# Summary Procedure

## General

1. Select surface on which $\bar{E}, \bar{H}$ are known.
   Choose $\bar{E}_a$, $\bar{H}_a$

2. Form equivalent current densities $\bar{J}_s, \bar{M}_s$
   over the chosen surface...
   $\bar{J}_s = \hat{n} \times \bar{H}_a$ , $\bar{M}_s = -\hat{n} \times \bar{E}_a$

3. Determine the potentials $\bar{A}$, $\bar{F}$ } in far-field

4. Determine $\bar{E}, \bar{H}$ } or near field

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## Summary Procedure - Far-Field Only

(refering to steps above)

1. ✓

2. ✓

3. $N_\theta, N_\phi, L_\theta, L_\phi$ as in example
   just completed

4. $\bar{E}, \bar{H}$, as in example

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## Principal Plane Patterns

**E - plane** (yz plane) $\phi = \pi/2$

$$E_\phi = E_r = H_r = 0$$

$$E_\theta = -j\frac{kabE_0}{2\pi r}e^{-jkr}\overset{1}{\cancel{\sin\phi}}\left[\overset{1}{\cancel{\frac{\sin X}{X}}}\cdot\frac{\sin Y}{Y}\right]$$

$$E_\theta = -j\frac{kabE_0}{2\pi r}e^{-jkr}\frac{\sin\left[\frac{bk\sin\theta}{2}\right]}{\left[\frac{bk\sin\theta}{2}\right]}$$

$$\vec{H} = +E/\eta\ \hat{a}_\phi$$

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/...

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H - plane (xz plane) $\phi = 0$

$$E_r = E_\theta = 0$$

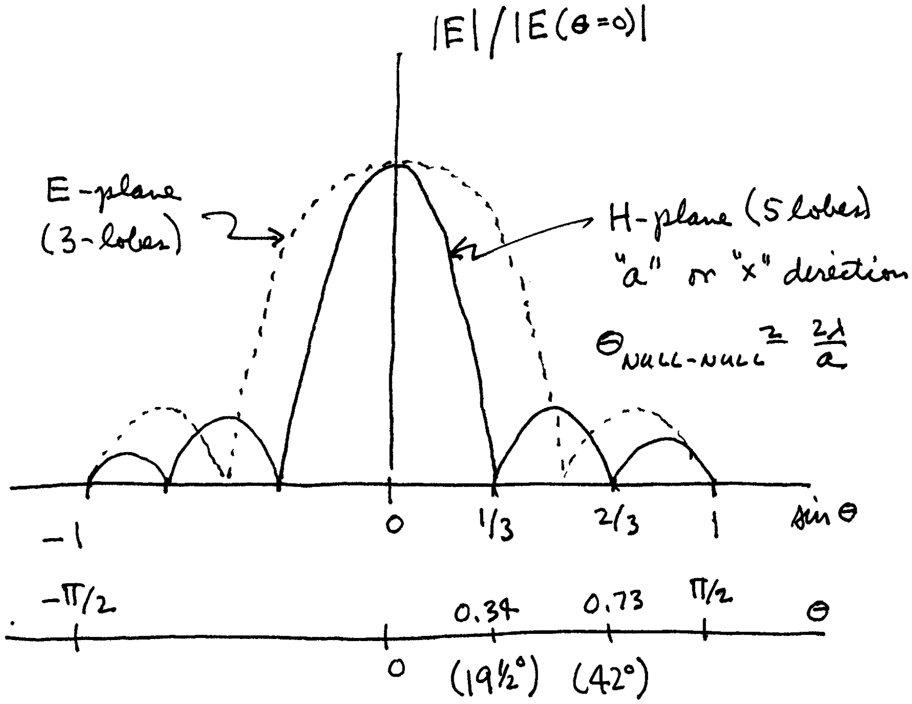
$$E_\phi = +j\frac{kabE_0}{2\pi r}e^{-jkr}\cos\theta\ \frac{\sin\left\{\frac{ka\sin\theta}{2}\right]}{\left[\frac{ka\sin\theta}{2}\right]}$$

$$H_\theta = ?$$

/...

Suppose $a = 3\lambda$, $b = 2\lambda$; $\frac{ka}{2} = 3\pi$, $\frac{kb}{2} = 2\pi$

<u>No grating lobes!</u>

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<u>Directivity</u> ?

$$D_o = U_{MAX} / P_{rad}/4\pi$$

$$U_{MAX} = r^2 W = \frac{\cancel{r^2} a^2 b^2 k^2 |E_o|^2}{(2\pi)^2 \cancel{r^2}\, 2\eta} = \left(\frac{ab}{\lambda}\right)^2 \frac{|E_o|^2}{2\eta}$$

$$P_{rad} = ? = \frac{|E_a|^2}{2\eta} \cdot a \cdot b$$ (integrate over aperture)

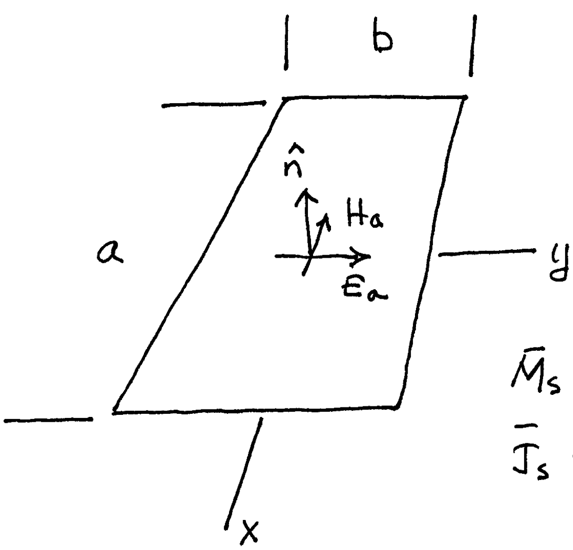
/...

Then

$$D_o = 4\pi \frac{\left(\frac{ab}{\lambda}\right)^2 \frac{|E_o|^2}{2\eta}}{\frac{|E_o|^2 ab}{2\eta}} = \frac{4\pi ab}{\lambda^2} = \frac{4\pi \text{ Area}}{\lambda^2}$$

$$\boxed{A_{eff} = A_{physical} \text{ for uniform aperture !}}$$

## An Aperture in Free-Space is a Bit Different ! 23-8




$$\bar{E}_a = \hat{a}_y E_o$$

$$\bar{H}_a = -\hat{a}_x E_o/\eta$$

$$\left.\begin{aligned} \bar{M}_s &= -\hat{n} \times \bar{E}_a = E_o \hat{a}_x \\ \bar{J}_s &= \hat{n} \times \bar{H}_a = -\frac{E_o}{\eta} \hat{a}_y \end{aligned}\right\} \quad \begin{aligned} |x| &\le a/2 \\ |y| &\le b/2 \end{aligned}$$

$\bar{M}_s = \bar{J}_s = 0$, otherwise.

<u>This is an approximation</u> !! /...

/...



<u>As before:</u>

$$E_\theta = -jkG(L_\phi + \eta N_\theta)$$

$$E_\phi = +jkG(L_\phi - \eta N_\theta)$$

$$H_\theta = -\frac{E_\phi}{\eta} \; ; \quad H_\phi = \frac{E_\theta}{\eta}$$

$$\bar{L}, \; \bar{N}, \; = ?$$



$$L_\phi = \iint (-M_x \sin\phi + \cancel{M_y}^{\,0} \cos\phi)\, e^{+jkr'\cos\xi}\, dx'dy'$$

$$= \iint (-E_0 \sin\phi)\, e^{+jk(x'\sin\theta\cos\phi + y'\sin\theta\sin\phi)}\, dx'dy'$$

$$= -ab E_0 \sin\phi \, \frac{\sin X}{X} \cdot \frac{\sin Y}{Y}$$

Similarly,

$$L_\theta = ab E_0 \cos\theta \cos\phi \, \frac{\sin X}{X} \, \frac{\sin Y}{Y}$$

/...

/...



$$N_\theta = -\frac{abE_0}{\eta} \sin\phi \cos\theta \; \frac{\sin X}{X} \cdot \frac{\sin Y}{Y}$$

$$N_\phi = -\frac{abE_0}{\eta} \cos\phi \; \frac{\sin X}{X} \cdot \frac{\sin Y}{Y}$$

$$E_\theta = +jkabE_0 \frac{e^{-jkr}}{4\pi r} (1+\cos\theta) \sin\phi \; \frac{\sin X}{X} \cdot \frac{\sin Y}{Y}$$

$$E_\phi = +jkabE_0 \frac{e^{-jkr}}{4\pi r} \cos\phi \, (1+\cos\theta) \; \frac{\sin X}{X} \cdot \frac{\sin Y}{Y}$$

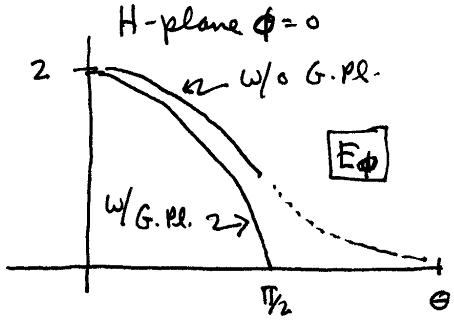


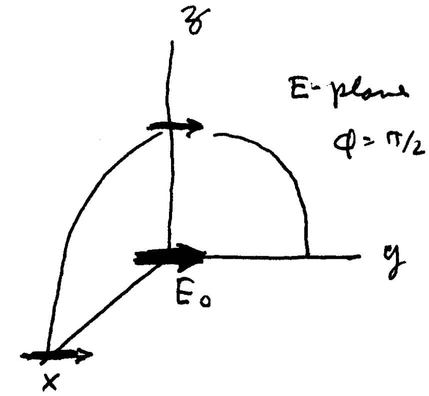
## Effect of Ground Plane — Comparison

$$\left.\begin{array}{lll} \text{with G.Pl} & E_\theta \sim 2\sin\phi \\ & E_\phi \sim 2\cos\theta\cos\phi \\ \text{without G-Pl} & E_\theta \sim (1+\cos\theta)\sin\phi \\ & E_\phi \sim (1+\cos\theta)\cos\phi \end{array}\right\} \begin{array}{l} \text{same} \\ \phi\text{-variation,} \\ \text{distinct} \\ \theta\text{-variations} \end{array}$$

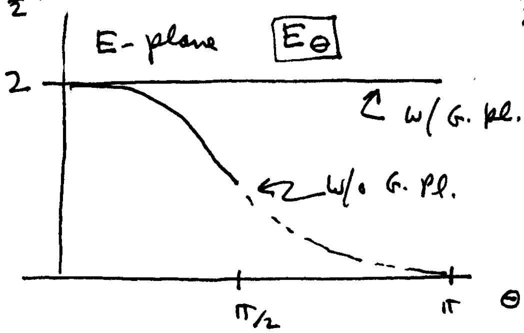
Note: These are not the patterns, just the differences in the "projection" factors.

/...




2:1 at $\theta = \frac{\pi}{2}$!




Principal Plane Cuts

<u>Again, these are not the radiation patterns!</u>

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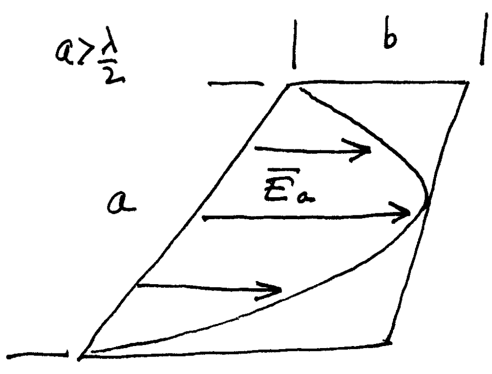
## <u>What about a realistic excitation?</u>




$TE_{10}$ wave guide excitation in an infinite ground plane.

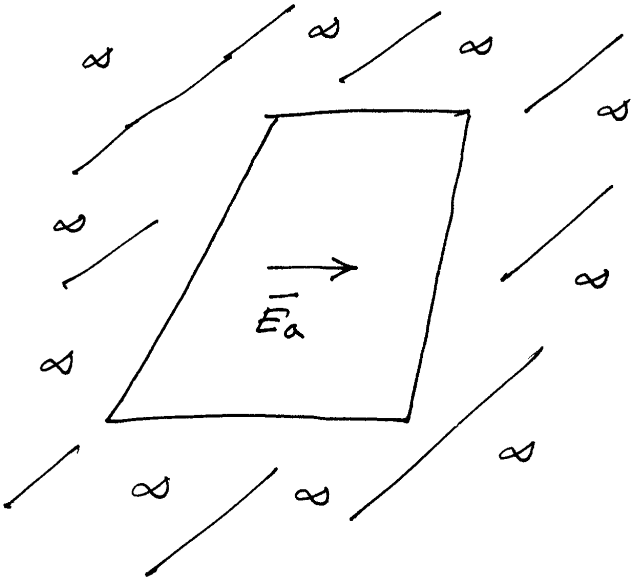
$$\bar{E}_a = E_0 \cos\left(\frac{\pi}{a} x'\right) \hat{a}_y$$

$$E_\theta = -\frac{\pi}{2} \frac{e^{-jkr}}{4\pi r} \sin\phi \; \frac{\cos X}{X^2 - (\pi/2)^2} \cdot \frac{\sin Y}{Y}$$

$$E_\phi = -\frac{\pi}{2} G \cos\theta \cos\phi \; \underbrace{\quad ''\quad}_{\text{modified H-plane}} \; \underbrace{\quad ''\quad}_{\text{same as before E-plane}}$$

$A_{eff} = 0.81\, A_{phys}$

Comments on Approximations 23-15




Aperture in a conducting plane

**Exact!**

Gnd plane eliminates effects of unknown $J_s$ in vicinity of aperture

I

$$E_\theta \propto 2 \sin\phi$$

$$E_\phi \propto 2 \cos\theta \cos\phi$$

$$M_s = -2\hat{n} \times \bar{E}_a$$

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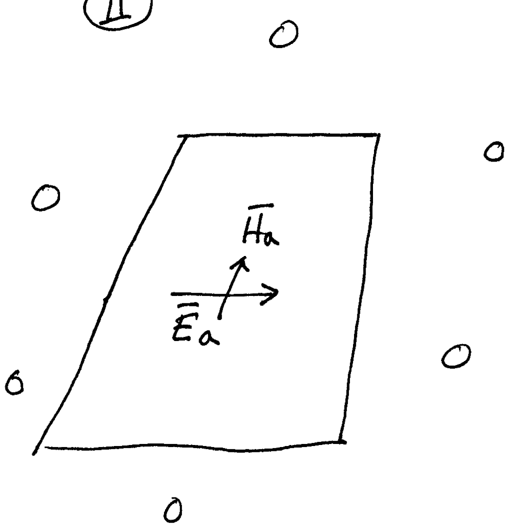
II




$\mathcal{F}$ Approximation

$$E_\theta \sim (1 + \cos\theta) \sin\phi$$

$$E_\phi \sim (1 + \cos\theta) \cos\phi$$

Ignores "currents"* outside of the aperture

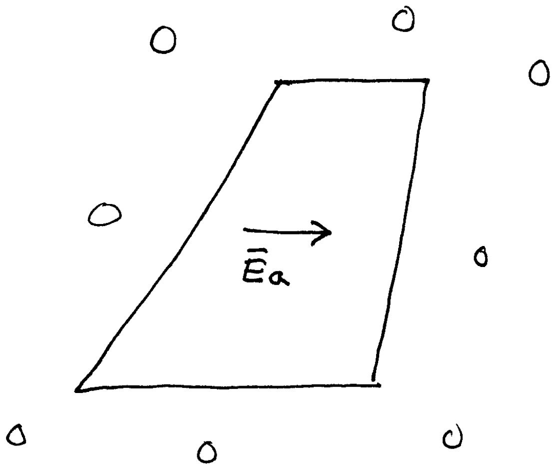
$$\bar{J}_s = \hat{n} \times \bar{H}_a$$

$$\bar{M}_s = -\hat{n} \times \bar{E}_a$$

* Equivalent currents

III

## S&T Approximation




$E_\theta \sim 2 \sin\phi$

$E_\phi \sim 2\cos\theta\cos\phi$

$\sigma_e = 0$ !

$\bar{M}_s = -2\hat{n} \times \bar{E}_a$

Must be in error by radiation from currents on the (effectively assumed) ground plane!