Inertial Measurement Units I

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EE 267 Virtual Reality
Lecture 9

stanford.edu/class/ee267/
Lecture Overview

- coordinate systems (world, body/sensor, inertial, transforms)
- overview of inertial sensors: gyroscopes, accelerometers, and magnetometers
- gyro integration aka *dead reckoning*
- orientation tracking in *flatland*
- pitch & roll from accelerometer
- overview of VRduino
primary goal: track orientation of head or other device

orientation is the rotation of device w.r.t. world/earth or inertial frame

rotations are represented by Euler angles (yaw, pitch, roll) or quaternions
• orientation tracked with IMU models relative rotation of sensor/body frame in world/inertial coordinates

• example: person on the left looks up \( \rightarrow \) pitch=90° or rotation around x-axis by 90°

• similarly, the world rotates around the sensor frame by -90° (inverse rotation)
from lecture 2:

vertex in clip space

\[ \mathbf{v}_{\text{clip}} = M_{\text{proj}} \cdot M_{\text{view}} \cdot M_{\text{model}} \cdot \mathbf{v} \]
from lecture 2:

vertex in clip space

\[ v_{\text{clip}} = M_{\text{proj}} \cdot M_{\text{view}} \cdot M_{\text{model}} \cdot v \]

projection matrix    view matrix    model matrix
from lecture 2:

\[ v_{\text{clip}} = M_{\text{proj}} \cdot M_{\text{view}} \cdot M_{\text{model}} \cdot v \]

vertex in clip space

projection matrix  
view matrix  
model matrix

rotation  
translation

\[ M_{\text{view}} = R \cdot T(-\text{eye}) \]
from lecture 2:

vertex in clip space

\[ v_{\text{clip}} = M_{\text{proj}} \cdot M_{\text{view}} \cdot M_{\text{model}} \cdot v \]

projection matrix

view matrix

model matrix

rotation

translation

\[ M_{\text{view}} = R \cdot T(-\text{eye}) \]

view matrix for stereo camera:

\[ M_{\text{view}}^{\text{stereo}} = T\left(\pm \frac{ipd}{2},0,0\right) \cdot R \cdot T(-\text{eye}) \]
from lecture 2:

vertex in clip space

\[ \mathbf{v}_{\text{clip}} = \mathbf{M}_{\text{proj}} \cdot \mathbf{M}_{\text{view}} \cdot \mathbf{M}_{\text{model}} \cdot \mathbf{v} \]

projection matrix

view matrix

model matrix

rotation

translation

\[ \mathbf{M}_{\text{view}} = \mathbf{R} \cdot \mathbf{T} (\mathbf{-eye}) \]

sensor/body frame

world/inertial frame
\[ M_{\text{view}} = R \cdot T(-\text{eye}) \]

\[ R = R_z(-\theta_z) \cdot R_x(-\theta_x) \cdot R_y(-\theta_y) \]

Order of rotations (world to body)
• this representation for a rotation is known as **Euler angles**

• need to specify order of rotation, e.g. yaw-pitch-roll

\[
R = R_z(-\theta_z) \cdot R_x(-\theta_x) \cdot R_y(-\theta_y)
\]

order of rotations (world to body)
ATTENTION!

- Euler angles are usually a terrible idea for orientation tracking with more than 1 axis

- one of several reasons: rotations are not commutative

\[
R = R_z(-\theta_z) \cdot R_x(-\theta_x) \cdot R_y(-\theta_y)
\]

order of rotations (world to body)
What do Inertial Sensors Measure?

- gyroscope measures angular velocity $\tilde{\omega}$ in degrees/sec
- accelerometer measures linear acceleration $\tilde{a}$ in m/s$^2$
- magnetometer measures magnetic field strength $\tilde{m}$ in uT (micro Tesla) or Gauss $\rightarrow$ 1 Gauss = 100 uT
What do Inertial Sensors Measure?

- Gyroscope measures angular velocity $\omega$ in degrees/sec.
- Accelerometer measures linear acceleration $\tilde{a}$ in m/s$^2$.
- Magnetometer measures magnetic field strength $\tilde{m}$ in uT (micro Tesla) or Gauss $\rightarrow$ 1 Gauss = 100 uT.

**ALL MEASUREMENTS TAKEN IN SENSOR/BODY COORDINATES!**
History of Gyroscopes

• critical for inertial measurements in ballistic missiles, aircrafts, drones, the mars rover, pretty much anything that moves!

WWII era gyroscope used in the V2 rocket
MEMS Gyroscopes

- today, we use microelectromechanical systems (MEMS)

Coriolis Force
MEMS Gyroscope
Gyroscopes

• gyro model: \( \tilde{\omega} = \omega + b + \eta \)
Gyroscopes

- gyro model: \( \tilde{\omega} = \omega + b + \eta \)
  \[ \eta \sim N\left(0, \sigma_{\text{gyro}}^2\right) \]
  true angular velocity
  additive, zero-mean Gaussian noise
  bias
Gyroscopes

- gyro model: \( \tilde{\omega} = \omega + b + \eta \quad \eta \sim N\left(0, \sigma_{\text{gyro}}^2\right) \)
  - true angular velocity
  - additive, zero-mean Gaussian noise
  - bias

- 3 DOF = 3-axis gyros that measures 3 orthogonal axes, assume no crosstalk
- bias is temperature-dependent and may change over time; can approximate as a constant
- additive measurement noise
Gyroscopes

• from gyro measurements to orientation – use Taylor expansion

\[ \theta(t + \Delta t) \approx \theta(t) + \frac{\partial}{\partial t} \theta(t) \Delta t + \varepsilon, \quad \varepsilon \sim O(\Delta t^2) \]
Gyrosopes

- from gyro measurements to orientation – use Taylor expansion

\[
\theta(t + \Delta t) \approx \theta(t) + \frac{\partial \theta(t)}{\partial t} \Delta t + \varepsilon, \quad \varepsilon \sim O(\Delta t^2)
\]

**have**: angle at last time step

**want**: angle at current time step

**have**: gyro measurement (angular velocity)

**have**: time step

**approximation error!**
Gyro Integration: linear motion, no noise, no bias
Gyro Integration: linear motion, noise, no bias
Gyro Integration: linear motion, no noise, bias
Gyro Integration: **nonlinear** motion, no noise, no bias
Gyro Integration: **nonlinear** motion, **noise**, **bias**
Gyro Integration aka *Dead Reckoning*

- works well for linear motion, no noise, no bias = unrealistic

- even if bias is know and noise is zero $\rightarrow$ drift (from integration)

- bias & noise variance can be estimated, other sensor measurements used to correct for drift (sensor fusion)

- accurate in short term, but not reliable in long term due to drift
Dead Reckoning for Ship Navigation

- can measure north with compass, ship’s speed, and time
- initial position known

map from game pirates!
Dead Reckoning for Ship Navigation

- can measure north with compass, ship’s speed, and time
- initial position known
- problem: drift! (similar to that observed in gyro integration)
Gyro Advice

Always be aware of what units you are working with, degrees per second v radians per second!
Accelerometers

• measure linear acceleration $\ddot{a} = a^{(g)} + a^{(l)} + \eta$, $\eta \sim N\left(0,\sigma_{acc}^2\right)$

• without motion: read noisy gravity vector $a^{(g)} + \eta$ pointing UP! with magnitude $9.81 \text{ m/s}^2 = 1\text{g}$

• with motion: combined gravity vector and external forces $a^{(l)}$
capacitive plates

spring

proof mass
MEMS Accelerometer
Accelerometers

- advantages:
  - points up on average with magnitude of 1g
  - accurate in long term because no drift and the earth’s center of gravity (usually) doesn’t move

- problem:
  - noisy measurements
  - unreliable in short run due to motion (and noise)

- complementary to gyro measurements!
Accelerometers

- fusing gyro and accelerometer data = 6 DOF sensor fusion

- can correct tilt (i.e., pitch & roll) only – no information about yaw
Orientation Tracking in *Flatland*

- problem: track angle $\theta$ in 2D space
- sensors: 1 gyro, 2-axis accelerometer
- goal: understand sensor fusion
Orientation Tracking in *Flatland*

- gyro integration via Taylor series as

$$\theta_{gyro}^{(t)} = \theta_{gyro}^{(t-1)} + \tilde{\omega} \Delta t$$

- get $\Delta t$ from microcontroller
- set $\theta_{gyro}^{(0)} = 0$

- biggest problem: drift!
Orientation Tracking in *Flatland*

- angle from accelerometer

\[ \theta_{acc} = \tan^{-1}\left( \frac{\tilde{a}_x}{\tilde{a}_y} \right) \]
Orientation Tracking in *Flatland*

- angle from accelerometer

\[
\theta_{\text{acc}} = \tan^{-1}\left(\frac{\tilde{a}_x}{\tilde{a}_y}\right) = \text{atan2}(\tilde{a}_x, \tilde{a}_y)
\]

handles division by 0 and proper signs, provided by most programming languages

\[\text{VRduino} \quad \text{IMU} \quad \text{y}^{(\text{world})} \quad \text{x}^{(\text{world})}\]
Orientation Tracking in *Flatland*

- angle from accelerometer

\[ \theta_{acc} = \tan^{-1}\left( \frac{\tilde{a}_x}{\tilde{a}_y} \right) = \text{atan2}(\tilde{a}_x, \tilde{a}_y) \]

- biggest problem: noise
Orientation Tracking in *Flatland*

- sensor fusion: combine gyro and accelerometer measurements

- intuition:
  - remove drift from gyro via high-pass filter
  - remove noise from accelerometer via low-pass filter
Orientation Tracking in *Flatland*

- sensor fusion with complementary filter, i.e. linear interpolation

\[
\theta^{(t)} = \alpha \left( \theta^{(t-1)} + \tilde{\omega} \Delta t \right) + (1 - \alpha) \text{atan2} \left( \tilde{a}_x, \tilde{a}_y \right)
\]

- no drift, no noise!
Orientation Tracking in *Flatland*

- gyro (dead reckoning)
- accelerometer
- complementary filter
Pitch and Roll from 3-axis Accelerometer

• problem: estimate pitch and roll angles in 3D, from 3-axis accelerometer

• together, pitch & roll angles are known as *tilt*

• goal: understand tilt estimation in 3D
**Pitch and Roll from 3-axis Accelerometer**

- use only accelerometer data – can estimate pitch & roll, not yaw
- assume no external forces (only gravity) – acc is pointing UP!

normalize gravity vector in inertial coordinates

\[
\hat{a} = \frac{\tilde{a}}{||\tilde{a}||} = R \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} = R_z(-\theta_z) \cdot R_x(-\theta_x) \cdot R_y(-\theta_y) \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}
\]

normalize gravity vector rotated into sensor coordinates
Pitch and Roll from 3-axis Accelerometer

• use only accelerometer data – can estimate pitch & roll, not yaw
• assume no external forces (only gravity) – acc is pointing UP!

\[ \hat{a} = \frac{\tilde{a}}{||\tilde{a}||} = R \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} = R_z(-\theta_z) \cdot R_x(-\theta_x) \cdot R_y(-\theta_y) \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \]

\[
= \begin{pmatrix} \cos(-\theta_z) & -\sin(-\theta_z) & 0 \\ \sin(-\theta_z) & \cos(-\theta_z) & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos(-\theta_x) & -\sin(-\theta_x) \\ 0 & \sin(-\theta_x) & \cos(-\theta_x) \end{pmatrix} \begin{pmatrix} \cos(-\theta_y) & 0 & \sin(-\theta_y) \\ 0 & 1 & 0 \\ -\sin(-\theta_y) & 0 & \cos(-\theta_y) \end{pmatrix} \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}
\]
Pitch and Roll from 3-axis Accelerometer

- use only accelerometer data – can estimate pitch & roll, not yaw
- assume no external forces (only gravity) – acc is pointing UP!

\[
\hat{a} = \frac{\tilde{a}}{||\tilde{a}||} = \begin{pmatrix} -\cos(-\theta_x)\sin(-\theta_z) \\ \cos(-\theta_x)\cos(-\theta_z) \\ \sin(-\theta_x) \end{pmatrix}
\]
Pitch and Roll from 3-axis Accelerometer

- use only accelerometer data – can estimate pitch & roll, not yaw
- assume no external forces (only gravity) – acc is pointing UP!

\[
\hat{a} = \frac{\tilde{a}}{|\tilde{a}|} = \begin{pmatrix}
-\cos(-\theta_x)\sin(-\theta_z) \\
\cos(-\theta_x)\cos(-\theta_z) \\
\sin(-\theta_x)
\end{pmatrix}
\]

- roll

\[
\frac{\hat{a}_x}{\hat{a}_y} = \frac{-\sin(-\theta_z)}{\cos(-\theta_z)} = -\tan(-\theta_z)
\]

\[
\theta_z = -\text{atan2}\left(-\hat{a}_x, \hat{a}_y\right) \text{ in rad } \in [-\pi, \pi]
\]
Pitch and Roll from 3-axis Accelerometer

- use only accelerometer data – can estimate pitch & roll, not yaw
- assume no external forces (only gravity) – acc is pointing UP!

\[
\hat{a} = \frac{\tilde{a}}{|\tilde{a}|} = \begin{pmatrix}
-\cos(-\theta_x)\sin(-\theta_z) \\
\cos(-\theta_x)\cos(-\theta_z) \\
\sin(-\theta_x)
\end{pmatrix}
\]

\[
\hat{a}_z = \frac{\hat{a}_z}{\sqrt{\hat{a}_x^2 + \hat{a}_y^2}} = \frac{\sin(-\theta_x)}{\sqrt{\cos^2(-\theta_x)\left(\sin^2(-\theta_z) + \cos^2(-\theta_z)\right)}} = 1
\]

\[
= \frac{\sin(-\theta_x)}{\cos(-\theta_x)} = \tan(-\theta_x)
\]
Pitch and Roll from 3-axis Accelerometer

- use only accelerometer data – can estimate pitch & roll, not yaw
- assume no external forces (only gravity) – acc is pointing UP!

\[
\hat{a} = \frac{\tilde{a}}{||\tilde{a}||} = \begin{pmatrix}
-\cos(-\theta_x)\sin(-\theta_z) \\
\cos(-\theta_x)\cos(-\theta_z) \\
\sin(-\theta_x)
\end{pmatrix}
\]

\[
\theta_x = -\text{atan2}(\hat{a}_z, \sqrt{\hat{a}_x^2 + \hat{a}_y^2}) \text{ in rad } \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]
\]

\[
\frac{\hat{a}_z}{\sqrt{\hat{a}_x^2 + \hat{a}_y^2}} = \frac{\sin(-\theta_x)}{\sqrt{\cos^2(-\theta_x)(\sin^2(-\theta_z) + \cos^2(-\theta_z))}} = 1
\]

\[
\hat{a}_z = \sqrt{\hat{a}_x^2 + \hat{a}_y^2}
\]
Pitch and Roll from 3-axis Accelerometer

- use only accelerometer data – can estimate pitch & roll, not yaw
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\[ \frac{\hat{a}_z}{\sqrt{\hat{a}_x^2 + \hat{a}_y^2}} = \frac{\sin(-\theta_x)}{\sqrt{\cos^2(-\theta_x)(\sin^2(-\theta_z) + \cos^2(-\theta_z))}} = 1 \]

\[ \theta_x = -\text{atan2}\left(\hat{a}_z, \text{sign}(\hat{a}_y) \cdot \sqrt{\hat{a}_x^2 + \hat{a}_y^2}\right) \text{ in rad} \in [-\pi, \pi] \]
Magnetometers
MEMS Magnetometer

Hall Effect

Magneto-resistive effect
Magnetometers

- measure earth’s magnetic field in Gauss or uT

- 3 orthogonal axes = vector pointing along the magnetic field

- actual direction depends on latitude and longitude!

- distortions due to metal / electronics objects in the room or in HMD
Magnetometers

difficult to work with magnetometers without proper calibration →
we will not use the magnetometer in the HW!

LaValle et al.
Magnetometers

• advantages:
  • complementary to accelerometer – gives yaw (heading)

• problems:
  • affected by metal, distortions of magnetic field
  • need to know location, even when calibrated (e.g. GPS)

• together with gyro + accelerometer = 9 DOF sensor fusion
Prototype IMU

- 9 DOF IMU: InvenSense MPU-9250 = updated model of what was in the Oculus DK2

- 3-axis gyro, 3-axis accelerometer, 3-axis magnetometer all on 1 chip (we’ll only use gyro and acc, but we’ll give you code to read mag if you want to use it in your project)

- interface with I2C (serial bus) from Arduino
Prototype IMU

to host:
serial via USB

e.g. Arduino

I2C

InvenSense MPU-9250
MPU-9250 Specs

- multi-chip module: 1 die houses gyro & accelerometer, the other the magnetometer
- magnetometer: Asahi Kasei Microdevices AK8963 ("3rd party device")
- 9x 16 bit ADCs for digitizing 9DOF data
MPU-9250 Specs

- gyro modes: ±250, ±500, ±1000, ±2000 °/sec
- accelerometer: ±2, ±4, ±8, ±16 g
- magnetometer: ±4800 uT

- configure using registers (see specs) via I2C
- also supports on-board Digital Motion Processing™ (DMP™)

sorry, we don’t have access

- we’ll provide starter code for Arduino in lab (easy to use for beginners, not consumer product grade!)
MPU-9250 Specs

- gyro modes: ±250, ±500, ±1000, ±2000 °/sec
- accelerometer: ±2, ±4, ±8, ±16 g
- magnetometer: ±4800 uT

$$\text{metric\_value} = \frac{\text{raw\_sensor\_value}}{2^{15} - 1} \cdot \text{max\_range}$$
MPU-9250 Coordinate Systems

gyro & accelerometer

magnetometer
How to read data from IMU

- I2C = serial interface with 2 wires (also see next lab)
- microcontroller to read, we’ll use Teensy 3.2, but any Arduino can be used, e.g. past offerings used Metro Mini

- schematics - which pins to connect where
- quick intro to Arduino
- Wire library to stream out data via serial
- serial client using node server
How to read data from IMU
How to read data from IMU

• connect power & ground

3.3V
How to read data from IMU

- connect power & ground
- connect I2C clock (SCL, pin19/A5) and data (SDA, pin18/A4) lines
How to read data from IMU

• connect power & ground

• connect I2C clock (SCL, pin19/A5) and data (SDA, pin18/A4) lines

• connect micro USB for power and data transfer
VRduino

- Teensy 3.2 & IMU already connected through PCB
- also has 4 photodiodes (more details next week)
- GPIO pins for additional sensors or other add-ons
Introduction to Arduino

- open source microcontroller hardware & software
- directly interface with sensors (i.e. IMU) and process raw data
- we will be working with Teensy 3.2 (Arduino compatible)
- use Arduino IDE for all software development, installed on all lab machines
- if you want to install it on your laptop, make sure to get:
  - IDE:  https://www.arduino.cc/en/Main/Software
  - Teensyduino:  https://www.pjrc.com/teensy/teensyduino.html
  - FTDI drivers:  http://www.ftdichip.com/Drivers/VCP.htm
Introduction to Arduino (Random Test Program)

```c
// Blink
// Turns on an LED on for one second, then off for one second, repeatedly.
// This example code is in the public domain.

// Pin 13 has an LED connected on most Arduino boards.
// give it a name:
int led = 13;

// the setup routine runs once when you press reset:
void setup() {
  // initialize the digital pin as an output.
  pinMode(led, OUTPUT);
}

// the loop routine runs over and over again forever:
void loop() {
  digitalWrite(led, HIGH);  // turn the LED on (HIGH is the voltage level)
  delay(1000);  // wait for a second
  digitalWrite(led, LOW);  // turn the LED off by making the voltage LOW
  delay(1000);  // wait for a second
}
```

- **variable definition**
- **setup function** = initialization
- **loop function** = runtime callback
- connected to COM1 serial port
Introduction to Arduino

- need to stream data from Arduino to host PC
- use Wire library for all serial & I2C communication

- use node server to read from host PC and connect to JavaScript (see lab)
Introduction to Arduino

- setup function = one time initialization
- open serial connection to communicate with host PC
- set registers to configure IMU
- read from I2C (connected to IMU)
- write to I2C (connected to IMU)
Read Serial Data in Windows

- serial ports called COMx (USB serial usually COM3-COM7)
  1. establish connection to correct COM port (choose appropriate baud rate)
  2. read incoming data (in a thread)
Summary

• coordinate systems (world, body/sensor, inertial, transforms)
• overview of inertial sensors: gyroscopes, accelerometers, and magnetometers
• gyro integration aka *dead reckoning*
• orientation tracking in *flatland*
• pitch & roll from accelerometer
• overview of VRduino
Next Lecture

- quaternions and rotations with quaternions
- 6 DOF sensor fusion with quaternions & complementary filtering
Must read: course notes on IMUs!
Additional Information


- S. LaValle, A. Yershova, M. Katsev, M. Antonov “Head Tracking for the Oculus Rift”, Proc. ICRA 2014

- http://www.chrobotics.com/library