution function as

$$f(v_x, v_y, v_z) = Ae^{-\frac{mv_x^2}{2k_\mathrm{b}T}} e^{-\frac{mv_y^2}{2k_\mathrm{b}T}} e^{-\frac{mv_z^2}{2k_\mathrm{b}T}}$$

   (a) Noting that the plasma density is N_0, find A. (b) By suitably weighting your distribution function with particle energy (in a manner similar to Equation [1.1] of Lecture 1) find the average energy of the plasma.

3. **Polarization drift.** An electron gyrates in a constant and uniform B-field with $\mathbf{B} = 0.1\hat{\mathbf{z}}$ T and $v_\parallel = 0$. A uniform electric field is slowly introduced into the system, with $\mathbf{E}_1 = 0.1\,t\,\hat{\mathbf{y}}$ V/m. At $t = 0$, the guiding center of the electron is located at the origin. (a) Calculate the displacement of the guiding center of the electron due only to polarization drift at $t = 10$ seconds. (b) What is the displacement of the guiding center at $t = 10$ seconds due to $\mathbf{E} \times \mathbf{B}$ drift? (c) At $t = 20$ seconds, the electric field switches to $\mathbf{E}_2 = \mathbf{E}_1(10) - 0.1\,t\,\hat{\mathbf{y}}$ V/m. Repeat part (a) at $t = 20$ seconds and at $t = 30$ seconds. (d) Repeat part (b) at

$t = 20$ seconds and at $t = 30$ seconds. (e) At $t = 30$ seconds, the electric field switches to $\mathbf{E}_3 = \mathbf{E}_2(30) + 0.1\,t\,\hat{\mathbf{y}}$ V-m^{-1}. Repeat part (a) and (b) at $t = 40$ seconds. (j) At $t = 10$ seconds, what was the work done by the field on each particle? (i.e., $\int q\,\mathbf{E}\cdot d\mathbf{l}$?) and what is the kinetic energy of the $\mathbf{E}\times\mathbf{B}$ drift?

HINT: This exercise is meant to demonstrate that the slowly increasing electric field is teh agent which imparts energy to the particle, which is then stored as kinetic energy in the $\mathbf{E}\times\mathbf{B}$ drift. In fact, it can be shown that the energy picked up due to the polarization drift defined in equation [5.6] of Lecture #5 Notes just suffices to cover the difference in zero-order electric drift energy, due to the change in the electric field.

4. **Conductivity tensor** Apply equation [4.17] from the Lecture #4 Notes to an electron under the influence of a time-harmonic electric field of $\mathbf{E} = E_x\hat{\mathbf{x}} + jE_y\hat{\mathbf{y}}$ where $E_x, E_y > 0$. Find expressions for the particle velocity and sketch its trajectory for (a) $\omega_c \gg \omega$. Does the expression look familiar? (b) for $\omega_c \ll \omega$, and (c) for ω_c slightly larger than ω. How would one analytically determine the velocity of the electron and its trajectory for a general time-varying electric field $\mathbf{E} = E_x(t)\hat{\mathbf{x}} + E_y(t)\hat{\mathbf{y}}$?

5. **Magnetic pumping.** A plasma confined by an axial magnetic field B_z is heated by magnetic pumping using collisional distribution of energy. For this purpose, B_z is increased from B_1 to B_2 in a time Δt_1, maintained at B_2 for a time Δt_2, decreased from B_2 to B_1 in a time Δt_1 and maintained at B_1 for a time Δt_2. This process is repeated periodically. The time interval Δt_1 is large compared to the gyroperiod but short compared to the time required for the plasma to attain thermal equilibrium. The time duration Δt_2 is large compared to that required to establish thermal equilibrium. Show that for each cycle of variation of the magnetic field as described, the plasma temperature increases by the factor

$$\frac{\left[2 + 5B_2/B_1 + 2(B_2/B_1)^2\right]}{9B_2/B_1}$$