1 Optimal operation of a two-state chemical reactor
(a) Explain how to choose $T_0$. (4 points)

(b) • Report the optimal value of $T_0$. (1 point)
   • Report the corresponding amount of compound $k$. (1 point)
   • Submit a plot showing $x(t)$ as a function of time. (2 points)

(c) Explain how to choose $T_1$ and $T_2$. (6 points)

(d) • Report the optimal values of $T_1$ and $T_2$. (2 points [1 each])
   • Report the corresponding amount of compound $k$. (2 points)
   • Submit a plot showing $x(t)$ as a function of time. (2 points)

2 Ranking teams in a round-robin tournament
(a) • According to our usual analysis, what is the value of $\bar{x}$? (2 points)
   • State the assumptions that we use to justify our analysis. (2 points)

(b) • According to our usual analysis, what is the value of $\rho$? (2 points)
   • State the usual assumptions that we use to justify our analysis. (3 points)

(c) (i) • Submit a plot showing $x_j(t)$ for $j = 10, 20, 30$. (1 point)
   • Indicate $\bar{x}$ on your plot. (1 point)

(ii) • Submit a plot showing $\|e(t)\|$. (1 point)
   • Report the value of $\rho$. (1 point)
   • Explain how the slope of the plot is related to $\rho$. (1 point)

(iii) • Are the assumptions that we usually use to justify our analysis of the asymptotic behavior of $x(t)$ satisfied? (1 point)
   • Does our usual conclusion hold? (1 point)

(iv) • Are the assumptions that we usually use to justify our analysis of the asymptotic behavior of $\|e(t + 1)\|/\|e(t)\|$ satisfied? (1 point)
   • Does our usual conclusion hold? (1 point)

(v) Which team is most overrated if we use win/loss record instead of our refined scoring system? (1 point)

(vi) Which team is most underrated if we use win/loss record instead of our refined scoring system? (1 point)

3 Minimum-sensitivity estimation of radiation levels
(a) Explain how to choose the measurement locations. (10 points)
(b) • Report the indices of your chosen measurement locations. (5 points)
  • Report the corresponding sensitivity. (2 points)
  • Submit a plot indicating the chosen measurement locations. (3 points)

4 Principal-components analysis of decathlon data
(a) • Submit a plot showing \( \sigma_j \) versus \( j \). (2 point)
  • Submit a plot of \( p_j \) versus \( j \). (2 point)
  • Report \( p_2 \). (1 point)
(b) • Submit a plot with the point \((v_1)_j, (v_2)_j\) labeled with the name of the \( j \)th event. (2 points)
  • Do similar events appear to be close together? (1 point)
(c) • Submit a spatial plot of \( r \). (2 points)
  • Which right singular vector seems to represent \( r \)? (2 points)
(d) • Submit a spatial plot of \( t \). (2 points)
  • Submit a spatial plot of \( \delta \). (2 points)
  • Give an intuitive interpretation of the first left singular vector of \( X \). (2 points)
  • Give an intuitive interpretation of the second left singular vector of \( X \). (2 points)

5 Stock-market prediction using maximum-correlation linear functions
(a) Show that
   \[
   r(u, v) = \frac{\alpha^T X \hat{Y}^T \beta}{\| X^T \alpha \| \| \hat{Y}^T \beta \|}.
   \]
   (4 points)
(b) • Explain how to choose \( a \) and \( b \) in order to maximize \( r(u, v) \). (3 points)
  • Explain how to find unit vectors \( \alpha \) and \( \beta \) that maximize \( r(u, v) \). (3 points)
(c) • Report your value of \( \alpha \). (2 points)
  • Report your value of \( \beta \). (2 points)
  • Report the corresponding value of \( r(u, v) \). (2 points)
(d) • Report the fraction of the time that we make money on the test set using the trading strategy based on \( \alpha \) and \( \beta \). (2 points)
  • Report the fraction of the time that we make money on the test set using the simple trading strategy. (2 points)