Announcements

• questions for HW1? post on piazza and zoom office hours!

• WIM workshop 1: this Friday 2-3 pm, zoom → if you are a WIM student, you must attend!

• WIM HW1 going out this Friday
Lecture Overview

• rasterization
• the rendering equation, BRDFs
• lighting: computer interaction between vertex/fragment and lights
  • Phong lighting
• shading: how to assign color (i.e. based on lighting) to each fragment
  • Flat, Gouraud, Phong shading
• vertex and fragment shaders
• texture mapping
Review of Vertex/Normal Transforms

https://www.ntu.edu.sg/home/ehchua/programming/opengl/CG_BasicsTheory.html
Rasterization
Rasterization

Purpose:
1. determine which fragments are inside the triangles
2. interpolate vertex attributes (e.g. color) to all fragments

https://www.ntu.edu.sg/home/ehchua/programming/opengl/CG_BasicsTheory.html
Rasterization / Scanline Interpolation

- grid of 6x6 fragments
Rasterization / Scanline Interpolation

- grid of 6x6 fragments
- 2D vertex positions after transformations
Rasterization / Scanline Interpolation

- grid of 6x6 fragments
- 2D vertex positions after transformations
  + edges = triangle

\[(x_1, y_1)\]
\[(x_2, y_2)\]
\[(x_3, y_3)\]
Rasterization / Scanline Interpolation

- grid of 6x6 fragments
- 2D vertex positions after transformations + edges = triangle
- each vertex has 1 or more attributes A, such as R/G/B color, depth, …
- user can assign arbitrary attributes, e.g. surface normals
Rasterization / Scanline Interpolation

• scanline moving top to bottom
Rasterization / Scanline Interpolation

- scanline moving top to bottom
Rasterization / Scanline Interpolation

- scanline moving top to bottom
- determine which fragments are inside the triangle
Rasterization / Scanline Interpolation

- scanline moving top to bottom
- determine which fragments are inside the triangle
- interpolate attribute along edges in y

\[
A^{(l)} = \left( \frac{y^{(l)} - y_2}{y_1 - y_2} \right) A_1 + \left( \frac{y_1 - y^{(l)}}{y_1 - y_2} \right) A_2
\]

\[
A^{(r)} = \left( \frac{y^{(r)} - y_3}{y_1 - y_3} \right) A_1 + \left( \frac{y_1 - y^{(r)}}{y_1 - y_3} \right) A_3
\]
Rasterization / Scanline Interpolation

- scanline moving top to bottom
- determine which fragments are inside the triangle
- interpolate attribute along edges in y
- then interpolate along x

\[ A = \left( \frac{x - x^{(l)}}{x^{(r)} - x^{(l)}} \right) A^{(r)} + \left( \frac{x^{(r)} - x}{x^{(r)} - x^{(l)}} \right) A^{(l)} \]
repeat:
• interpolate attribute along edges in $y$
• then interpolate along $x$
repeat:
- interpolate attribute along edges in $y$
- then interpolate along $x$
Rasterization / Scanline Interpolation

A1

A2

A3
output: set of fragments inside triangle(s) with interpolated attributes for each of these fragments
Lighting & Shading
(how to determine color and what attributes to interpolate)
The Rendering Equation

- **direct (local) illumination**: light source $\rightarrow$ surface $\rightarrow$ eye

- **indirect (global) illumination**: light source $\rightarrow$ surface $\rightarrow$ ... $\rightarrow$ surface $\rightarrow$ eye

$$L_o(x, \omega_o, \lambda, t) = L_e(x, \omega_o, \lambda, t) + \int_{\Omega} f_r(x, \omega_i, \omega_o, \lambda, t) L_i(x, \omega_i, \lambda, t) (\omega_i \cdot n) \, d\omega_i$$

Kajija “The Rendering Equation”, SIGGRAPH 1986
The Rendering Equation

- **direct (local) illumination:**
  light source → surface → eye

- **indirect (global) illumination:**
  light source → surface → … → surface → eye

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\]

- \(L_o\): radiance towards viewer
- \(L_e\): emitted radiance
- \(f_r\): BRDF
- \(L_i\): incident radiance from some direction

Kajija “The Rendering Equation”, SIGGRAPH 1986
The Rendering Equation

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L_o(x, \omega_o, \lambda, t) = L_e(x, \omega_o, \lambda, t) + \int_{\Omega} f_r(x, \omega_i, \omega_o, \lambda, t) L_i(x, \omega_i, \lambda, t) (\omega_i \cdot n) \, d\omega_i
\]

\( \omega_o \rightarrow \Omega \rightarrow \omega_i \)

- **direct (local) illumination:**
  light source \( \rightarrow \) surface \( \rightarrow \) eye

- **indirect (global) illumination:**
  light source \( \rightarrow \) surface \( \rightarrow \) ... \( \rightarrow \) surface \( \rightarrow \) eye
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\]

- $L_o$ is the radiance towards the viewer.
- $L_e$ is the emitted radiance.
- $f_r$ is the BRDF.
- $L_i$ is the incident radiance from some direction.

Kajija “The Rendering Equation”, SIGGRAPH 1986
The Rendering Equation

- **direct (local) illumination:**
  light source $\rightarrow$ surface $\rightarrow$ eye

- **indirect (global) illumination:**
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\]

- radiance towards viewer
- emitted radiance
- BRDF
- incident radiance from some direction

Kajija “The Rendering Equation”, SIGGRAPH 1986
The Rendering Equation

- direct (local) illumination:
  light source $\rightarrow$ surface $\rightarrow$ eye

- indirect (global) illumination:
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$$L_o(x, \omega_o, \lambda, t) = L_e(x, \omega_o, \lambda, t) + \int_{\Omega} f_r(x, \omega_i, \omega_o, \lambda, t) L_i(x, \omega_i, \lambda, t) (\omega_i \cdot n) \, d\omega_i$$

Kajija “The Rendering Equation”, SIGGRAPH 1986
The Rendering Equation

- drop time, wavelength (RGB), emission & global illumination to make it simple

\[ L_0(x, \omega_0) = \sum_{k=1}^{\text{num\_lights}} f_r(x, \omega_k, \omega_o) L_i(x, \omega_k)(\omega_k \cdot n) \]

- direct (local) illumination:
  - light source \(\rightarrow\) surface \(\rightarrow\) eye

- indirect (global) illumination:
  - light source \(\rightarrow\) surface \(\rightarrow\) ... \(\rightarrow\) surface \(\rightarrow\) eye

\[ L_o(x, \omega_o, \lambda, t) = L_e(x, \omega_o, \lambda, t) + \int_{\Omega} f_r(x, \omega_i, \omega_o, \lambda, t) L_i(x, \omega_i, \lambda, t)(\omega_i \cdot n) \, d\omega_i \]

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The Rendering Equation

- drop time, wavelength (RGB), emission & global illumination to make it simple

\[ L_0(x, \omega_0) = \sum_{k=1}^{num\_lights} f_r(x, \omega_k, \omega_o) L_i(x, \omega_k)(\omega_k \cdot n) \]
Bidirectional Reflectance Distribution Function (BRDF)

- many different BRDF models exist: analytic, data driven (i.e. captured)

\[ \rho(\theta_r, \phi_r, \theta_i, \phi_i) \]
Bidirectional Reflectance Distribution Function (BRDF)

- can approximate BRDF with a few simple components

\[ \rho(\theta_r, \phi_r, \theta_i, \phi_i) \]

incident light direction

normal

diffuse component

specular component

surface

Phong Lighting

• emissive part can be added if desired
• calculate separately for each color channel: RGB

Ambient + Diffuse + Specular = Phong Reflection
Phong Lighting

- simple model for direct lighting
- ambient, diffuse, and specular parts
- requires:
  - material color $m_{RGB}$ (for each of ambient, diffuse, specular)
  - light color $l_{RGB}$ (for each of ambient, diffuse, specular)

\[
R = 2(N \cdot L)N - L
\]

$L$ normalized vector pointing towards light source

$N$ normalized surface normal

$V$ normalized vector pointing towards viewer

normalized reflection on surface normal
Phong Lighting: Ambient

- independent of light/surface position, viewer, normal
- basically adds some background color

\[
\mathbf{m}_{\text{ambient}} \cdot \mathbf{l}_{\text{ambient}} \quad \{R,G,B\} \cdot \{R,G,B\}
\]
Phong Lighting: Diffuse

- needs normal and light source direction
- adds intensity cos-falloff with incident angle

\[ m_{\text{diffuse}} \cdot l_{\text{diffuse}} \cdot \max(L \cdot N, 0) \]

dot product

https://www.ntu.edu.sg/home/ehchua/programming/opengl/CG_BasicsTheory.html
Phong Lighting: Specular

- needs normal, light & viewer direction
- models reflections = specular highlights
- shininess – exponent, larger for smaller highlights (more mirror-like surfaces)

\[ \mathbf{m}_{\text{specular}} \cdot \mathbf{l}_{\text{specular}} \cdot \max(\mathbf{R} \cdot \mathbf{V}, 0)^{\text{shininess}} \]
Phong Lighting: Attenuation

- models the intensity falloff of light w.r.t. distance
- The greater the distance, the lower the intensity

\[
\frac{1}{k_c + k_l d + k_q d^2}
\]

constant, linear, quadratic attenuation
Phong Lighting: Putting it all Together

• this is a simple, but efficient lighting model
• has been used by OpenGL for ~25 years
• absolutely NOT sufficient to generate photo-realistic renderings (take a computer graphics course for that)

\[
\text{color}_{R,G,B} = m_{\text{ambient}} \cdot l_{\text{ambient}} + \sum_{i=1}^{\text{num\_lights}} \frac{1}{k_c + k_id_i + k_qd_i^2} \left( m_{\text{diffuse}} \cdot l_{\text{diffuse}} \cdot \max(L_i \cdot N, 0) + m_{\text{specular}} \cdot l_{\text{specular}} \cdot \max(R_i \cdot V, 0)^{\text{shininess}} \right)
\]
Lighting Calculations

• **all lighting calculations happen in camera/view space!**

• transform vertices and normals into camera/view space
• calculate lighting, i.e. per color (i.e., given material properties, light source color & position, vertex position, normal direction, viewer position)
Lighting v Shading

- **lighting**: interaction between light and surface (e.g. using Phong lighting model; think about this as “what formula is being used to calculate intensity/color”)
- **shading**: how to compute color of each fragment (e.g. what attributes to interpolate and where to do the lighting calculation)
  1. Flat shading
  2. Gouraud shading (per-vertex lighting)
  3. Phong shading (per-fragment lighting) - different from Phong lighting

courtesy: Intergraph Computer Systems
Flat Shading

- compute color only once per triangle (i.e. with Phong lighting)
- pro: usually fast to compute; con: creates a flat, unrealistic appearance
- we won’t use it
Gouraud or Per-vertex Shading

- compute color once per vertex (i.e. with Phong lighting)
- interpolate per-vertex colors to all fragments within the triangles!
- pro: usually fast-ish to compute; con: flat, unrealistic specular highlights

surface approximation by triangles
per-vertex normal
target surface
Gouraud Shading or Per-vertex Lighting

- compute color once per vertex (i.e. with Phong lighting)
- interpolate per-vertex colors to all fragments within the triangles!
- pro: usually fast-ish to compute; con: flat, unrealistic specular highlights
Gouraud Shading or Per-vertex Lighting

- compute color once per vertex (i.e. with Phong lighting)
- *interpolate per-vertex colors to all fragments within the triangles!*
- pro: usually fast-ish to compute; con: flat, unrealistic specular highlights
Gouraud Shading or Per-vertex Lighting

- compute color once per vertex (i.e. with Phong lighting)
- interpolate per-vertex colors to all fragments within the triangles!
- pro: usually fast-ish to compute; con: flat, unrealistic specular highlights

https://en.wikipedia.org/wiki/Gouraud_shading
Phong Shading or Per-fragment Lighting

- compute color once per fragment (i.e. with Phong lighting)
- need to interpolate per-vertex normals to all fragments to do the lighting calculation!
- pro: better appearance of specular highlights; con: usually slower to compute
Shading

Flat Shading

Gouraud Shading

Phong Shading

http://www.decew.net/OSS/timeline.php
Back to the Graphics Pipeline
Per-vertex Lighting v Per-fragment Lighting

- **vertex shader**: lighting calculations done for each vertex
- **fragment shader**: lighting calculations done for each fragment

https://www.ntu.edu.sg/home/ehchua/programming/opengl/CG_BasicsTheory.html
Vertex and Fragment Shaders

• shaders are small programs that are executed in parallel on the GPU for each vertex (vertex shader) or each fragment (fragment shader)

• vertex shader (before rasterizer):
  • modelview projection transform of vertex & normal (see last lecture)
  • if per-vertex lighting: do lighting calculations here (otherwise omit)

• fragment shader (after rasterizer):
  • assign final color to each fragment
  • if per-fragment lighting: do all lighting calculations here (otherwise omit)
Fragment Processing

• **lighting and shading** (per-fragment) – same calculations as per-vertex shading, but executed for each fragment
• **texture mapping**

these also happen, but don’t worry about them (we won’t touch these):
• fog calculations
• alpha blending
• hidden surface removal (using depth buffer)
• scissor test, stencil test, dithering, bitmasking, …
Depth Test

- oftentimes we have multiple triangles behind each other, the depth test determines which one to keep and which one to discard
- if depth of fragment is smaller than current value in depth buffer \(\rightarrow\) overwrite color and depth value using current fragment; otherwise discard fragment
Texture Mapping

- texture = 2D image (e.g. RGBA)
- we want to use it as a “sticker” on our 3D surfaces
- mapping from vertex to position on texture (texture coordinates \( u,v \))

https://blogs.msdn.microsoft.com/danlehen/2005/11/06/3d-for-the-rest-of-us-texture-coordinates/ (sorry, this website seems to be discontinued)
Texture Mapping

- texture = 2D image (e.g. RGBA)
- we want to use it as a “sticker” on our 3D surfaces
- mapping from vertex to position on texture (texture coordinates $u,v$)

![Diagram of texture mapping]

Normalized Texture Coordinates

Non-normalized Texture Coordinates
Texture Mapping

- same texture, different texture coordinates

https://blogs.msdn.microsoft.com/danlehen/2005/11/06/3d-for-the-rest-of-us-texture-coordinates/ (sorry, this website seems to be discontinued)
Texture Mapping

- texture mapping faces

Bermano et al. 2013
Texture Mapping

- texture filtering: fragments don’t align with texture pixels (texels) → interpolate

![Magnification – Nearest Point Sampling](image1)
![Magnification – Bilinear Interpolation](image2)

https://www.ntu.edu.sg/home/ehchua/programming/opengl/CG_BasicsTheory.html
Next Lecture: Vertex & Fragment Shaders, GLSL

vertex shader
- transforms & (per-vertex) lighting

fragment shader
- texturing
- (per-fragment) lighting

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Summary

- rasterization
- the rendering equation, BRDFs
- lighting: computer interaction between vertex/fragment and lights
  - Phong lighting
- shading: how to assign color (i.e. based on lighting) to each fragment
  - Flat, Gouraud, Phong shading
- vertex and fragment shaders
- texture mapping
Further Reading

- good overview of OpenGL (deprecated version) and graphics pipeline (missing a few things):
  https://www.ntu.edu.sg/home/ehchua/programming/opengl/CG_BasicsTheory.html


- WebGL / three.js tutorials: https://threejs.org/