Course information

Overview
Modeling
Least-squares problems
The singular-value decomposition
Course components
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- lecture: Tuesday/Thursday, 3.15pm – 5.05pm (Gates B3)
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- lecture: Tuesday/Thursday, 3.15pm – 5.05pm (Gates B3)
- office hours: TBA
Prerequisites

▶ necessary:
  ▶ linear algebra (as in MATH104)
  ▶ speaking vocabulary versus reading vocabulary
  ▶ The Karate Kid analogy
  ▶ differential equations and Laplace transforms (as in EE102A)
▶ not necessary (but may increase appreciation):
  ▶ control systems
  ▶ circuits and systems
  ▶ dynamics
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Course materials

Everything you need is on the course website.

Some additional references (not necessary):
- Linear algebra: Calafiore/El Ghaoui, Meyer, Axler
- Dynamical systems: Luenberger

A living document on the Piazza forum.

Grades (and only grades) on CourseWork.

Alex Lemon

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Grading

- weekly problem sets: 20% (usually due on Fridays at 5pm)
- midterm exam: 30% (24-hour take-home)
- final exam: 50% (24-hour take-home)
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"All models are wrong, but some are useful." – George Box

"Nothing at all takes place in the universe in which some rule of maximum or minimum does not appear." – Leonhard Euler
Modeling

- convert a practical problem into a mathematical model
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- most important and most difficult part of the course
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Linear dynamical systems
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- discrete-time linear dynamical system:
Linear dynamical systems

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\[ x(t + 1) = A(t)x(t) + B(t)u(t) \]
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    y(t) &= C(t)x(t) + D(t)u(t)
\end{align*}
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- \(u(t) \in \mathbb{R}^p\) is the input
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- \(A(t) \in \mathbb{R}^{n \times n}\) is the dynamics matrix
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- \(D(t) \in \mathbb{R}^{m \times p}\) is the feedthrough matrix
Least-squares problems
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- system identification
Least-squares problems

- system identification
- minimum-energy control
Least-squares problems

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- linear-filter design
Least-squares problems

\[
\text{minimize } \|Ax - b\| \quad \text{subject to } Cx = d
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The singular-value decomposition: extremal-gain problems
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- minimum-residual subspace
The singular-value decomposition: extremal-gain problems

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The singular-value decomposition: extremal-gain problems

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- analysis of robustness
The singular-value decomposition: extremal-gain problems

\[
\begin{align*}
\text{minimize} & : \|Ax\| \\
\text{subject to} & : \|x\| = 1
\end{align*}
\]

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The singular-value decomposition: low-rank approximation
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▶ latent-semantic indexing
The singular-value decomposition: low-rank approximation

- latent-semantic indexing
- recommendation systems
The singular-value decomposition: low-rank approximation

- latent-semantic indexing
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The singular-value decomposition: low-rank approximation

\[
\begin{align*}
\text{minimize} & : \| A - X \| \\
X & \in \mathbb{R}^{m \times n} \\
\text{subject to} & : \text{rank}(X) \leq r
\end{align*}
\]

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