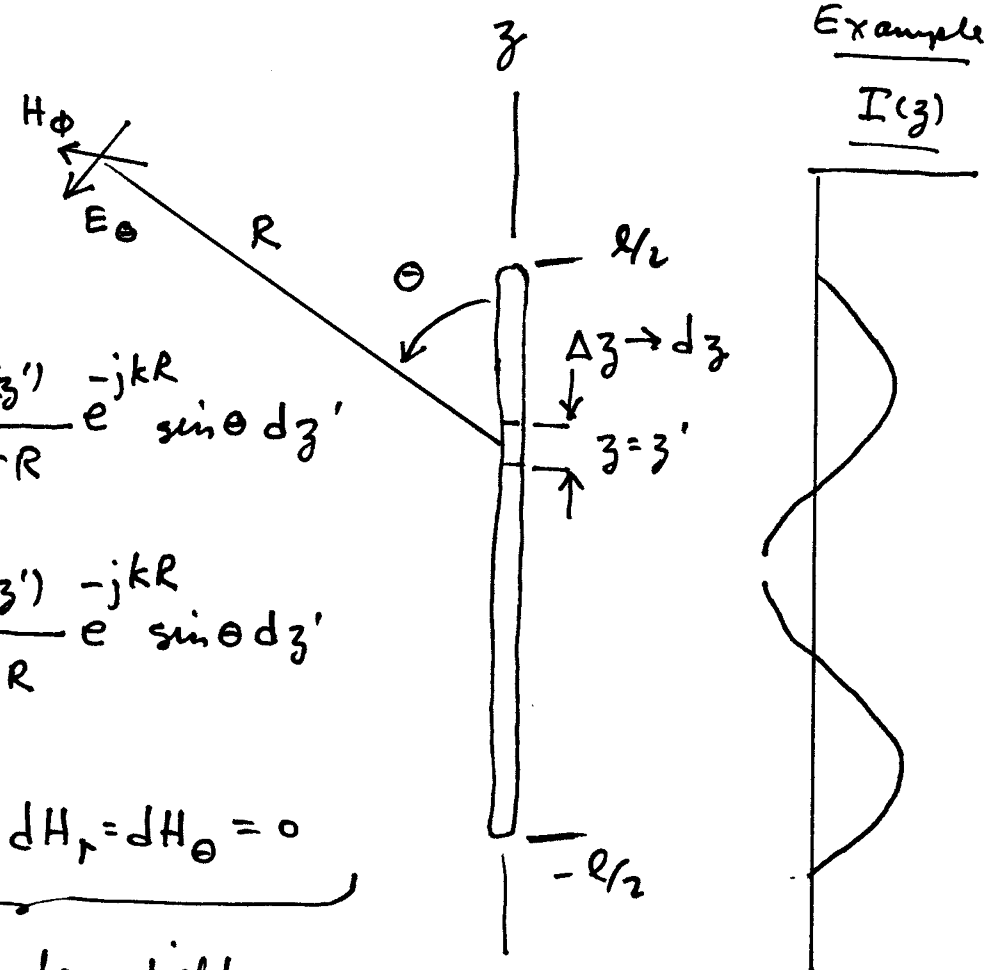
Dipole Antennas EE252

$$\bar{A} = \frac{\mu}{4\pi}\int_{vol} \bar{J}(\bar{r}')\frac{e^{-jkR}}{R}\,dv' \;\widetilde{\rightarrow}\; \underbrace{\frac{\mu}{4\pi}\int_{z=-\ell/2}^{z=+\ell/2} I(z')\frac{e^{-jkR}}{R}\,dz'}_{\text{for a fine wire element}}$$

$$\bar{E} = -j\omega\bar{A} + \frac{\nabla(\nabla\cdot\bar{A})}{j\omega\mu_0\epsilon_0} \underbrace{\longrightarrow -j\omega\bar{A}_T}_{\text{far-field}}$$

/...



$$dE_\theta = j\eta k \frac{I(z')}{4\pi R} e^{-jkR} \sin\theta \, dz'$$

$$dH_\phi = jk \frac{I(z')}{4\pi R} e^{-jkR} \sin\theta \, dz'$$

$$\underbrace{dE_r = dE_\phi = dH_r = dH_\theta = 0}_{\text{all in the far-field}}$$

/ . . .

Far-Field Approximations

$$E_\theta = \int_{-\ell/2}^{\ell/2} dE_\theta = \underbrace{j\eta \frac{k e^{-jkr}}{4\pi r} \sin\theta}_{\text{element factor}} \underbrace{\int_{-\ell/2}^{\ell/2} I(z') e^{+jkz'\cos\theta} \, dz'}_{\text{space factor}}$$

$$I(z') = \begin{cases} I_0 \sin[k(\ell/2 - z')] & 0 \le z' \le \ell/2 \\ I_0 \sin[k(\ell/2 + z')] & -\ell/2 \le z' < 0 \end{cases}$$

/ ...

$$\int I(z')\, e^{jk\cos\theta\, z'}\, dz' = ?$$

Form is $\int e^{\alpha z} \sin(\beta z + \gamma)\, dz$ with

$\alpha = jk\cos\theta$, $\beta = \pm k$, $\gamma = \frac{kl}{2} = \frac{\pi l}{\lambda}$

Use two step integration by parts ($u\,dv = uv - v\,du$)

Begin with ... $u = \sin(\beta z + \gamma)$ $du = \beta\cos(\beta z + \gamma)$

$dv = e^{\alpha z} dz$ $v = \frac{1}{\alpha} e^{\alpha z}$

/ . . .   A short while later . . .

. . .

. . .

. . .

$$E_\theta = j\eta \frac{I_o e^{-jkr}}{2\pi r} \underbrace{\left[\frac{\cos\left(\frac{kl}{2}\cos\theta\right) - \cos\left(\frac{kl}{2}\right)}{\sin\theta} \right]}_{F(\theta)}$$

$$E_\theta = j\eta \frac{I_o e^{-jkr}}{2\pi r} F(\theta)$$

$$H_\phi = \frac{E_\theta}{\eta} = j \frac{I_o e^{-jkr}}{2\pi r} F(\theta)$$

{ all valid in far-field with Dipole length "l" }

what about power radiated from dipole

$$\bar{W}_{AVE} = \frac{1}{2\eta} |E_\theta|^2 \bar{a}_r = \eta \frac{|I_0|^2}{8\pi^2 r^2} \cdot |F(\theta)|^2$$

$$\bar{U} = r^2 \bar{W}_{AVE} = \eta \frac{|I_0|^2}{8\pi^2} |F(\theta)|^2$$

$$F(\theta)\Big|_{\theta=0} = \text{?} \ (\text{L'Hopital's Rule}) \Rightarrow 0$$

<u>Never</u> any radiation off the end !!

/...

How about broadside to the dipole?

$$\frac{d}{d\theta}\left[|F(\theta)|^2\right]\Bigg|_{\theta=\frac{\pi}{2}} = 0 \qquad \forall\ kl\,,$$

So broadside $|F(\theta)|^2$ is <u>either</u> a

maximum <u>or</u> a minimum

Short Dipole

$$\bar{W}_{AVE} \rightarrow \frac{\eta |I_0|^2}{8\pi^2 r^2} \left[\frac{\overbrace{\left(1 - \gamma^2 \frac{\cos^2\theta}{2}\right) - \left(1 - \gamma^2/2\right)}^{\cos\left[\frac{kl}{2}\cdot\cos\theta\right] - \cos\left[\frac{kl}{2}\right]}}{\sin\theta} \right]^2$$

$$\frac{kl}{2} = \gamma << 1$$

$$\bar{W}_{AVE} \approx \eta \frac{|I_0|^2}{8\pi^2 r^2} \cdot \frac{\gamma^4}{4} \left(\frac{1 - \cos^2\theta}{\sin^2\theta}\right)^2 = \eta \frac{|I_0|^2}{8\pi^2 r^2} \frac{\gamma^4}{4} \sin^2\theta$$

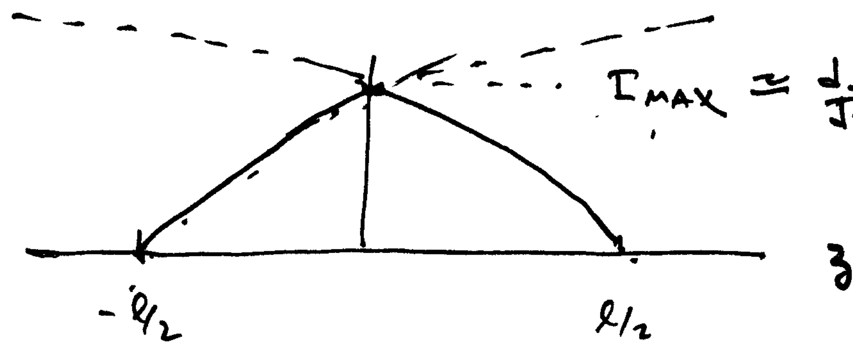
$$\frac{kl}{2} << 1$$

$$= \frac{\eta}{8} \left|\frac{I_0 l}{\lambda}\right|^2 \left(\frac{2\pi l}{\lambda}\right)^2 \frac{\sin^2\theta}{r^2}$$

What is the difference between the "Short Dipole" & "Ideal Dipole"?

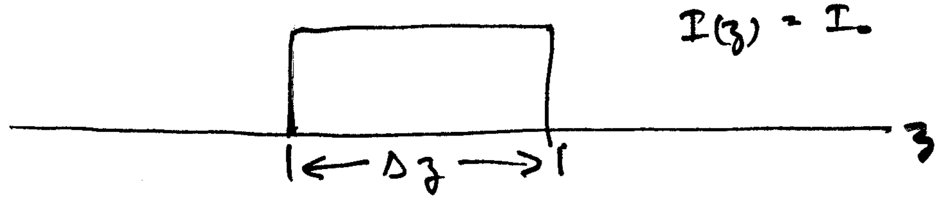
Ans: the current distribution

Short



* $I_{MAX} \simeq \frac{dI}{dz} \cdot \frac{\ell}{2} = I_0 \gamma$

Ideal



$I(z) = I_0$

* $I(z) = I_0 \sin\left[\frac{k\ell}{2} - k|z|\right]$

$$\left|\overline{W}_{AVE}\right| = \frac{\eta}{8\pi^2 r^2} \cdot \left|\frac{I_0 \gamma}{2}\right|^2 \gamma^2 \sin^2\theta$$

$$= \frac{\eta}{8} \left|\frac{I_{AVE}\,\ell}{\lambda}\right|^2 \frac{\sin^2\theta}{r^2}\,; \quad I_{AVE} = \frac{I_0 \gamma}{2}$$

Short dipole is not sensitive to the form of the current distribution, only to

$$\int_{-\ell/2}^{\ell/2} I(z)\, dz\,, \quad \text{for } \underline{\frac{\ell}{2} << \lambda}$$

If $\left[\frac{I_{AVE}\,\ell}{\lambda}\right] = \left[\frac{I_0 \Delta z}{\lambda}\right]$, Short dipole is equivalent to an ideal dipole

Short Dipole : Radiation Resistance

$$W_{AVE} = \eta/8 \ \frac{1}{r^2} \left| \frac{I_{AVE}\, l}{\lambda} \right|^2 \sin^2\theta$$

$$= \frac{15\,\pi}{r^2} \left| \frac{I_{AVE}\, l}{\lambda} \right|^2 \sin^2\theta$$

$$P_{rad} = \int_0^{2\pi} \int_0^{\pi} W_{AVE}\, r^2 \sin\theta \, d\theta \, d\phi = 30\pi^2 \left[\frac{I_{AVE}\, l}{\lambda} \right]^2 \cdot \frac{4}{3}$$

$$= 40\pi^2 \left| \frac{I_{AVE}\, l}{\lambda} \right|^2 = \frac{1}{2} \underbrace{R_{eff}}_{\text{short dip.}} \left| I_{MAX} \right|^2 = 2 R_{eff} \left| I_{AVE} \right|^2$$

/...

$$\underbrace{R_{eff}}_{\text{short dipole}} = 20\pi^2 \left(\frac{\ell}{\lambda}\right)^2 = \frac{1}{4} \underbrace{R_{eff}}_{\text{I.D.}} \text{ for an ideal dipole.}$$

Current is not used as effectively in a short dipole as it is in an ideal dipole!

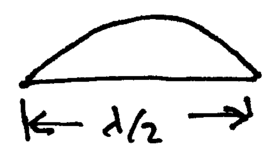
Aside: The 'trick' of 'top-loading' an antenna to increase the current on the radiating portion of the structure is based on the effect illustrated by the difference between the short and ideal dipoles. You can see this in practice at KNBR (near Foster City).

Half-wave Dipole $\left(\ell = \frac{\lambda}{2};\ \frac{k\ell}{2} = \frac{2\pi}{\lambda}\cdot\frac{\ell}{2} = \frac{\pi}{2}\right)$

$$E_\theta \simeq j\eta \frac{I_0 e^{-jkr}}{2\pi r} \frac{\cos\left(\frac{\pi}{2}\cos\theta\right)}{\sin\theta}$$

$I_0 = I_{max}$, for assumed distribution

$$H_\phi \simeq j \frac{I_0 e^{-jkr}}{2\pi r} \frac{\cos\left(\frac{\pi}{2}\cos\theta\right)}{\sin\theta}$$



$$W_{AVE} \simeq \eta/2 \frac{|I_0|^2}{4\pi^2 r^2} \frac{\cos^2\left(\frac{\pi}{2}\cos\theta\right)}{\sin^2\theta}$$

/... $\lambda/2$ - Dipole

$$P_{rad} \simeq \eta \frac{|I_0|^2}{4\pi} \cdot \int_0^{\pi} \frac{\cos^2\left(\frac{\pi}{2}\cos\theta\right)}{\sin\theta}\, d\theta = \eta \frac{|I_0|^2}{4\pi}(1.219)$$

$$\simeq \eta \frac{|I_0|^2}{8\pi} \cdot (2.4)$$

numerically determined factor

$$\text{Directivity} = \frac{4\pi\, U_{MAX}}{P_{rad}} = 4\pi \frac{\eta_0 |I_0|^2 / 8\pi^2}{\eta_0 \frac{|I_0|^2 \cdot (2.4)}{8\pi}} = \frac{4}{2.4} = 1.64$$

Which is only slightly greater ($9\frac{1}{3}$%) than that of an ideal dipole

Radiation Resistance of $\lambda/2$ - Dipole

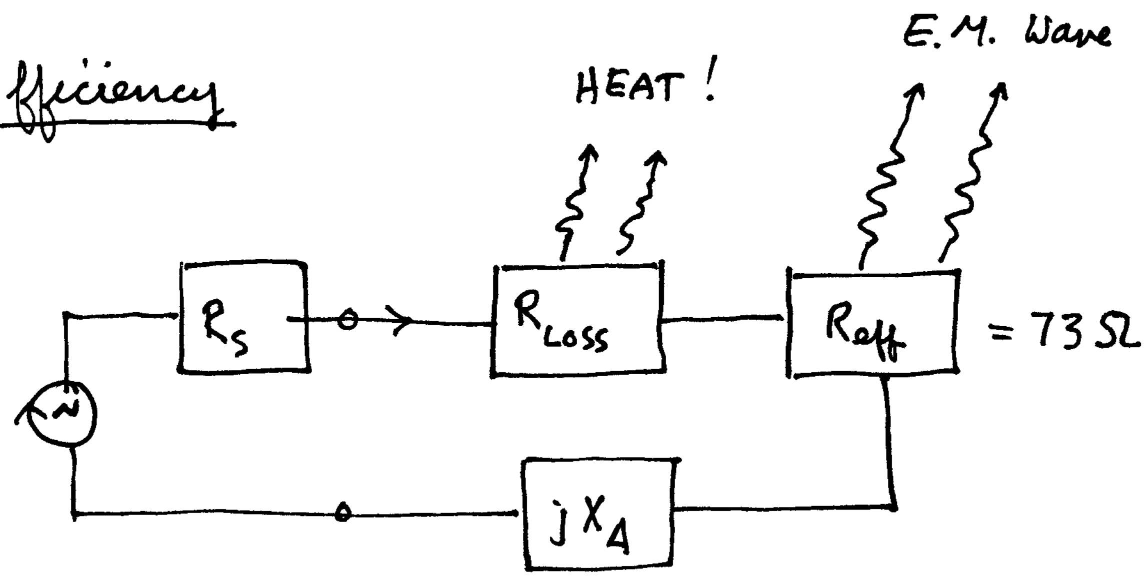
$$\frac{1}{2}|I_o|^2 R_{eff} = \eta \frac{|I_o|^2}{4\pi} (1.219)$$

$$R_{eff} = \frac{120\pi}{2\pi}(1.219) = 60 \cdot 1.219 = \underline{\underline{73\,\Omega}} \quad !$$

$$Z_{IN} = 73\,\Omega + j\,X(f) \quad \text{near} \quad \frac{\lambda}{2} = \ell .$$

Consider the field diagrams in Kraus, pp 56-57.

Efficiency



$$\frac{P_{rad}}{P_{TOT}} = \left[\frac{R_{eff}}{R_{Loss} + R_{eff}}\right] = \epsilon_A$$ antenna efficiency

for $\ell \lesssim \lambda/2$,

P_{rad} and $R_{rad} = \ell_{eff}$ are strongly dependent

on length $\left(\frac{\Delta z}{\lambda}\right)^2$ or $\left(\ell/\lambda\right)^2$,

while D_0 and A_{eff} are indifferent

to the length!

Radiation Characteristics Compared

$$\frac{\eta}{\pi}\left|\frac{I_0}{2}\right|^2 (1.22)$$

	Ideal	Short	Half-wave
	$\Delta z \ll \lambda$	$l \ll \lambda$	$\frac{\lambda}{2} = l$
P_{RAD}	$\frac{\pi}{3}\eta\left\|\frac{I_0 \Delta z}{\lambda}\right\|^2$	$\frac{\pi}{3}\eta\left\|\frac{I_0 l}{2\lambda}\right\|^2$	$\frac{\eta}{4\pi}\left\|I_0\right\|^2(1.22)$
D_0	$3/2$	$3/2$	1.643
A_{eff}	$3\lambda^2/8\pi$	$3\lambda^2/8\pi$	$\frac{3.2}{8\pi}\lambda^2$
$R_{eff} = R_{RAD}$	$80\pi^2\left(\frac{\Delta z}{\lambda}\right)^2 \Omega$	$20\pi^2\left(\frac{l}{\lambda}\right)^2 \Omega$	$73\,\Omega$
	$789 \cdot (\text{"})^2$	$197 \cdot (\text{"})^2$	