1 Temperatures in a multicore processor

We are concerned with the temperature of a processor at two critical locations. These temperatures, denoted \( T = (T_1, T_2) \) (in \( ^\circ \text{C} \)), are affine functions of the power dissipated by three processor cores, denoted \( P = (P_1, P_2, P_3) \) (in W). We make four measurements. In the first measurement, all three cores are idling, and dissipate 10 W. In the next three measurements, one of the processors is set to full power (100 W), and the other two are idling. In each experiment, we measure the temperatures at the two critical locations. The results of these experiments are summarized in the following table.

<table>
<thead>
<tr>
<th>( P_1 )</th>
<th>( P_2 )</th>
<th>( P_3 )</th>
<th>( T_1 )</th>
<th>( T_2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>10 W</td>
<td>10 W</td>
<td>10 W</td>
<td>27 ( ^\circ \text{C} )</td>
<td>29 ( ^\circ \text{C} )</td>
</tr>
<tr>
<td>100 W</td>
<td>10 W</td>
<td>10 W</td>
<td>45 ( ^\circ \text{C} )</td>
<td>37 ( ^\circ \text{C} )</td>
</tr>
<tr>
<td>10 W</td>
<td>100 W</td>
<td>10 W</td>
<td>41 ( ^\circ \text{C} )</td>
<td>49 ( ^\circ \text{C} )</td>
</tr>
<tr>
<td>10 W</td>
<td>10 W</td>
<td>100 W</td>
<td>35 ( ^\circ \text{C} )</td>
<td>55 ( ^\circ \text{C} )</td>
</tr>
</tbody>
</table>

Suppose we operate all three cores at the same power, \( p \). What is the largest value of \( P \) such that neither \( T_1 \) nor \( T_2 \) exceeds \( T_{\text{crit}} = 70 \( ^\circ \text{C} \) ?

2 Analysis of a layered medium

In this problem we consider a model for (incoherent) transmission in a layered medium. The medium is modeled as a set of \( n \) layers, separated by \( n \) dividing interfaces, shown as shaded rectangles in figure 1.

For \( i = 1, \ldots, n \), let \( x_i \in \mathbb{R} \) denote the amplitude of the right-traveling wave in layer \( i \), and let \( y_i \in \mathbb{R} \) denote the amplitude of the left-traveling wave in layer \( i \). The right-traveling and left-traveling waves in the first layer are called the incident and reflected waves, respectively. The scattering coefficient for the medium is defined to be the ratio \( S = y_1/x_1 \) (assuming \( x_1 \neq 0 \)).

The right- and left-traveling waves on each side of an interface are related by the transmission and reflection equations:

\[
\begin{align*}
    x_{i+1} &= t_i x_i + r_i y_{i+1}, \\
    y_i &= r_i x_i + t_{i+1} y_{i+1},
\end{align*}
\]

\( i = 1, \ldots, n - 1 \).
where \( t_i \in [0,1] \) is the transmission coefficient of the \( i \)th interface, and \( r_i \in [0,1] \) is the reflection coefficient of the \( i \)th interface. We assume that the last interface is totally reflective, so that \( y_n = x_n \).

(a) Explain how to find the scattering coefficient \( S \) given the transmission and reflection coefficients for the first \( n - 1 \) layers.

(b) Carry out your method on the instance of the problem with

\[
n = 20, \quad t_1 = \cdots = t_{n-1} = 0.96, \quad \text{and} \quad r_1 = \cdots = r_{n-1} = 0.02.
\]

Plot the amplitudes of the left- and right-traveling versus \( i \), and report the value of \( S \) that you compute.

(c) Fault location. Suppose that there is a fault at interface \( k \), so that \( t_k = 0.02 \) and \( r_k = 0.96 \) (the other interfaces still have \( t_i = 0.96 \) and \( r_i = 0.02 \)). You are told that the scattering coefficient of the medium with the fault is \( S = S_{\text{fault}} \). Explain how to determine which interface contains the fault. Carry out your method with \( S_{\text{fault}} = 0.70 \), and report the interface \( k \) that contains the fault. You may assume that the last layer does not contain the fault.