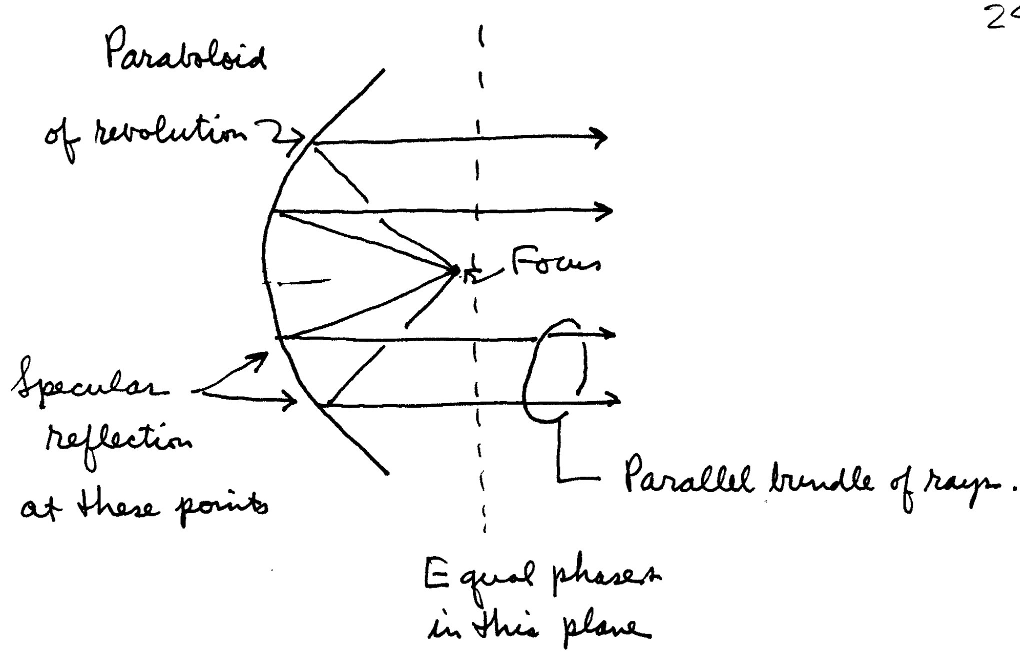
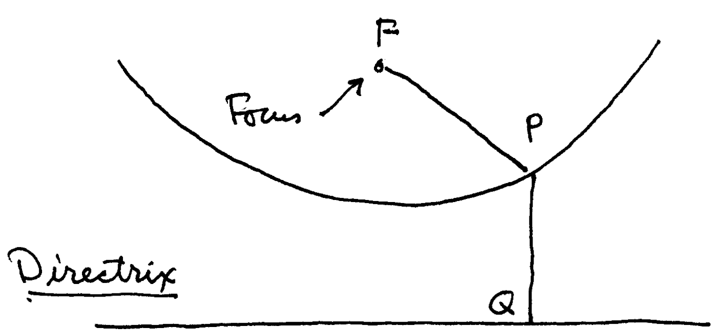
Focused Apertures : Most Common Application of aperture antennas

24-1



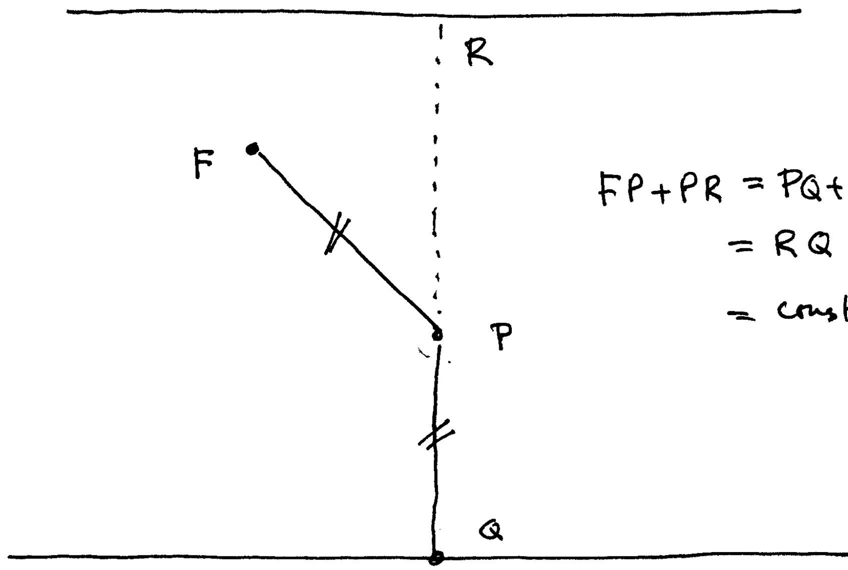
24-2

Parabola FP = PQ



Curve is equally distant from the focus and the directrix

24-3

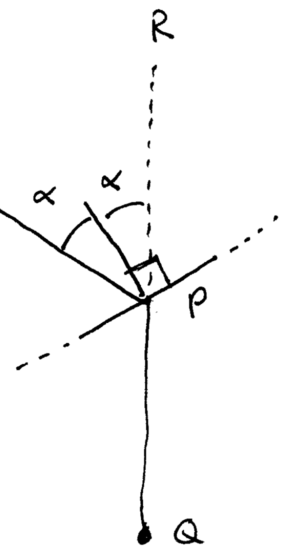


$$FP + PR = PQ + PR$$
$$= RQ$$
$$= \text{constant!}$$

24-4

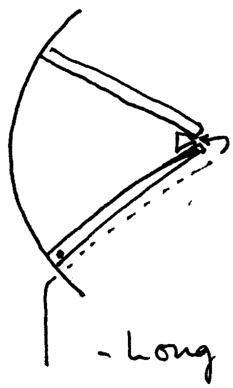
Specular Reflection

& Parallel Rays



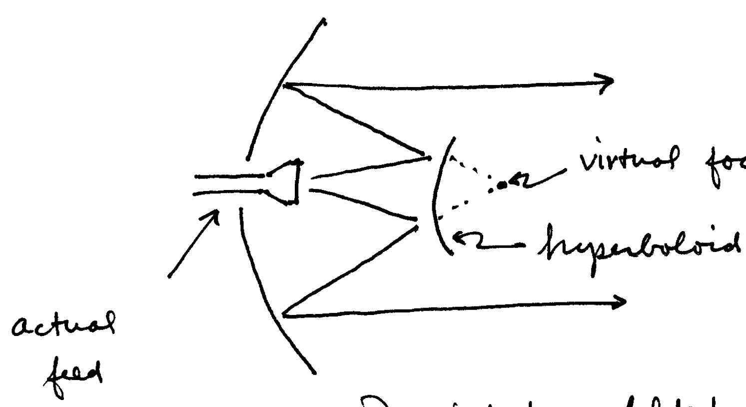
RF Applications : Feed arrangement carries energy to & from focal point

This geometrical arrangement called Newtonian



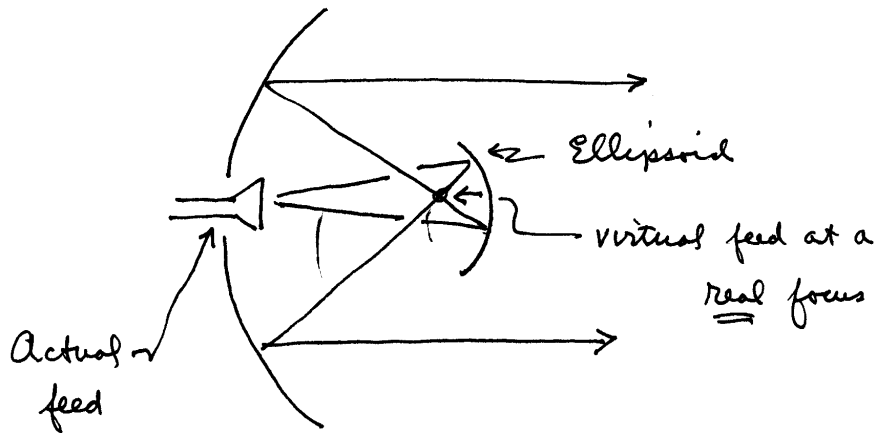
- Long transmission line
- difficult access
- limited space near focus

Alternative #1 Cassegrainian

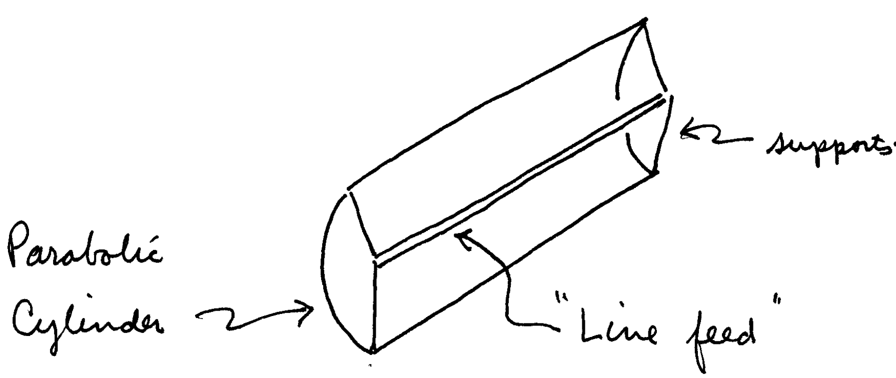


Derived from folded telescope technology / terminology

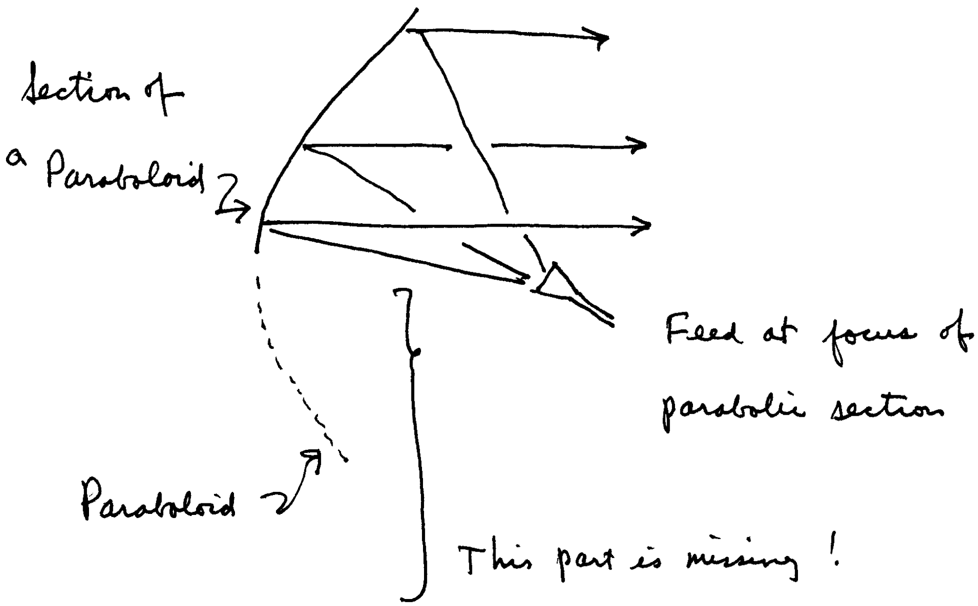
Alternative #2 : Gregorian



Right cylinders can be used, also.

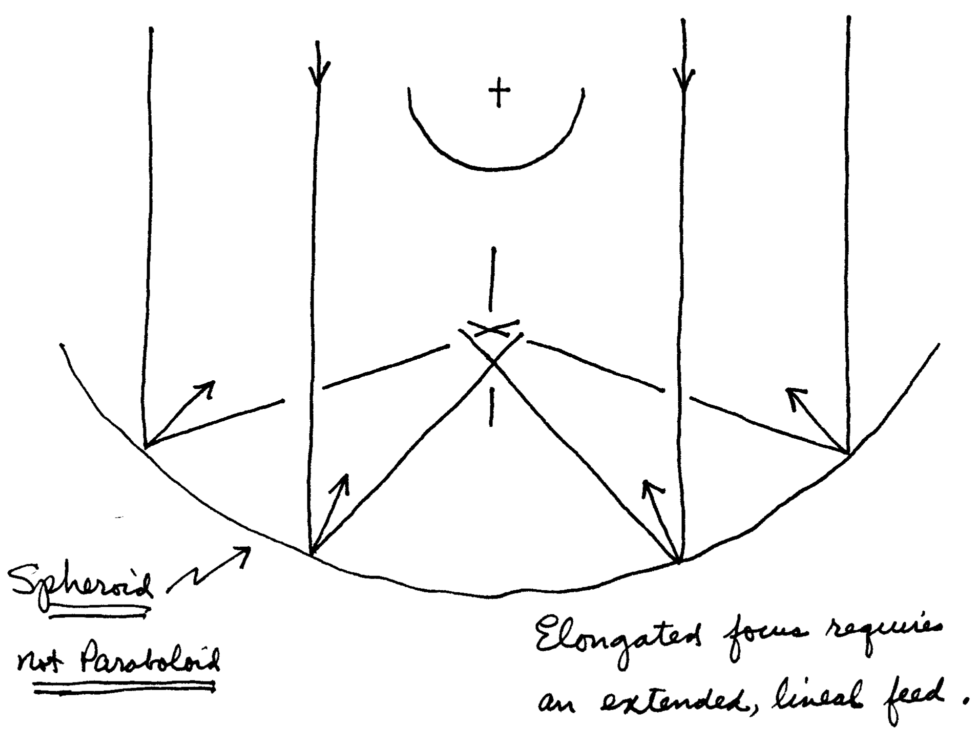


Offset Paraboloidal Section

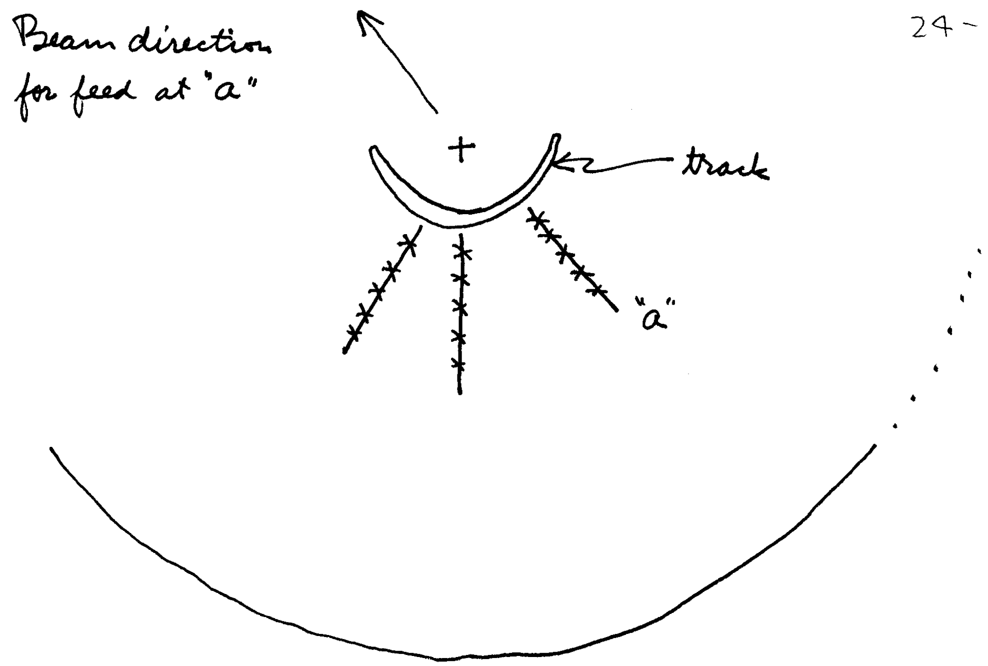


There are many, many other possibilities !

Not all reflector antennas are paraboloids

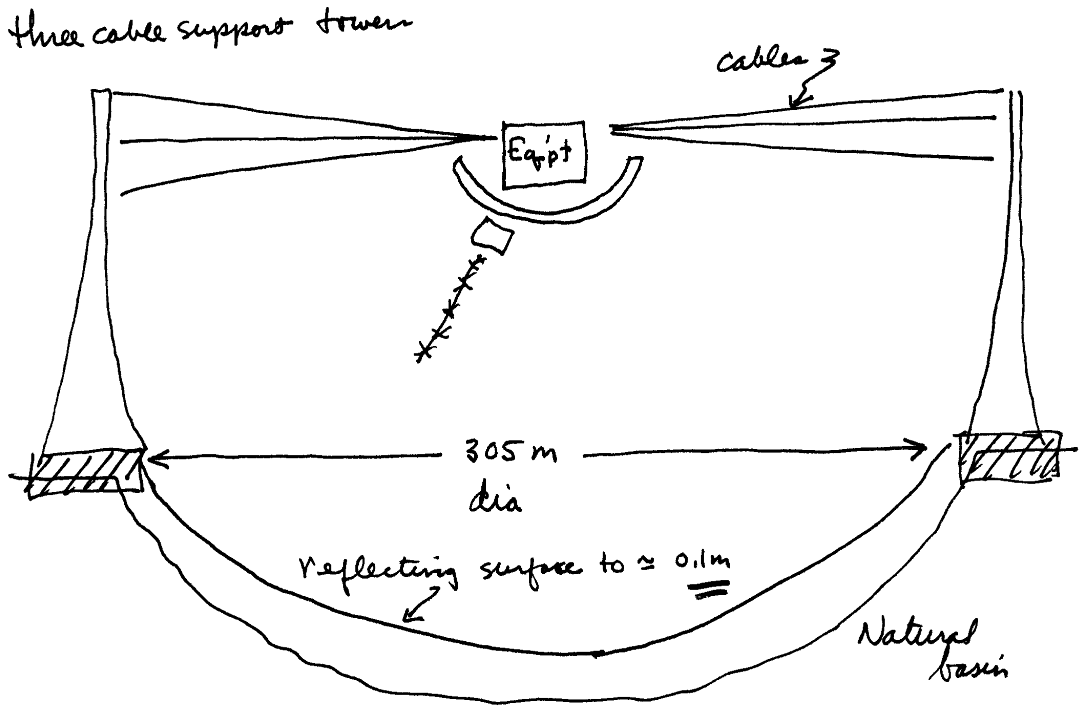


24-11



Arecibo Implementation

24-12



Other Large Antennas

Effielsberg	100 m - dia
DSN - NASA	70 m (three sites)
Arecibo	305 m
Usuda	64 m
Parkes	64 m

Numerous 30 to 40 m diameter antennas

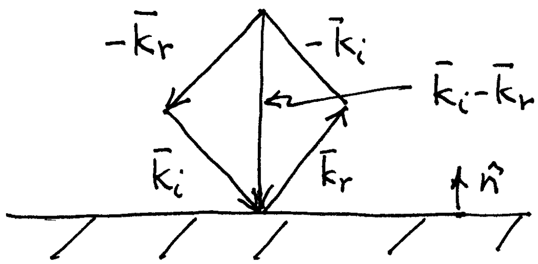
Methods of Analysis - Aperture Antennas

1. Aperture Distribution Method
   - use Geometric Optics to find field distribution over an aperture, then
   - use Equivalence Thm to find effective source distribution, and then
   - calculate the far-field.

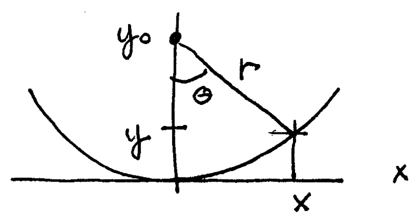
Geometric Optics : Use ray tracing to define tubes of flux (constant total power). At boundaries the rule is 24-15

$$\hat{n} \times (\bar{k}_i - \bar{k}_r) = 0$$

$\Rightarrow$ bisector of $\bar{k}_i, \bar{k}_r$ is $\perp$ to surface



An aside 24-16



$r(1+\cos\theta) = 2f$?

$r^2 = (y_0 - y)^2 + x^2 = \Delta y^2 + x^2$

$r\cos\theta = y_0 - y = \Delta y$

substituting:

$(\Delta y^2 + x^2)^{1/2} + \Delta y = 2f$

$\cancel{\Delta y^2} + x^2 = (2f - \Delta y)^2 = 4f^2 - 4f\Delta y + \cancel{\Delta y^2}$

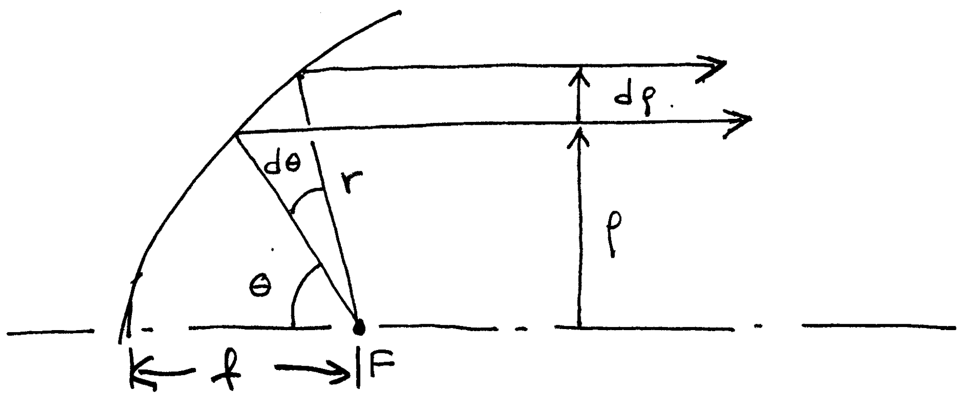
$4f\Delta y = 4f^2 - x^2$

$y_0 - y = f - x^2/4f \rightarrow y = \frac{x^2}{4f} + (y_0 - f)$ Eq. of a parabola

$y = 0, x = 0$ if $y_0 = f$

Energy is assumed to travel in straight lines along rays, "tubes" of energy flow are bounded by surfaces of rays. 24-17

$r(1+\cos\theta) = 2f$

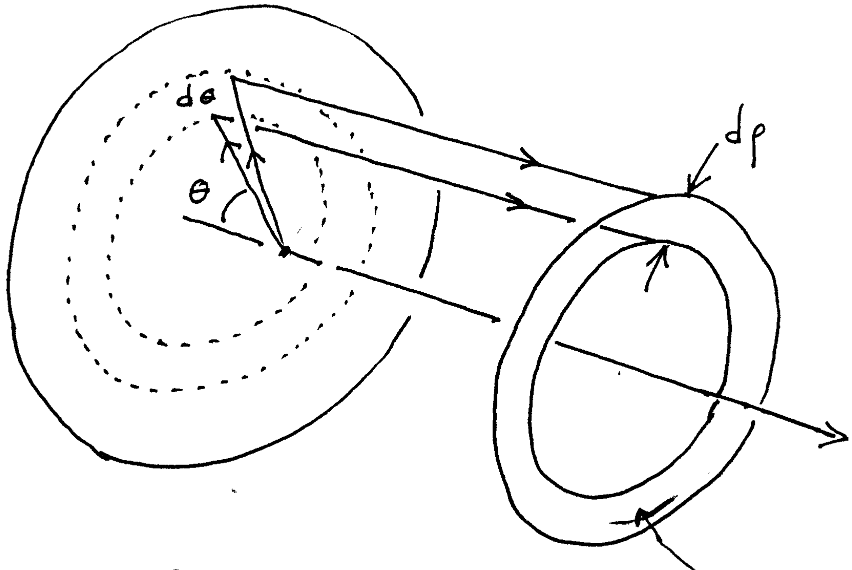


Energy flowing in $d\theta$ ends up in $d\rho$

/...

Redrawing Ray Paths

24-18



Annulus corresponding to $d\theta$

What is the power in the annulus?

$U(\theta)$ watts/steradian = radiation intensity from Newtonian feed point

Power in annulus / unit area =

$$\frac{U(\theta)\cdot 2\pi\cdot\sin\theta\, d\theta}{2\pi\rho\, d\rho} = ?$$

/...

Note: $U(\theta) = P(\theta)/4\pi$: Total power in annulus =

$$\frac{P(\theta)}{4\pi r^2}\left(2\pi r\sin\theta\, r d\theta\right) = U(\theta)\, 2\pi\sin\theta\, d\theta$$

$\rho = r\sin\theta$; $r(1+\cos\theta) = 2f \Rightarrow r = 2f(1+\cos\theta)^{-1}$

$$d\rho = \left(\frac{\partial\rho}{\partial r}\cdot\frac{dr}{d\theta} + \frac{\partial\rho}{\partial\theta}\right)d\theta$$

$$\frac{\partial\rho}{\partial\theta} = r\cos\theta \;;\quad \frac{\partial\rho}{\partial r} = \sin\theta$$

$$\frac{dr}{d\theta} = \frac{d}{d\theta}\left(\frac{2f}{1+\cos\theta}\right) = \frac{r\sin\theta}{1+\cos\theta}$$

$$\therefore\ \frac{d\rho}{d\theta} = \left(\frac{r\sin^2\theta}{1+\cos\theta} + r\cos\theta\right) = \left[\frac{\sin^2\theta+\cos\theta+\cos^2\theta}{1+\cos\theta}\right]r = r$$

(!)

/...

/...

So, $$U(\theta) \frac{2\pi \sin\theta \, d\theta}{2\pi \rho \, d\rho} = \underbrace{\frac{U(\theta)}{r^2} \text{ watts}/m^2}_{\text{value in the annulus}}$$

(and) So, the "illumination" of the aperture has a "natural"* $\frac{1}{r^2}$ taper across the radiating "surface", with respect to a simple mapping of $U(\theta)$ from the feed point radiation.

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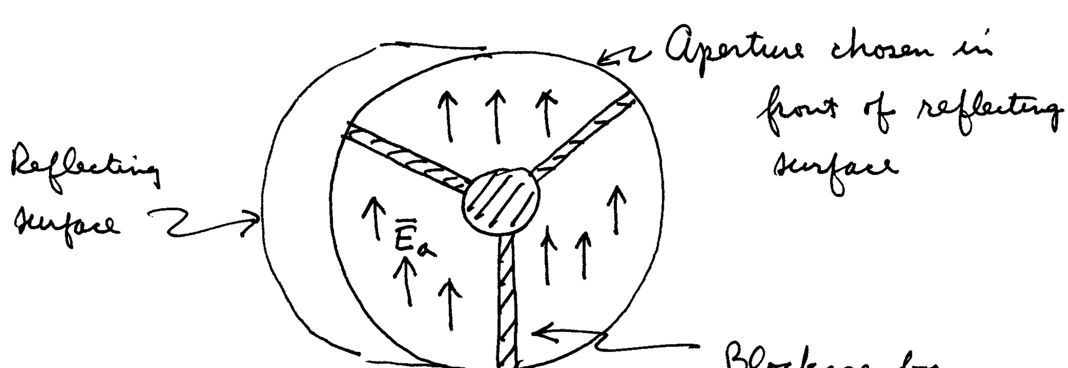
* "geometric" might be a more apt term.

Geometric Optics can be used to connect the fields radiated by the feed to a field distribution over the aperture.



Equivalence Principle can then be used to transfer aperture

2. Current Distribution Method

- Physical Optics used to find currents on surface of antenna
- Radiation Integral used to find far-field from surface distribution of currents
- req's assumptions – (typ)
  zero currents on back side of reflector
  discontinuity at rim is negligible
  blockage is negligible, or find $\bar{J}$ on blocked area, or use GTD, etc.

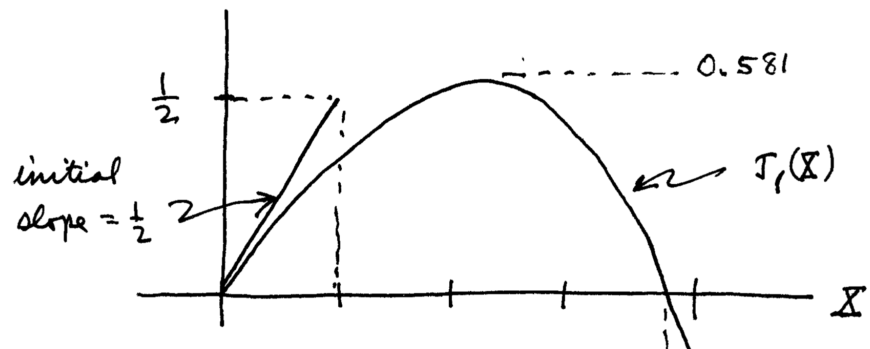
Either the Aperture Distribution Method or the Current Distribution Method is OK for the main lobe and the first few sidelobes

Current Distribution Method with corrections for edges, blockage, etc, is better, but can be difficult.

Aperture Distribution Method is an easy first cut at the problem.

Circular Aperture w/ Uniform Illumination

$$P(\theta) \rightarrow \pi a^2 \, 2 \, \frac{J_1(X)}{X} \; ; \quad X = \frac{2\pi a}{\lambda} \sin\theta$$



$Ka \sin\theta = 3.83$ is first null

$$\theta_{H.P.}\Big|_{\text{Full width}} = 1.02 \frac{\lambda}{2a} \; ;$$

$$G = \frac{4\pi A_{eff}}{\lambda^2} \, , \quad A_{eff} = A$$

/...

/...

Actually, it is very difficult to obtain uniform illumination, as this requires

$$\sqrt{\frac{U(\theta)}{r}} = \left[\sqrt{U(\theta)} \; \frac{(1+\cos\theta)}{2f} \right] = \text{constant}$$

So a typical parabolic antenna has a tapered illumination, giving somewhat lower sidelobes than is obtained for the uniformly illuminated case.