Information Theory

EE 276

INSTRUCTOR

Tsachy Weissman

TA

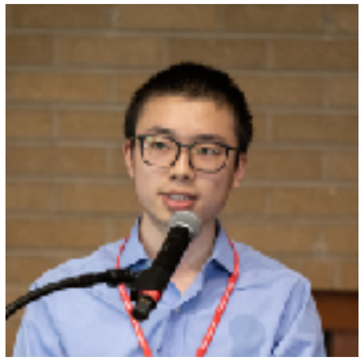
Divija Hasteer

TA

Jiwon Jeong

TA

Yifan Zhu



goal

- expose the elements, beauty and utility of the science of information (and, specifically, information theory)
- information scientific thinking (seeing the world through the lens of information)
- whet your appetite for subsequent learning

it's all information

- A book you write
- Ship of Theseus
- You

what is information?

information

noun | in·for·ma·tion | \ˌin-fər-ˈmā-shən\

Popularity: Top 1% of lookups | Updated on: 3 Sep 2018

TRENDING NOW: hirsute op-ed collegiality mistrial hogwash SEE ALL ›

Tip: Synonym Guide | Examples: INFORMATION in a Sentence

Definition of INFORMATION

1 : the communication or reception of knowledge or intelligence

2 a (1) : knowledge obtained from investigation, study, or instruction (2) : INTELLIGENCE, NEWS (3) : FACTS, DATA
b : the attribute inherent in and communicated by one of two or more alternative sequences or arrangements of something (such as nucleotides in DNA or binary digits in a computer program) that produce specific effects
c (1) : a signal or character (as in a communication system or computer) representing data (2) : something (such as a message, experimental data, or a picture) which justifies change in a construct (such as a plan or theory) that represents physical or mental experience or another construct
d : a quantitative measure of the content of information; *specifically* : a numerical quantity that measures the uncertainty in the outcome of an experiment to be performed

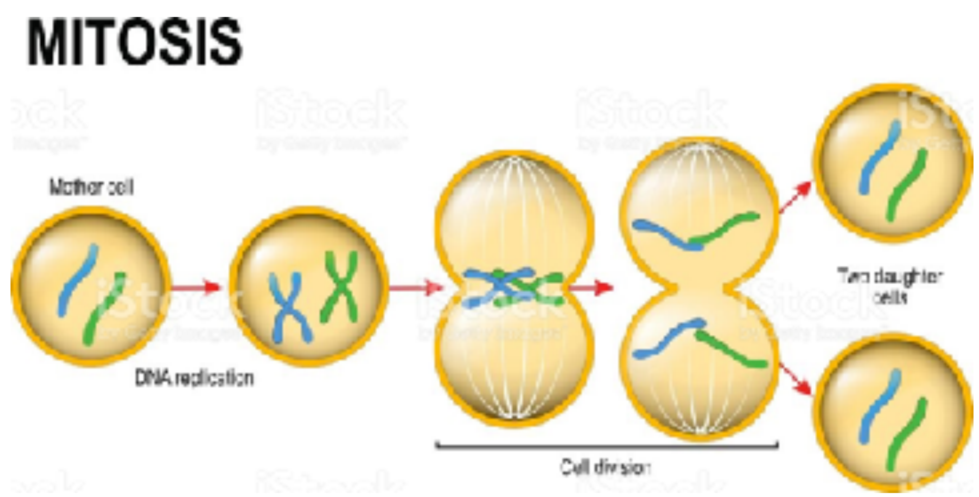
3 : the act of informing against a person

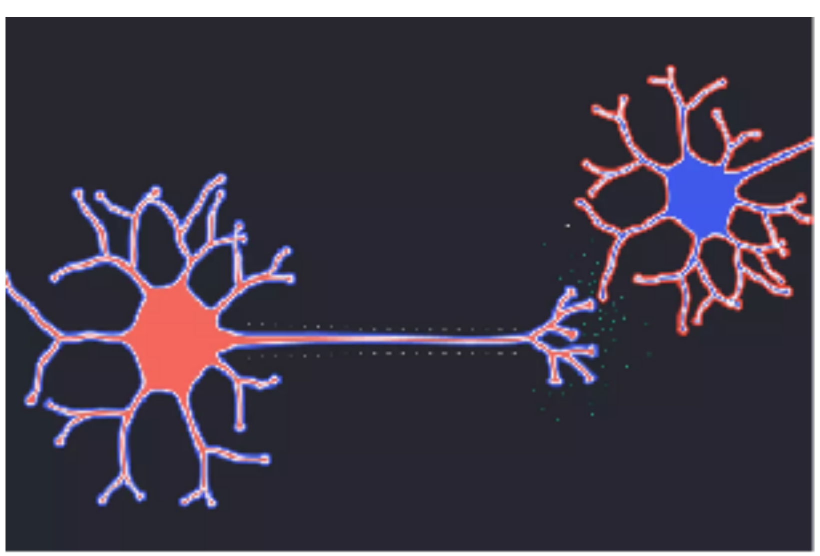
4 : a formal accusation of a crime made by a prosecuting officer as distinguished from an indictment presented by a grand jury

—informational \ˌin-fər-ˈmā-shnəl, -shə-nᵊl\ *adjective*

—informationally *adverb*

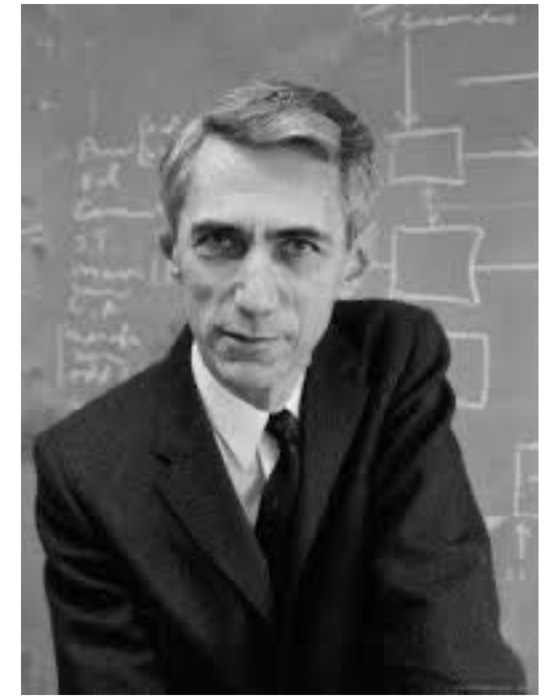
what is communication?

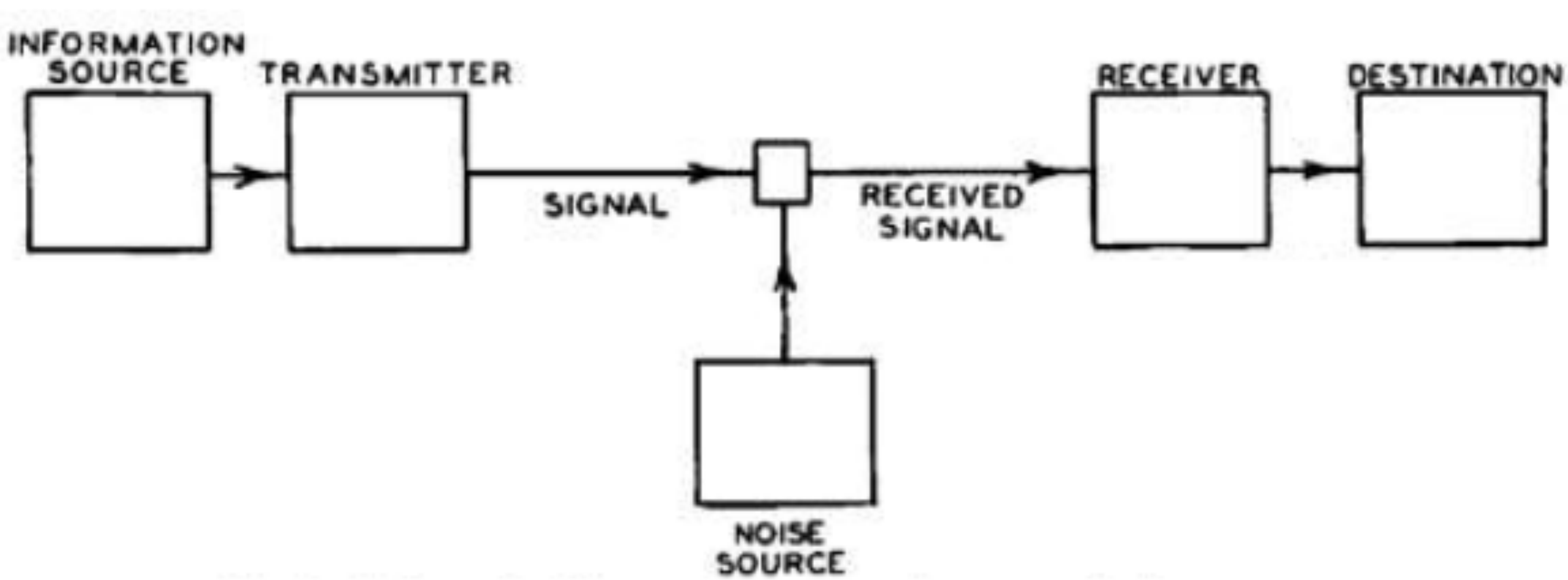


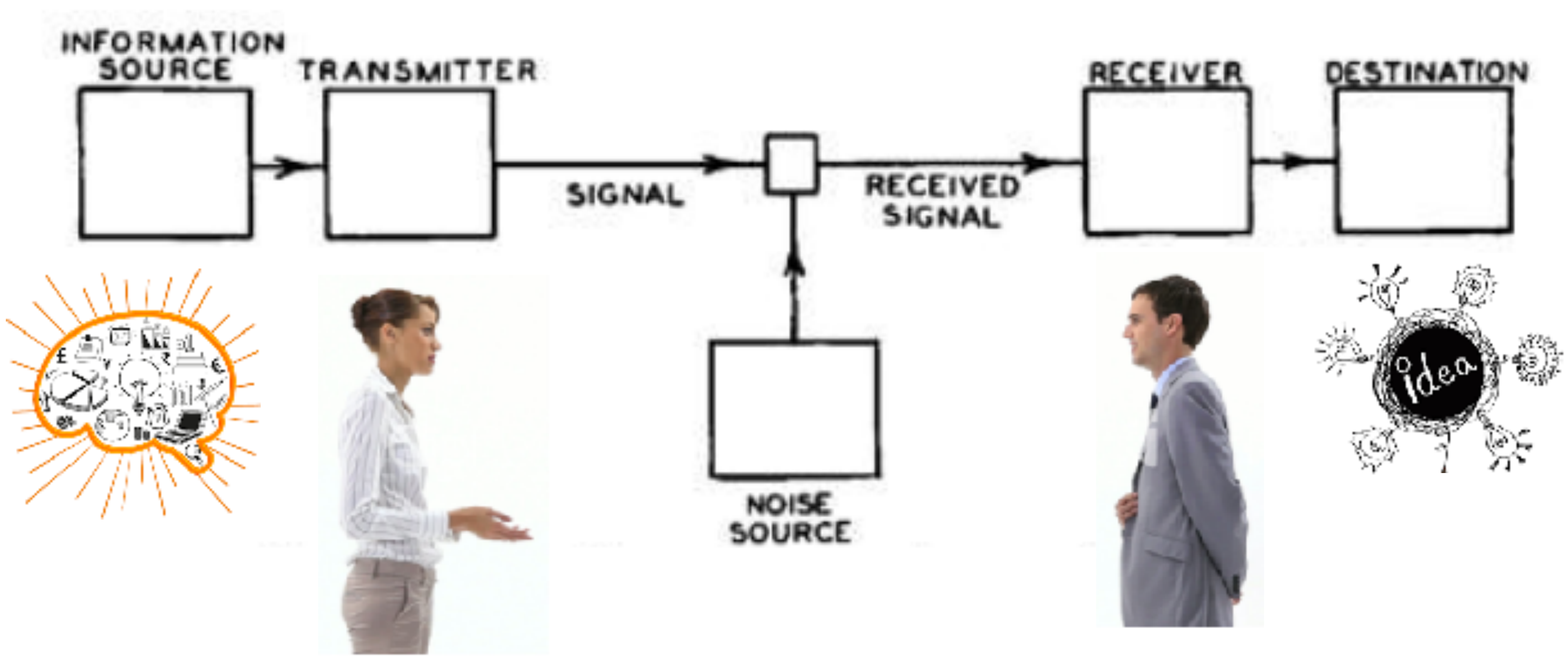


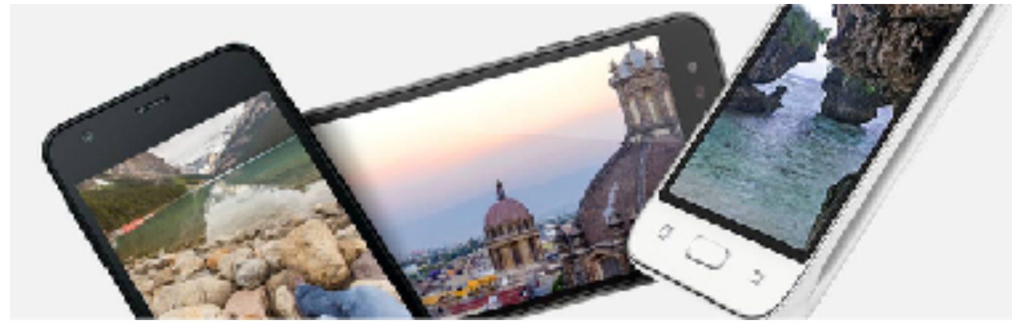
Claude Elwood Shannon
1948

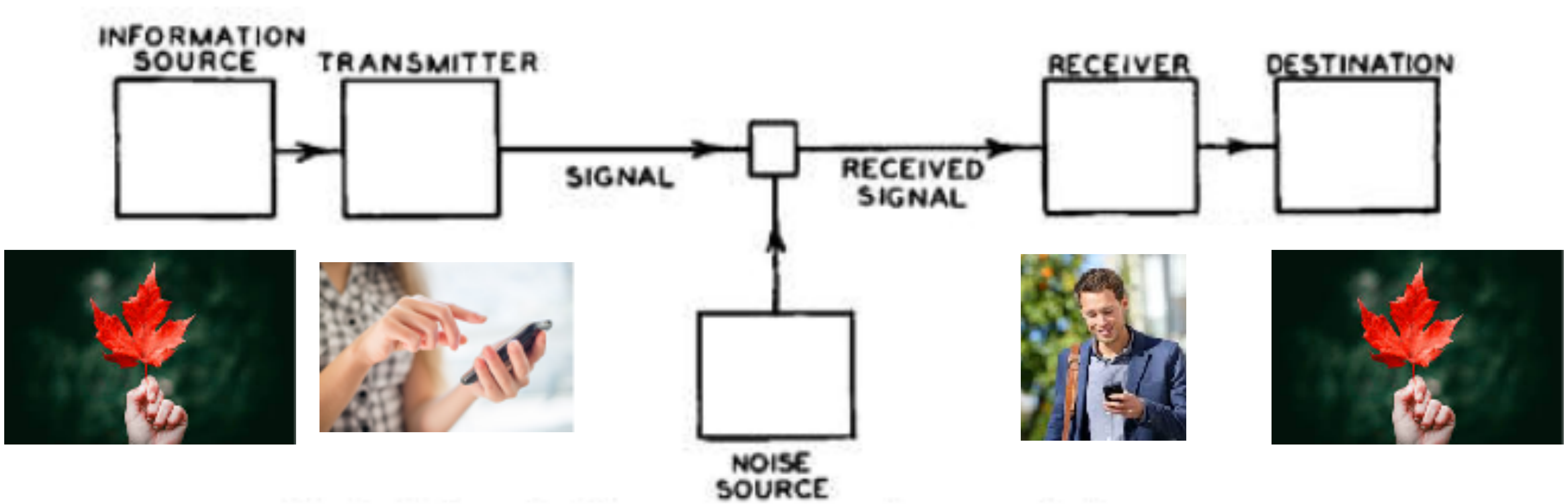
“A Mathematical Theory of Communication”






INFORMATION SOURCE
TRANSMITTER
SIGNAL
RECEIVED SIGNAL
RECEIVER
DESTINATION
NOISE SOURCE
idea

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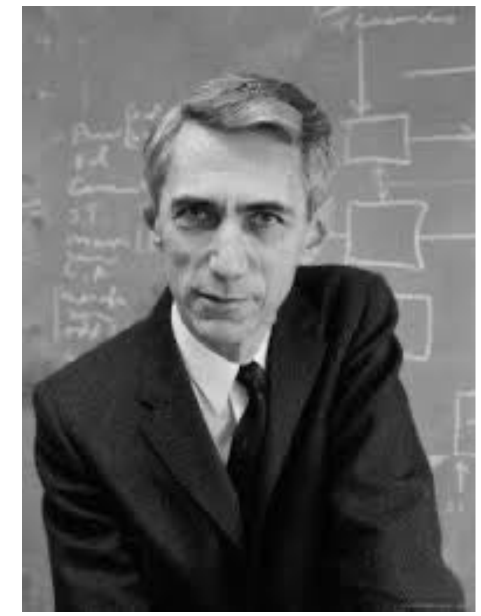
Shannon’s genius: I

- the question
- the answer

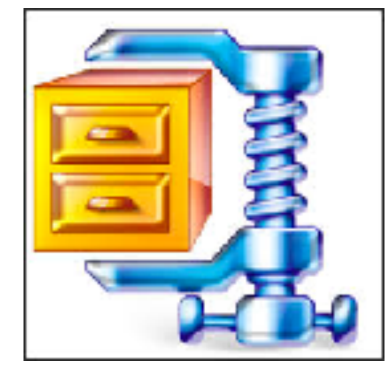
a bit about the bit

0 **or** **1**

in other words: digitization!



2 pillars of the science of information



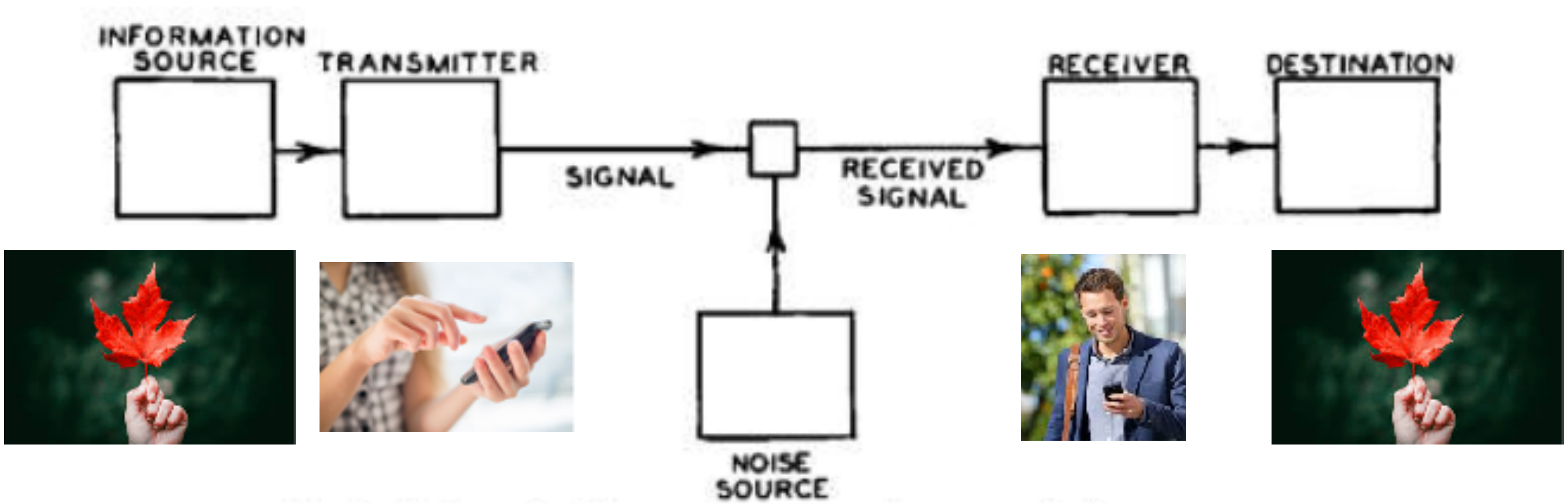
- succinct representation of in **bits** (compression)

- effective and reliable communication of **bits** (across unreliable media)

Shannon discovered the two,
showed reliable communication of bits is generally possible,
and that combining the two is optimal

Shannon's genius: II

- Characterizing what is the best that can be achieved with bits
- Showing that bits can be communicated reliably over unreliable channels
- Showing that this bit paradigm is optimal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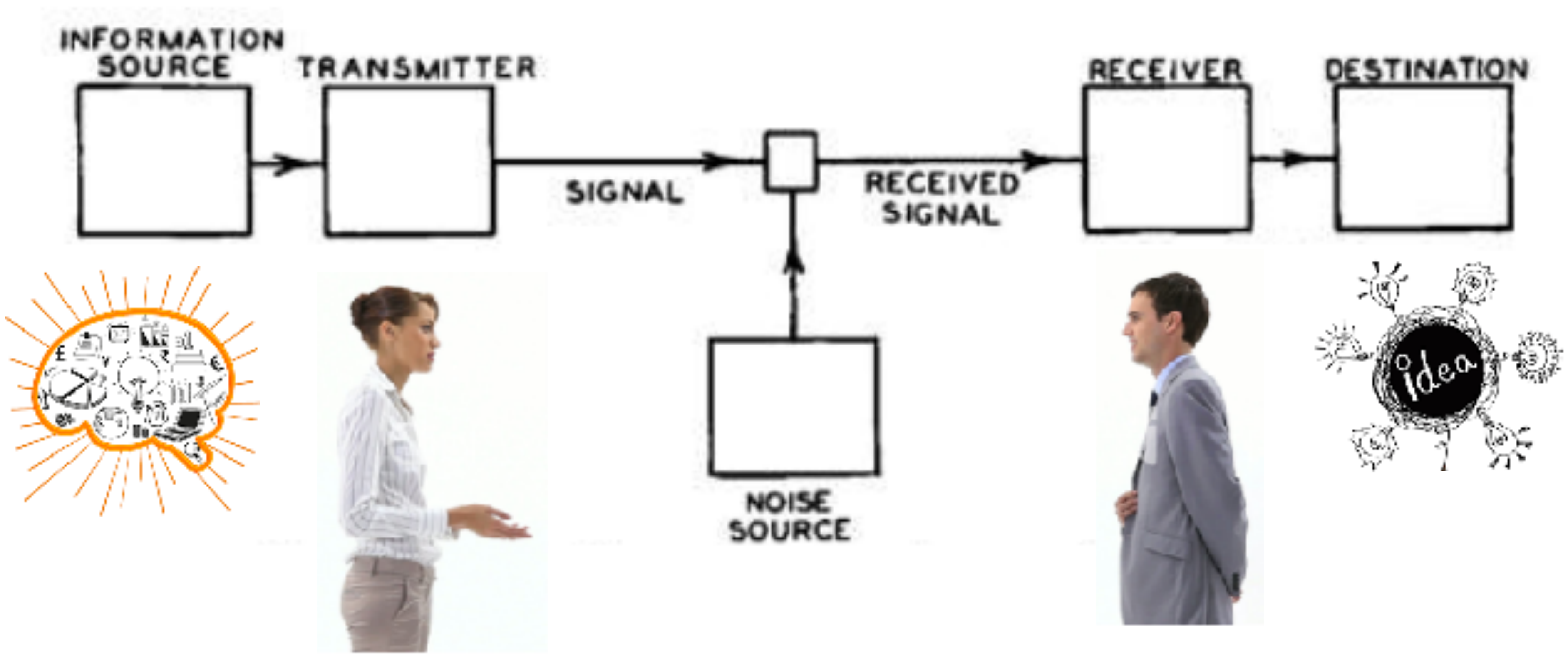

INFORMATION SOURCE
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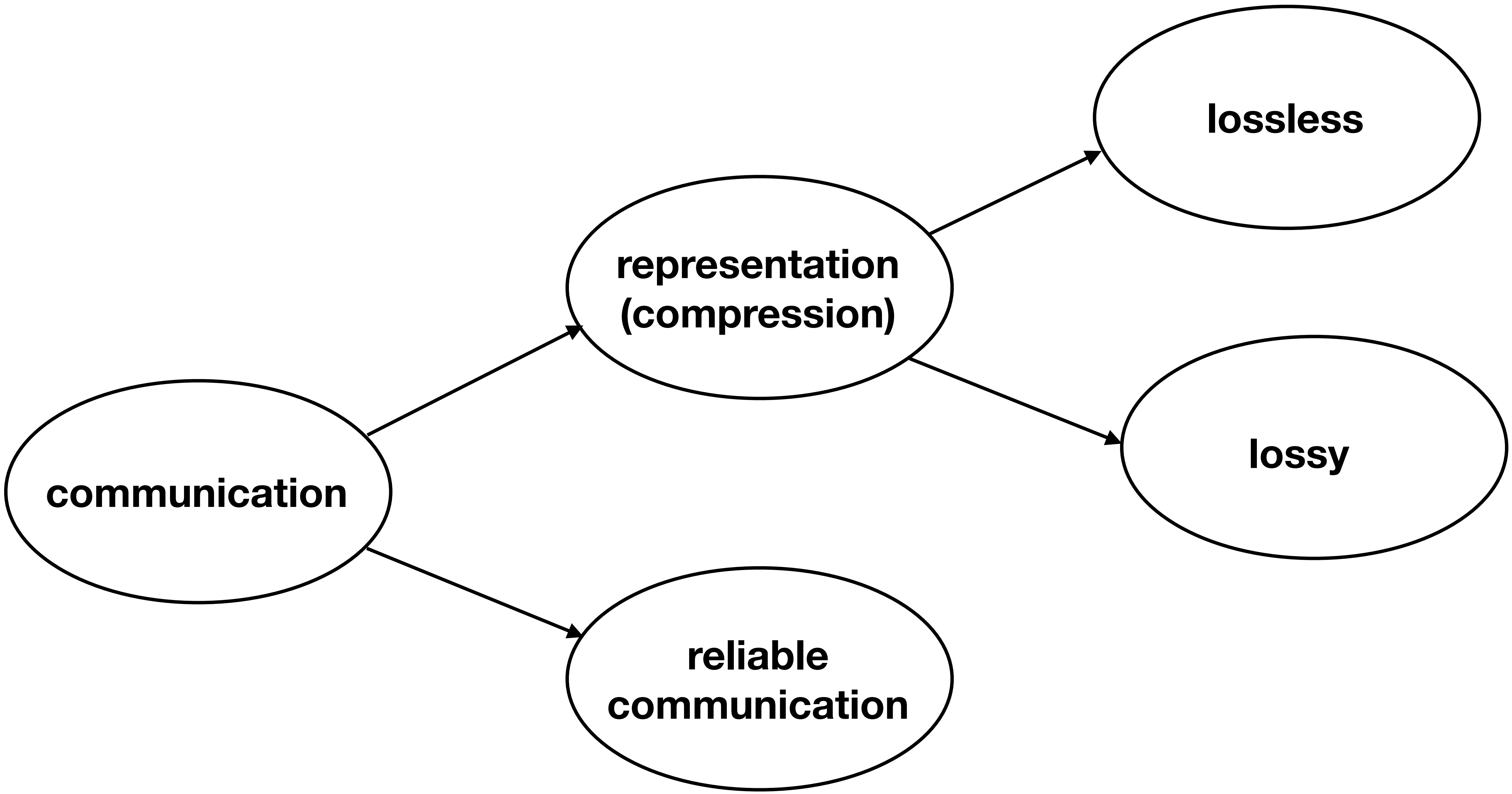
NOISE SOURCE
idea

and everything else

- neurons
- genetics/genomics
- language
- essentially all our technologies for: storage, communication, streaming, computation, …
- etc.

course theme I: communication



course theme II: concrete schemes

- Shannon
- Huffman
- Arithmetic
- Lempel-Ziv (GZIP)
- JPEG
- Polar codes for reliable communication (5G)

course theme III: measures of information

- entropy
- relative entropy
- mutual information
- Shannon capacity
- rate-distortion function

approximate lecture schedule

- Introduction and motivating examples
- Information measures: entropy, relative entropy and mutual information
- AEP and typicality
- Variable length lossless compression: prefix and Shannon codes
- Kraft inequality and Huffman coding
- Lempel Ziv compression
- Reliable communication and channel capacity
- Information measures for continuous random variables
- AWGN channel
- Joint AEP and Channel coding theorem
- Channel coding theorem converse
- Polar codes
- Lossy compression and rate-distortion theory
- Method of types and Sanov's theorem
- Strong, conditional and joint typicality
- Direct and converse in rate distortion theorem
- **Joint source-channel coding and the separation theorem**
- Distributed compression and multi-terminal information theory
- A bit about relations to other areas, quantum information theory, etc. if time remains

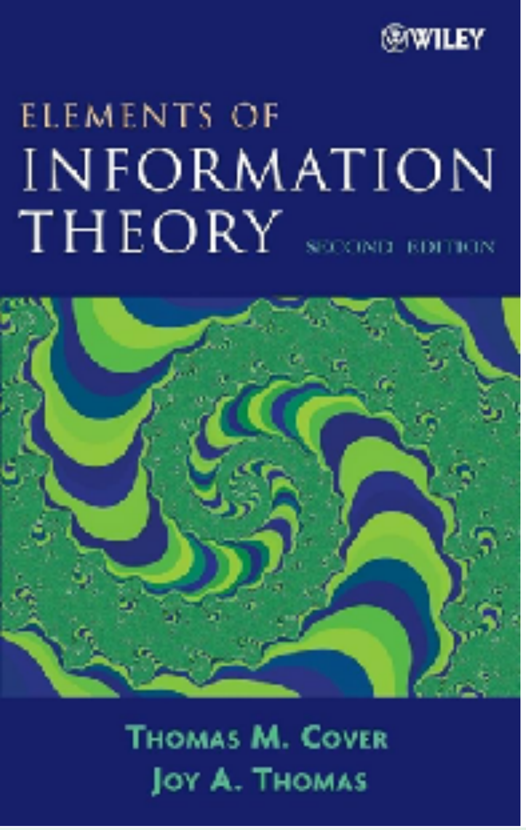
course elements

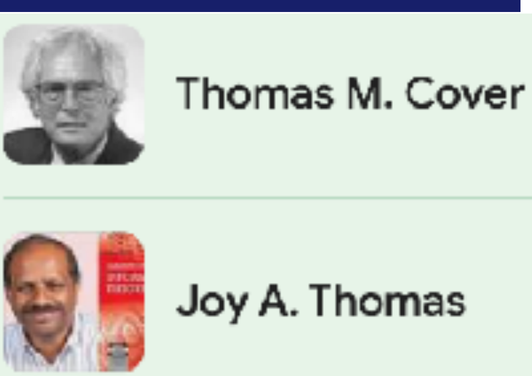
- lectures (Tue, Thu, 12:00-1:20pm)
- HW (6pm Fridays, submitted on Gradescope)
- recitations (please fill in survey form on Ed by tomorrow)
- midterm (Friday, February 7th, 5-7pm)
- final (Friday, March 21st, 12:15-3:15pm)

re the lectures and material

- formal prereq.: first course in probability
- you’ll be held ‘accountable’ only to material covered in lectures and HWs
- course website will provide additional resources, including videos of lectures from previous years, lectures and material from EE274, and a book
- parts of these will be referred to for further reading/ viewing

book





Abbas El Gamal

- 3rd edition close to completion
- By Prof. Abbas El Gamal
- Substantial revision, distillation and modernization of the material
- We are giving you access
- Please keep to yourselves
- Prof. El Gamal will appreciate your comments, suggestions, typo catches, etc. (up to 5 bonus points..)

grade elements

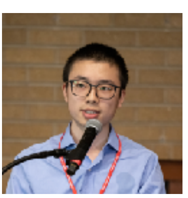
- HW: 20%
- midterm: 35%
- final: 45%
- up to 5% bonus for feedback on book

staff

- Instructor: Tsachy Weissman, OH Thursdays 1:30-3pm or by appointment (starting next week)

- TA: Divija Hasteer

- TA: Jiwon Jeong



- TA: Yifan Zhu

- course supporter: Abhiram Rao Gorle
- details including emails, office hours, etc. on the course website
- Gradescope and Ed for the course accessible via website

questions?

example I: lossless compression of a ternary source

$$\text{Source is } U_1, U_2, \cdots \overset{\text{i.i.d}}{\sim} U \in \mathcal{U} = \{A, B, C\}$$

$$P(U = A) = 0.7, \qquad P(U = B) = 0.15,\ P(U = C) = 0.15$$

how can/should we represent the source succinctly with bits?

first code suggestion:

$$A \to \text{`0'}$$
$$B \to \text{`10'}$$
$$C \to \text{`11'}$$

Let $\bar{L}$ denote the average number of bits per symbol. For the coding above,

$$\bar{L} = 0.7 \times 1 + 0.15 \times 2 + 0.15 \times 2 = 1.3 \text{ bits/symbol}$$

note how easily we can decode, e.g.:

$$0011010011101011$$

(thanks to the "prefix condition" satisfied by this code)

second code suggestion:

pair	probability	Code word	Num. Bits Used
AA	0.49	0	1
AB	0.105	100	3
AC	0.105	111	3
BA	0.105	101	3
CA	0.105	1100	4
BB	0.0225	110100	6
BC	0.0225	110101	6
CB	0.0225	110110	6
CC	0.0225	110111	6

$$\bar{L} = \frac{1}{2}\left(0.49 \times 1 + 0.105 \times 3 \times 3 + 0.105 \times 4 + 0.0225 \times 6 \times 4\right)$$
$$= 1.1975 \text{ bits/symbol}$$

we'll see:

source "entropy":

$$H(U) = \sum_{u \in \mathcal{U}} p(u) \log_2 \frac{1}{p(u)} \simeq 1.1829$$

"converse" result: for any compressor $$H(U) \leq \bar{L}$$

"direct" result: for any eps>0 there exists a compressor satisfying

$$\bar{L} \leq H(U) + \epsilon$$

example ii: binary source and channel

Source: $\mathbb{U} = \{U_1, U_2, ...\}$ where $Pr[U_i = 0] = Pr[U_i = 1] = \frac{1}{2}$. The U_i's are i.i.d.

Channel: The channel flips each bit given to it with probability $q < \frac{1}{2}$. We define the channel input to be $\mathbb{X} = \{X_i\}$, the channel noise to be $\mathbb{W} = \{W_i\}$ and the channel output to be $\mathbb{Y} = \{Y_i\}$ such that:

$$W_i \sim Ber(q)$$
$$Y_i = X_i \oplus_2 W_i$$

The W_i are i.i.d. and the X_i are functions of the input source sequence $\mathbb{U}$.

Probability of error per source bit: P_e

encoding scheme 1:

trivial encoding: $$X_i = U_i$$

yields: $$P_e = q$$

the ***rate*** of this scheme is 1 information bits/channel use

Encoding Scheme 2: We can repeat each source bit three times:

$$\mathbb{U} = 0\ 1\ 1\ 0\ \ldots$$
$$\mathbb{X} = 0\ 0\ 0\ 1\ 1\ 1\ 1\ 1\ 1\ 0\ 0\ 0\ \ldots$$

$$P_e = 3q^2(1-q) + q^3 < q$$

the rate of this scheme is 1/3 information bits/channel use can repeat K times (repetition coding)

as K grows we'll get:

arbitrarily small Pe

at the cost of vanishing rate

Shannon 1948: $\exists\ R > 0$ and schemes with rate $\geq R$ satisfying $P_e \to 0$

C = "Channel Capacity" = largest such R

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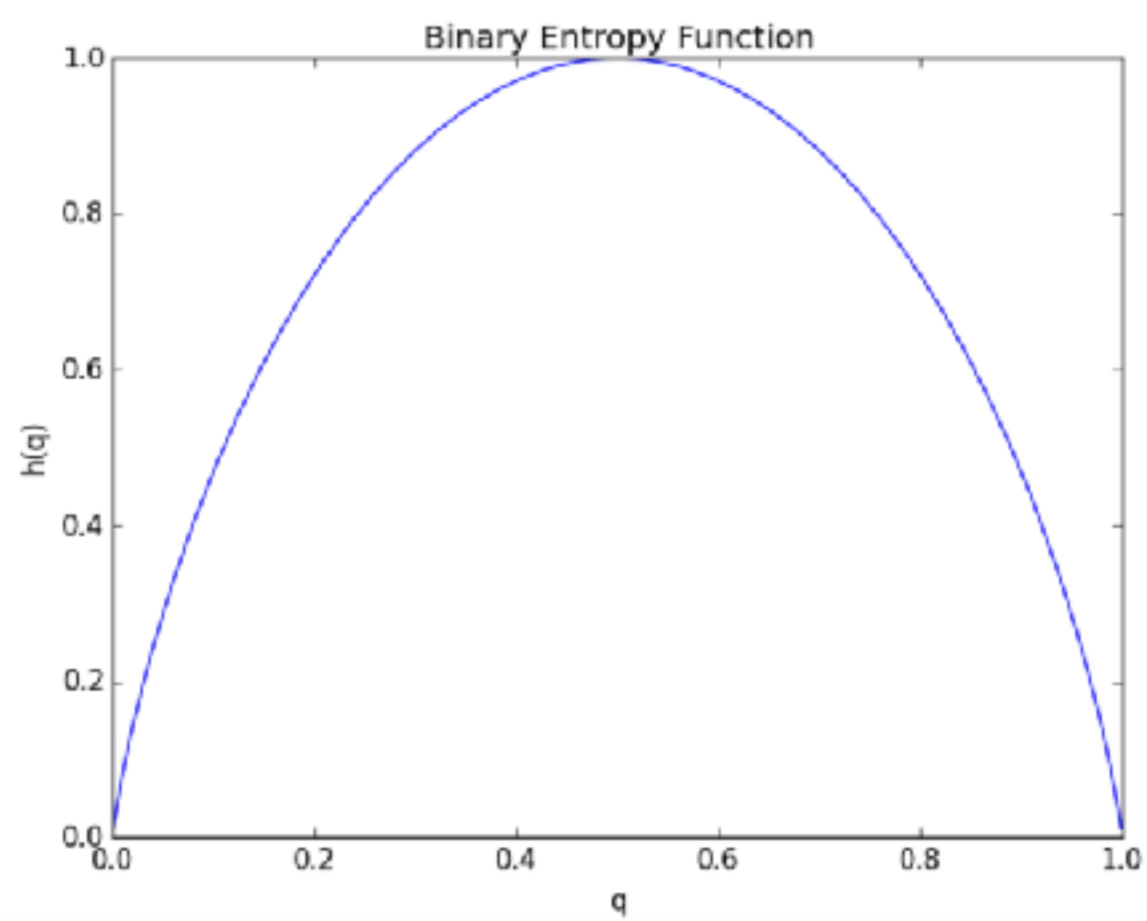
C = "Channel Capacity" = largest such R

in our example:

$$C(q) = 1 - h(q)$$

$$h(q) \triangleq H(Ber(q)) = q \log \frac{1}{q} + (1-q) \log \frac{1}{1-q}$$

The figure below plots $h(q)$ for $q \in [0, 1]$.



Shannon 1948: $\exists\, R > 0$ and schemes with rate $\geq R$ satisfying $P_e \to 0$

C = “Channel Capacity” = largest such R

Here too we’ll see:

a “converse” part:
no scheme can communicate reliably
at a rate above C(q)

a “direct” part:
for any rate below C(q), there exist
schemes that can communicate
reliably at that rate

practical schemes that deliver on the promise

questions?