EE356 Elementary Plasma Physics H.O. #5

Inan 5 April 2002

Spring 2002

HOMEWORK ASSIGNMENT # 1

(due Friday, April 12th)

1. **Short Review Questions:**

   a. *Metal sphere.* A metal sphere of radius, $r = a$, with charge, Q, is placed in a neutral plasma with number density, N_0 and temperature, T. Calculate is the effective capacitance of the system. Compare this with the capacitance of the same sphere placed in free space.

   b. *Magnetic moment.* Given a background magnetic field, $\mathbf{B} = B_0\,\hat{\mathbf{z}}$, calculate is the magnetic moment of an electron with temperature, T and no velocity component along the direction of the magnetic field? Calculate the magnetic moment for a proton with the same properties? How does the magnetic moment of each affect the ambient field?

   c. *Plasma rocket.* Given a plasma rocket at rest within a space plasma, it turns on its $\mathbf{E}\times\mathbf{B}$ thruster (Both **E** and **B** are constants in space and time) which spits plasma out the back and accelerates the rocket. Plasma enters the rocket through an opening in the nose. Assume the flow of plasma entering is equal to that leaving. What will happen to the rocket?

   d. *Heat and Temperature.* Suppose that on a routine trip around the galaxy, your dog puts his head out of the spacecraft window (as dogs often do). From your reading (p.13 of Bittencourt) you know that the solar wind has a temperature of approximately $10^{4\circ}$K. Would your dog's head burn up immidiately? Explain briefly in the context of temperature versus heat (assume your dog holds his breath during the exercise)

   e. *Plasma criteria.* What are the 4 criteria for an ionized gas to be considered a plasma? Give a brief rationale for each one.

   f. *Debye length in a flame.* Calculate the Debye length for a typical flame, assume: $N_e = 10^{15}\text{m}^{-3}$, $T = 10^{4\circ}$K What is the value of λ_D if T is changed to $10^{3\circ}$K, $10^{2\circ}$K, $10^\circ K$, and in the limit as $T \to 0^\circ$ K ? Explain qualitatively what happens to the Debye length when $T \to 0$.

   g. *Gravitational baseball.* A regulation-sized baseball is placed at an altitude of 1 Earth radius (6370 km) on the magnetic equatorial plane, where the magnitude of the earth's magnetic field is $B = 3.9 \times 10^{-6}$ Teslas. The baseball quickly acquires a charge of -1 C due to free electron attachment. Calculate the drift velocity due to gravitional force.

2. **Metal cylinder.** A metal coaxial cylinder of inner radius, a, and outer radius b, contains a fully ionized gas with number density, N_0. The cylinder is placed within a constant magnetic field, $\mathbf{B}_0 = B_0\,\hat{\mathbf{z}}$ (perpendicular to the radial direction). A voltage V_0 is placed on the inner conductor while the outer conductor is grounded. Assume that gyroradius $r_c \ll (b-a)$, how would the plasma react? Now assume that the gas is only 10 percent ionized and that the collision frequency is ν_c for both electrons and ions. Describe (qualitatively) how the response of the plasma would be different. What would happen if the electrons and ions have different collision frequencies?

3. **Debye length.** An alternative derivation of λ_D provides further insight into its meaning. Consider two infinite, parallel plates located at $x = \pm d$, kept at a potential of $\Phi = 0$. The space between the plates is uniformly filled with a gas of density N of particles of charge q. (a) Using Poisson's equation, show that the potential distribution between the plates is $\Phi(x) = [Nq/(2\epsilon_0)](d^2 - x^2)$. (b) Show that for $d > \lambda_\mathrm{D}$, the energy needed to transport a particle from one of the plates to the mid-point (i.e., $x = 0$) is greater than the average kinetic energy of the particles. (Assume a Maxwellian distribution of particle speeds.)

4. **Particle motion in E and B fields.** Consider two infinite , perfectly conducting plates A_1 and A_2 occupying the planes $y = 0$ and $y = d$, respectively. The potential difference between the plates A_1 and A_2 is positive and is given by V_0. An electron of charge q_e and mass m_e enters the plate A_1 through a small hole and has an initial velocity $\mathbf{v} = v_0\hat{\mathbf{y}}$. (a) Find the minimum value of the potential difference V_0 necessary to prevent the electron from reaching the plate A_2. (b) Suppose that the region between the plaets is permeated uniformly by a static magnetic field of $\mathbf{B} = B_0\hat{\mathbf{z}}$. Let an ion of charge q_i and mass m_i enter with zero initial velocity through a small hole in the plate A_1. As in part (a), the potential difference between plates A_1 and A_2 is positive and is given by V_0. Find the minimum value of B_0 necessary to prevent the ion from reaching the plate A_2.

5. **Magnetic mirror.** We set out to compare two different magnetic field configurations:

   A: Uniform B-field, i.e. $\mathbf{B} = B_0\,\hat{\mathbf{z}}$, and

   B: A 'magnetic mirror' geometry, i.e. $\mathbf{B} = B_0[1 + (z/a_0)^2]\,\hat{\mathbf{z}}$, where $B_0 > 0$ and $a_0 > 0$.

   (a) For a charged particle of mass m and charge q calculate $v_{||}(z)$ for Cases A and B. [HINT: Assume that the particle mirrors at $z = \pm z_m$ and remember that $\mu = \frac{1}{2}mv_\perp^2/|B|$ is conserved, as is the total energy of the particle.] (b) Given that the average force acting on the particle guiding center is given by

   $$< F_{||}(z) >= -|m|\frac{\partial B}{\partial z}\hat{\mathbf{z}}$$

   calculate $< F_{||}(z) >$ for Cases A and B and qualitatively describe what this force tends to do in each case as the guiding center of a particle moves along the z-axis. (c) With the force you have calculated for Case B above, show that the guiding center of a particle executes harmonic motion along the z-axis. Calculate the period of this harmonic motion. This type of motion is the underlying principle of magnetic trapping of energetic particles in a magnetic mirror system.

6. **HVDC line.** High Voltage Direct Current (HVDC) transmission lines are a new development in transmission line technology and are capable of carrying large amounts of electrical power. Suppose a neutral hydrogen atom travels vertically down with a velocity of 10^5 m-s^{-1} until it is horizontally level with a HVDC line. At this point it splits into a positive ion and an electron both travelling with the same inital velocity and direction as the original atom. The HVDC line is a typical one, carrying 2800 A at 250 kV. (a) Calculate the gyroradius, r_c, and gyrofrequency, f_c, of the electron and ion. (b) Calculate ∇BB and compare to $1r_c$ for both electron and ion. Comment on the relationship of these two terms, i.e why do we compare them? (c) Calculate $E \times B$, gradient and curvature drifts for the electron. Why don't we do this for the ion?

7. **Charged particle near a current carrying wire.** Consider a particle of charge q and mass m moving in the neighborhood of an infinitely long straight filamentary wire carrying a constant current I and extended along the z-axis. At time t = 0, the particle is at z = 0 and at a radial distance from the wire of r = r_0, and has a velocity $\mathbf{v} = v_0\hat{\mathbf{z}}$ parallel to the wire. (a) Determine and plot the trajectory (in other words the spatial path followed by the particle as time progresses) of the particle. (b) Assuming that the magnitude of the current to be such that $q\,\mu_0 I$ = $m\,2\pi$, r_0 = 1 m, and v_0 = 0.5 m-s^{-1}, find the location of the particle(i.e., numerical values of its coordinates r and z) at t = 2 s.