Summary:

Fields of an Ideal, Infinitesimal Dipole

$$\bar{H} = jk \frac{I \Delta z}{4\pi r}\left[1 + \frac{1}{jkr}\right] e^{-jkr} \sin\theta \, \bar{a}_\phi$$

$$E_r = \eta \frac{I \Delta z}{2\pi r^2}\left[1 + \frac{1}{jkr}\right] e^{-jkr} \cos\theta$$

$$E_\theta = jk\eta \frac{I \Delta z}{4\pi r}\left[1 + \frac{1}{jkr} + \frac{1}{(jkr)^2}\right] e^{-jkr} \sin\theta$$

$$\bar{E} = E_r \bar{a}_r + E_\theta \bar{a}_\theta$$

$$E_\phi = H_\theta = H_r = 0 \; ; \quad k = \frac{2\pi}{\lambda} \; ; \quad \eta = \sqrt{\frac{\mu}{\epsilon}}$$

Ideal Dipole - What are fields like?

There are <u>three</u> readily identifiable regions

$kr \ll 1$ Reactive (Near-field) Region

$kr > 1$ Transition (Intermediate) Region

$kr \ggg 1$ Propagating (Far-field) Region

$$\left(\frac{r}{\lambda} \ll \frac{1}{2\pi} \; ; \; \frac{r}{\lambda} > \frac{1}{2\pi} \; ; \; \frac{r}{\lambda} \ggg \frac{1}{2\pi} \right)$$

Summary

Far-Field. $\left\{ \begin{array}{l} r/\lambda >> \frac{1}{2\pi} \\ kr >> 1 \end{array} \right\}$ $\left. \begin{array}{l} r/\lambda << \frac{1}{2\pi} \\ kr << 1 \end{array} \right\}$ Near-field

$$\underline{\underline{H_\phi}} = jk\left[\frac{I\Delta z}{4\pi r} e^{-jkr}\right] \sin\theta \left[\underline{\underline{1}} + \frac{1}{\underline{\underline{jkr}}}\right]$$

$$E_r = \eta \left[\frac{I\Delta z}{2\pi r^2} e^{-jkr}\right] \cos\theta \left[1 + \frac{1}{j\underline{\underline{kr}}}\right]$$ drops out in far-field

$$\underline{\underline{E_\theta}} = jk\eta\left[\frac{I\Delta z}{4\pi r} e^{-jkr}\right] \sin\theta \left[\underline{\underline{1}} + \frac{1}{jkr} + \frac{1}{\underline{\underline{(jkr)^2}}}\right]$$

$$E_\phi = H_r = H_\theta = 0$$

$kr >> 1$, say 10, $r/\lambda = \frac{10}{2\pi} = 1.7$

$r/\lambda = 2$, $kr = 12$ /...

/ ...

Now, for $kr \ll 1$, $e^{-jkr} \simeq 1$ so

E_r, E_θ are same forms as for a static charged dipole

H_ϕ is same as for a constant current element - compare with Biot - Savart Law.
(it's the same!)

Fields are quasi-static.

/ ...

Summary

$r/\lambda \ll \frac{1}{2\pi}$
$kr \ll 1$ } Near-field

$$\underline{\underline{H_\phi}} = jk\left[\frac{I\Delta z}{4\pi r} e^{-jkr}\right] \sin\theta \quad \left[\frac{1}{\underline{\underline{jkr}}}\right]$$

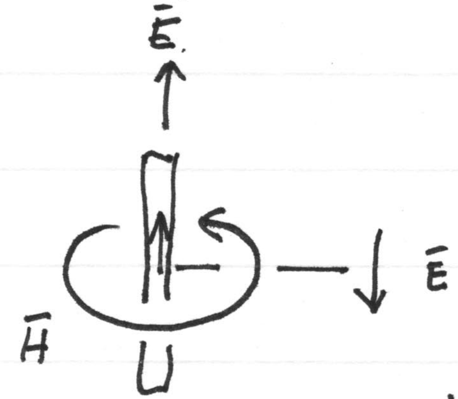
$$E_r = \eta\left[\frac{I\Delta z}{2\pi r^2} e^{-jkr}\right] \cos\theta \quad \left[\frac{1}{j\underline{\underline{kr}}}\right]$$ } drops out in far-field

$$\underline{\underline{E_\theta}} = jk\eta\left[\frac{I\Delta z}{4\pi r} e^{-jkr}\right] \sin\theta \quad \left[\frac{1}{jkr} + \frac{1}{\underline{\underline{(jkr)^2}}}\right]$$

$$E_\phi = H_r = H_\theta = 0$$

Reactive Region, $kr \ll 1$ $\left(\frac{r}{\lambda} \ll \frac{1}{2\pi} \approx 0{,}16 \right)$

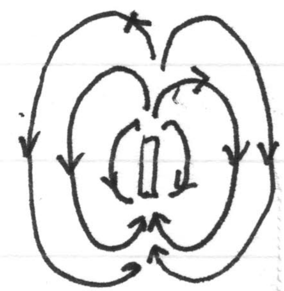
$$E_r \simeq -j\eta \frac{I\Delta z}{2\pi k r^3} e^{-jkr} \cos\theta$$



$$E_\theta \simeq -j\eta \frac{I\Delta z\, e^{-jkr}}{4\pi k r^3} \sin\theta$$

$\Delta z \ll r, \lambda$!!

$$H_\phi \simeq \frac{I\Delta z}{4\pi r^2} e^{-jkr} \sin\theta$$



CAPACITIVE ("E" lags "H") /...

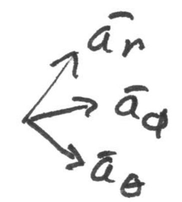
$\bar{E}$, $\bar{H}$ are in phase and space quadrature

fields are highly reactive ! (recall that there are neglected terms, tho)

H_ϕ is in time phase with I, while E is $\pi/2$ 'behind' I. "Voltage" lags "current", so fields are said to be capacitive.

E_r, E_θ have the same r^{-3} variation, although their magnitudes differ by a factor of 2.

Is there real power flow?



$$\bar{W} = \frac{\bar{E}\times\bar{H}^*}{2} = \bar{a}_r E_\theta H_\phi^* - \bar{a}_\theta E_r H_\phi^* + 0\cdot\bar{a}_\phi$$

$$Re(\bar{W}) = 0$$

$$\bar{a}_r E_\theta H_\phi^* = -j\eta\frac{(I\Delta z)^2}{(4\pi)^2 k r^5}\sin^2\theta\,\bar{a}_r$$

$$-\bar{a}_\theta E_r H_\phi^* = j\eta\frac{(I\Delta z)^2}{8\pi^2 k r^5}\sin\theta\cos\theta\,\bar{a}_\theta$$

Think about this!

? !

Reactive Field Region - Poynting's Vector

$$\left.\begin{aligned}\overline{W} = & -j\eta\,\frac{(I\Delta z)^2}{(4\pi)^2 k r^5}\sin^2\theta\,\bar{a}_r \\ & + j\eta\,\frac{(I\Delta z)^2}{8\pi^2 k r^5}\sin\theta\cos\theta\,\bar{a}_\theta\end{aligned}\right\}$$

note very rapid fall-off of reactive power with r.

"Like" standing waves - no net transmission of power. Energy is stored in $\bar{E}$, $\bar{H}$, in turn.

(compare with standing waves on a transmission line)

Transition Region $kr > 1 \quad \left(\frac{r}{\lambda} > \frac{1}{2\pi}\right)$

$$E_r \simeq \eta \frac{I \Delta z}{2\pi r^2} e^{-jkr} \cos\theta$$

out of phase

$$E_\theta \simeq jk\eta \frac{I \Delta z}{4\pi r} e^{-jkr} \sin\theta$$

in phase

$$H_\phi \simeq jk \frac{I \Delta z}{4\pi r} e^{-jkr} \sin\theta$$

$$W_r = \frac{1}{2} (E \times H^*)_r \neq 0$$, outward power flow

more important (?)

Transition Region is confusing – rewrite eqs. for E_r, E_θ as

$$E_r = jk\eta\left(\frac{I\,\Delta z}{2\pi r}\,e^{-jkr}\right)\left[\frac{1}{jkr} + \frac{1}{(jkr)^2}\right]\cos\theta$$

$$E_\theta = jk\eta\left(\frac{I\,\Delta z}{4\pi r}\,e^{-jkr}\right)\left[1 + \frac{1}{jkr} + \frac{1}{(jkr)^2}\right]\sin\theta$$

Then, even two wavelengths from the infinitesimal dipole, $r/\lambda = 2$, $kr \simeq 12$

$$\left|\frac{E_r}{E_\theta}\right| \simeq 2\cdot\frac{1}{kr}\Bigg|_{kr=12} \simeq 0.16 \quad \text{which is significant}$$

Example of Transition Region $kr = 1$,

$$H_\phi = jk\left(\frac{I\Delta z}{4\pi r}e^{-j}\right)\left[1-j\right]\sin\theta \quad \propto \underline{\underline{\sqrt{2}e}}^{-j\pi/4}$$

$$E_r = jk\eta\left(\frac{I\Delta z}{2\pi r}e^{-j}\right)\left[-j-1\right]\cos\theta \quad \propto \underline{\underline{\eta\sqrt{2}e}}^{-j\frac{3\pi}{4}}\cdot 2$$

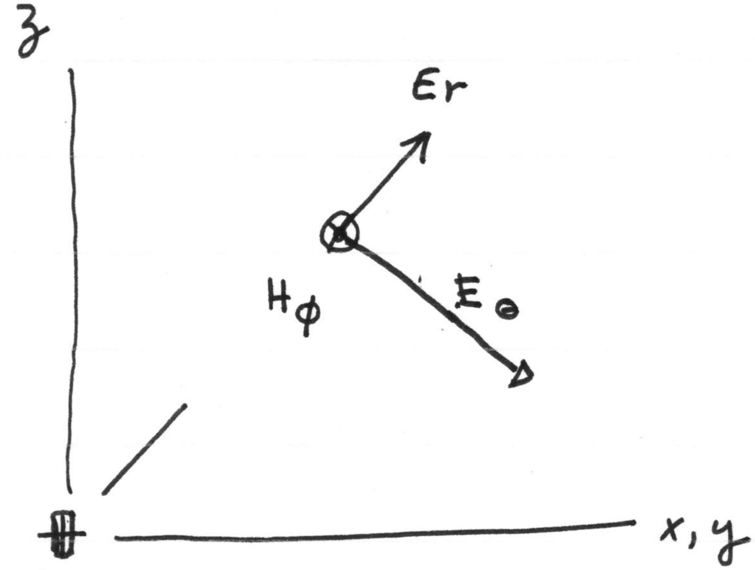
$$E_\theta = jk\eta\left(\frac{I\Delta z}{4\pi r}e^{-j}\right)\left[-j\right]\sin\theta \quad \underline{\underline{\propto \eta\cdot 1\cdot e}}^{-j}$$

$\bar{a}_\theta \times \bar{a}_\phi = \bar{a}_r$

$\bar{a}_r \times \bar{a}_\phi = -a_\theta$

$$\left|\frac{E_r}{E_\theta}\right| = \sqrt{2}\cdot 2$$

So in the transition region E_r is still very important!



H_ϕ, E_r always time quadrature! – never contributes to real power flow

H_ϕ, E_θ time quadrature to in-phase $kr = 1 \sim 100$

Summary

Far-Field. $\left\{ \begin{array}{l} r/\lambda >> \frac{1}{2\pi} \\ kr >> 1 \end{array} \right\}$

$$\underline{\underline{H_\phi}} = jk\left[\frac{I\Delta z}{4\pi r} e^{-jkr}\right]\sin\theta\left[\underline{\underline{1}}\right]$$

$$E_r \sim E_\theta (2/jkr)\cot\theta \to 0, r\to\infty$$

$$\underline{\underline{E_\theta}} = jk\eta\left[\frac{I\Delta z}{4\pi r} e^{-jkr}\right]\sin\theta\left[\underline{\underline{1}}\right]$$

$$E_\phi = H_r = H_\theta = 0$$

$kr >> 1$, say 10, $r/\lambda = \frac{10}{2\pi} = 1.7$

$r/\lambda = 2$, $kr = 12$ /...

Propagating Region

$kr >>> 1 \qquad \frac{r}{\lambda} >>> \frac{1}{2\pi}$

$$E_\theta = j\eta k \frac{I\Delta z}{4\pi r} e^{-jkr} \sin\theta$$

$$H_\phi = jk \frac{I\Delta z}{4\pi r} e^{-jkr} \sin\theta$$

$\propto \frac{1}{r}$

time phase,

space quadrature

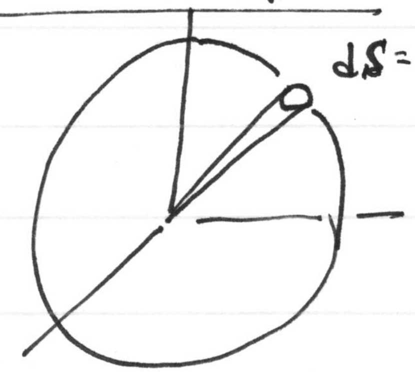
$E_\phi = H_\theta = H_r = 0$; E_r negligible

Locally, waves resemble plane waves.

$$\bar{W}_{ave} = \frac{1}{2} Re(E \times H^*) = \eta \frac{k^2}{2} \frac{(I\Delta z)^2}{(4\pi r)^2} \sin^2\theta \ \bar{a}_r$$

/...

How much power "leaves" an Ideal Dipole ?



$$\text{Power} = \int_0^{2\pi}\int_0^{\pi} \bar{W}\cdot d\bar{S}$$

$$\text{Power} = \int_0^{2\pi}\int_0^{\pi} \frac{\eta}{8}\left(\frac{I\Delta z}{\lambda}\right)^2 \frac{\sin^2\theta}{r^2} \cdot r^2 \sin\theta \, d\theta \, d\phi$$

$$= \int_0^{2\pi}\int_0^{\pi} \frac{\eta}{8}\left(\frac{I\Delta z}{\lambda}\right)^2 (1-\cos^2\theta)\sin\theta \, d\theta \, d\phi \quad =$$

/...

Simplifying the above

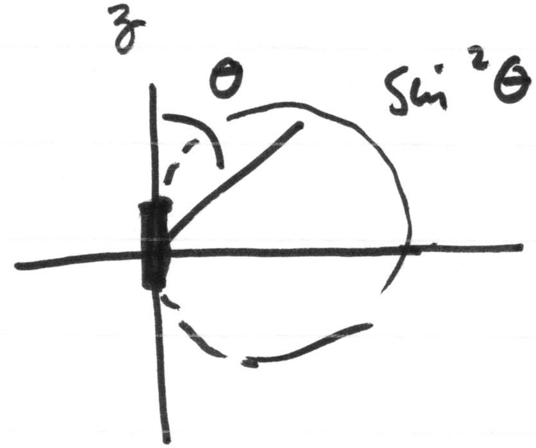
$$W_{ave} = \eta/8 \left(\frac{I \Delta z}{\lambda}\right)^2 \frac{\sin^2\theta}{r^2}$$

$\propto I^2 \eta$

$\propto \left(\frac{\Delta z}{\lambda}\right)^2$

$\propto \sin^2\theta$

$\propto \frac{1}{r^2}$



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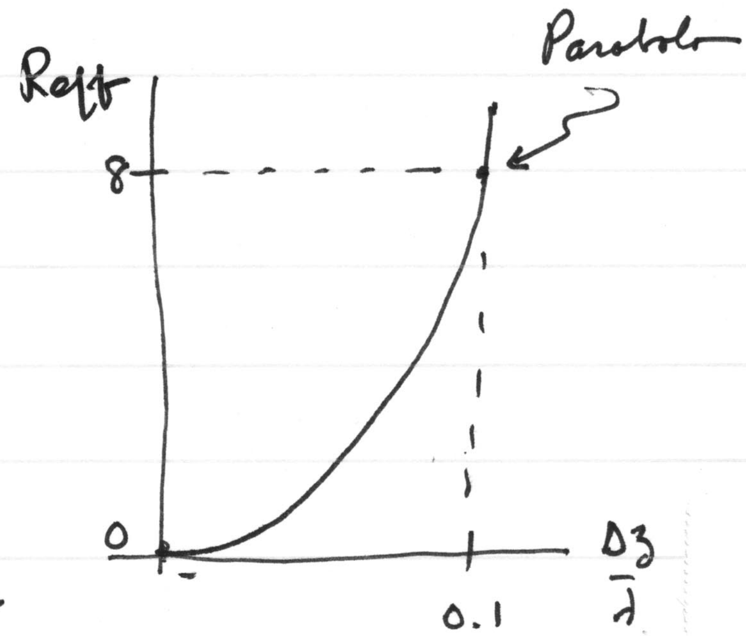
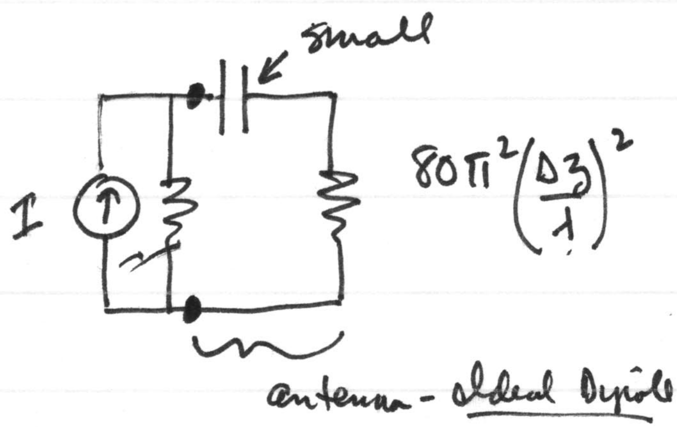
$$\frac{I^2}{2} R_{eff} = \text{Power} = \frac{\pi\eta}{3}\left(I\frac{\Delta z}{\lambda}\right)^2$$

$$R_{eff} = 80\pi^2\left[\frac{\Delta z}{\lambda}\right]^2$$

$$= 789\left[\frac{\Delta z}{\lambda}\right]^2$$



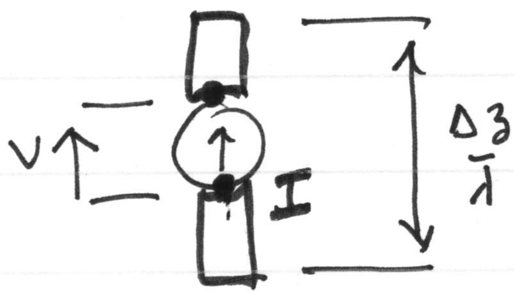


Why a series circuit ?

/...

/...

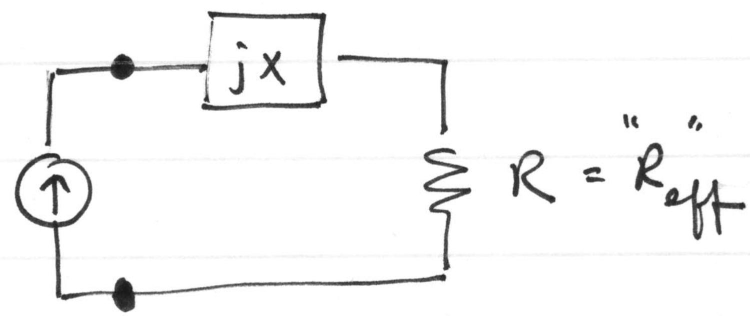
$$\text{Power} = \frac{\eta}{8}\left[\frac{I\,\Delta z}{\lambda}\right]^2 \cdot 2\pi \cdot \frac{4}{3} = \frac{\pi\eta}{3}\left[\frac{I\,\Delta z}{\lambda}\right]^2$$



$$V = (R + jX)\,I$$

$$R = \mathrm{Re}\left(\frac{V}{I}\right)$$

Why the series equivalent circuit?



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<u>Same</u> current effects . C, R

$R \to 0$ as $\lambda \to \infty$, but circuit goes

to an open.