# EE 252 YAGI-UDA ARRAYS

S&T Y-U ARRAYS, SECTION 5.4

MUTUAL "Z," SECTION 3.6

KRAUS Y-U ARRAYS, SECTION 11.9

MUTUAL "Z," SECTIONS 10.4-10.9

All that we say about this you can confirm by MoM methods!

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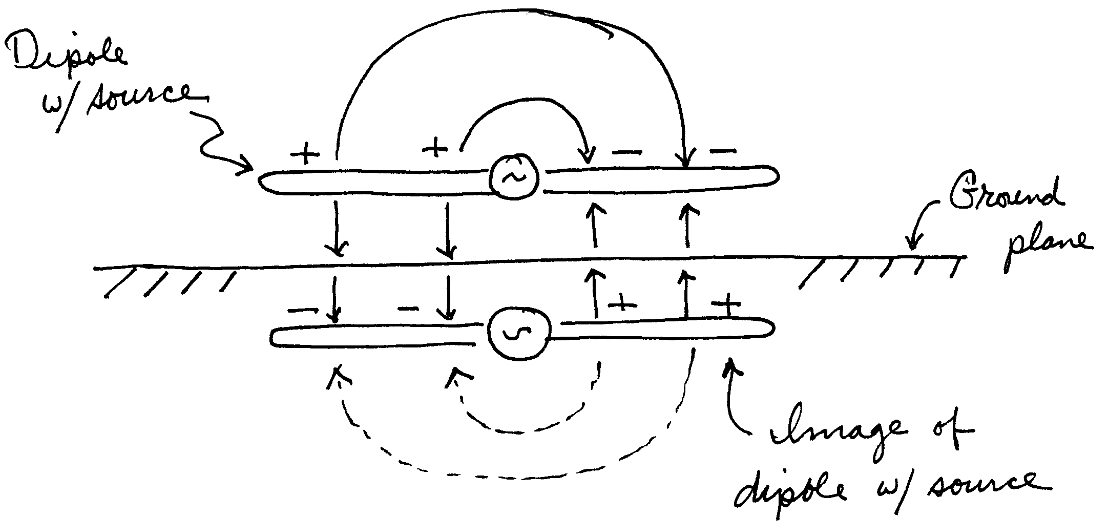
Yagi-Uda Arrays are both commonly employed antennas and an instructive lesson in applied electromagnetics. Beyond this, they also form a jumping-off point for understanding a large class of very wideband antennas known as log-periodic antennas or log-periodic structures. For these reasons we will attempt to develop a firm physical understanding of the Yagi-Uda example.

Consider first the situation in which an active dipole element is brought close to a large, plane conductor.

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In the above, the "image" ~~is~~ behaves as a "coupled" parasitic element coupled by fields to the source. Above the ground plane the effective radiation is from the combination of the dipole <u>and</u> its image. While in the figure we only represent the charge on the dipole, note that the <u>sign</u> of the charge is reversed in the lateral direction. Hence, the <u>directions</u> of the <u>horizontal</u> <u>dipole</u> <u>current</u> and the <u>horizontal</u> <u>image</u> current are <u>reversed</u>!

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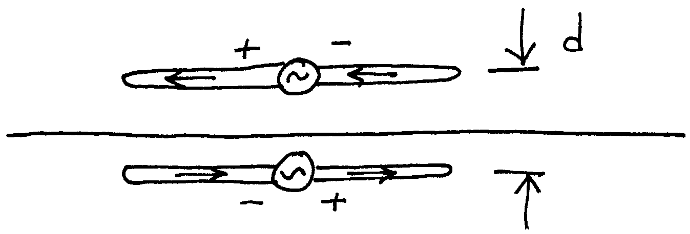
What do you expect will occur as the separation between the dipole and ground plane is reduced to zero?




As $d \to 0$

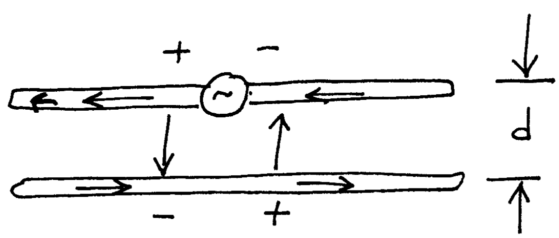
i) Radiation fields from dipole and image cancel.

ii) $Z_{11}$ decreases (input impedance to dipole)

iii) $R_{rad}$ decreases (effectively, for dipole + image)



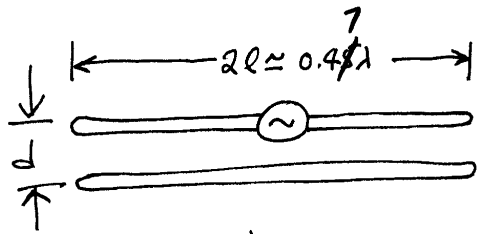
What about two identical dipoles — one with and one without a source?




We expect ~ equal and opposite currents on the second member relative to the driven element; for very close spacing the currents become almost equal. (Recall Pocklington's Equation and apply it in this situation.) Also expect symmetry to develop!

## Pattern of Two Closely Spaced Elements

Assume resonance, identical elements.




$d \ll \lambda$

These two elements form a very closely spaced array driven by very nearly equal but oppositely directed currents. Perpendicular to the array axis the effects of these currents cancel; along the axis they do not quite do so.

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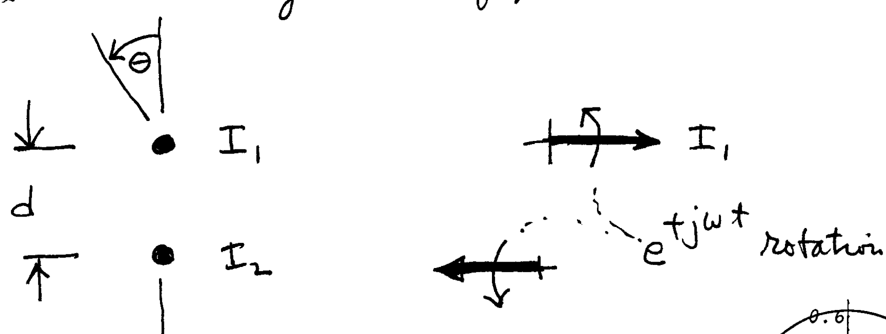
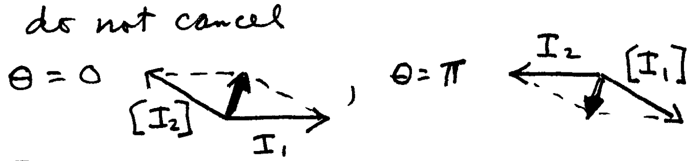
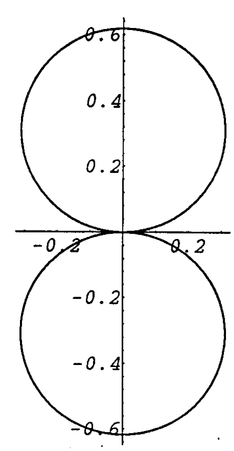
~~So~~ the array is configured like*




For $\theta = \pm \frac{\pi}{2}$, the effects of $I_1, I_2$ cancel. The phasor diagram is $\rightleftarrows$.

For $\theta = 0, \pi$, the effects of $I_1\ I_2$ do not cancel




Array Pattern Factor

* the elements would be horizontal in this figure

In the above [ ] signifies a retarded quantity. For purpose of illustration $I_1$ is taken as the reference phasor for $\theta = 0$, while $I_2$ is used as the reference for $\theta = \pi$. Note that the $\theta = \pm \pi/2$ nulls are associated with the array factor. There is also a null in the element patterns at $\theta = \pm\pi/2$, because this is the direction of orientation of the dipoles and there is no radiation off the ends.

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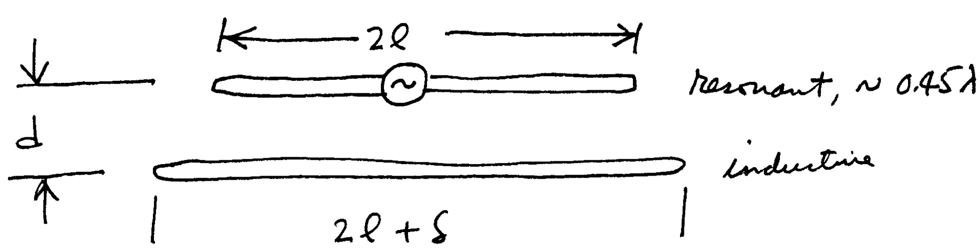
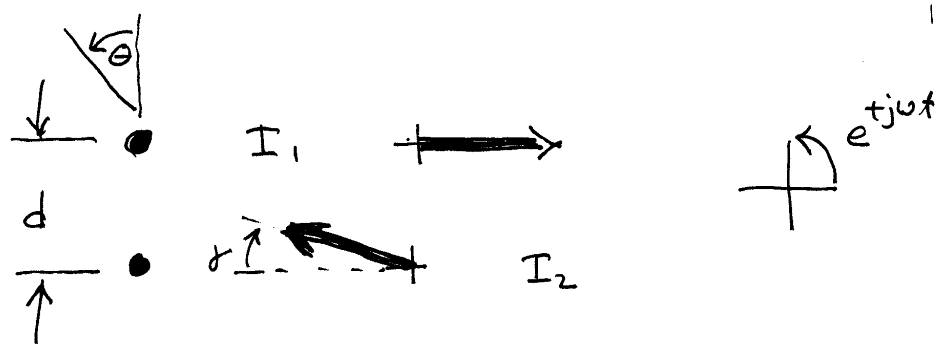
Now suppose that we change the length of the parasitic, or passive element? The new situation is




An element slight longer than the resonant length is "inductive" in that the current lags the applied $\bar{E}$ field. The excitation is primarily through $\bar{E}$ coupling, so $I_2$ now lags its previous phase. /...

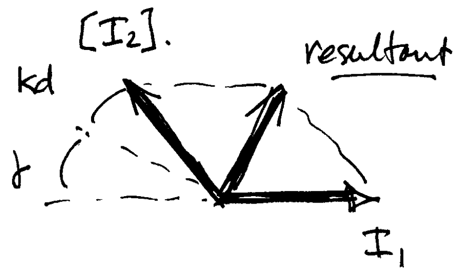
So now our array and its excitation "look like" 19-11



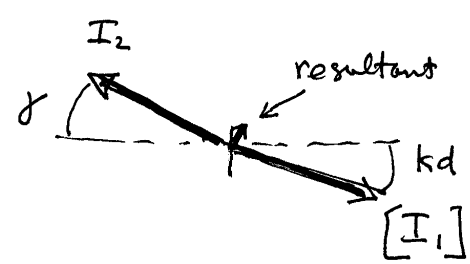

$I_2$ lags by $\gamma$.

The result of this is to introduce asymmetry into the pattern, thus creating a "beaming" effect in the $\theta = 0$ direction relative to $\theta = \pi$.

To see how this works, draw the phasor diagrams for $\theta = 0, \pi$ 19-12



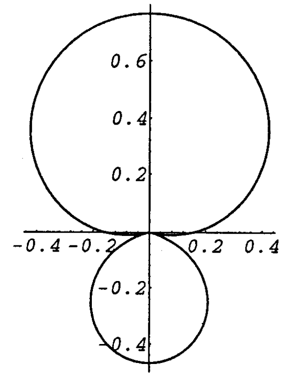

$\theta = 0$




$\theta = \pi$

For $\theta = \pi$, the kd retardation of $I_1$ moves $I_1$ toward the $-I_2$ location. The "sign" of the resultant depends on relative values of $\gamma$ and kd.

The figure at the right depicts the magnitude of the array factor for $\gamma = -9°$, $kd = 36°$, and equal magnitude $I_1$ and $I_2$

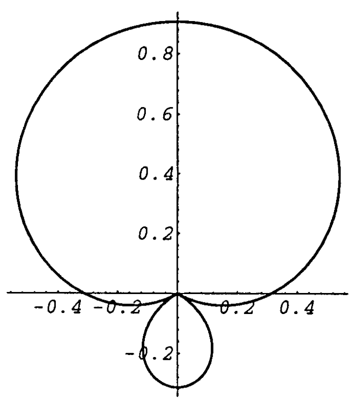



Note that the null is located at $\theta \simeq \pm 100°$, it need not be at $\pm 90$. (For $|\gamma| > kd$ there is no null!).

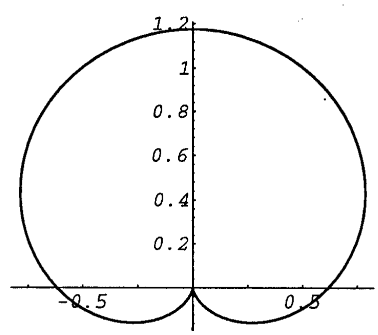
Because the beam is formed in the direction opposite the location of the passive element, this element is called a reflector.



Here are two more examples of dipole + reflector array patterns



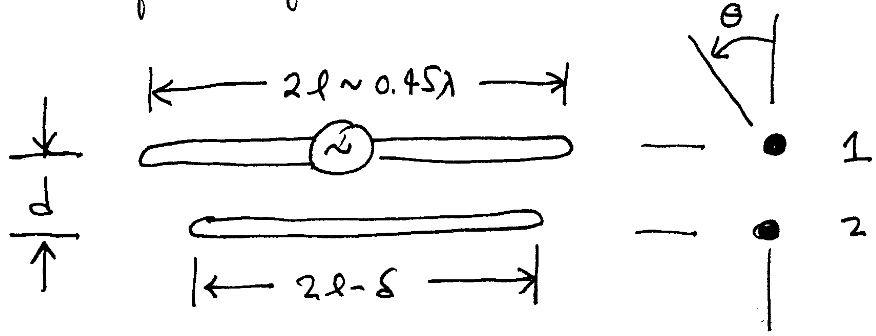

$\gamma = -18°$



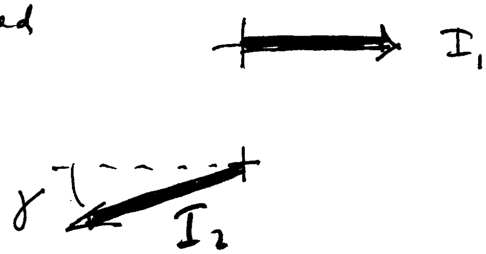

$\gamma = -36°$

$kd = 36°$

What if we repeat the analysis, this time for a passive element that is shorter than that required for resonance?

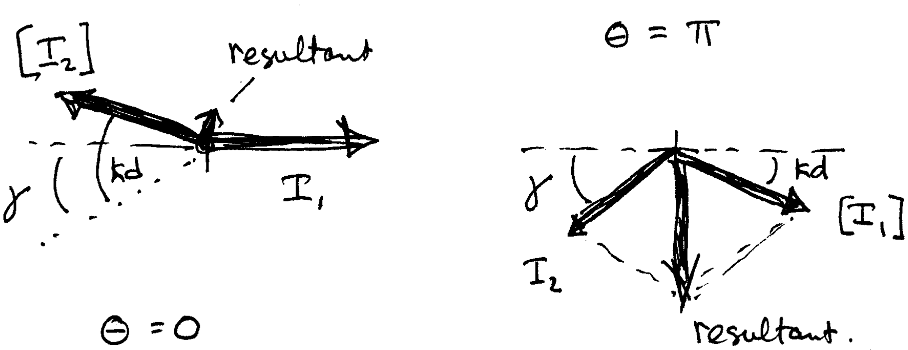



Now we find the relative currents to have $I_2$ phase advanced by $\gamma$



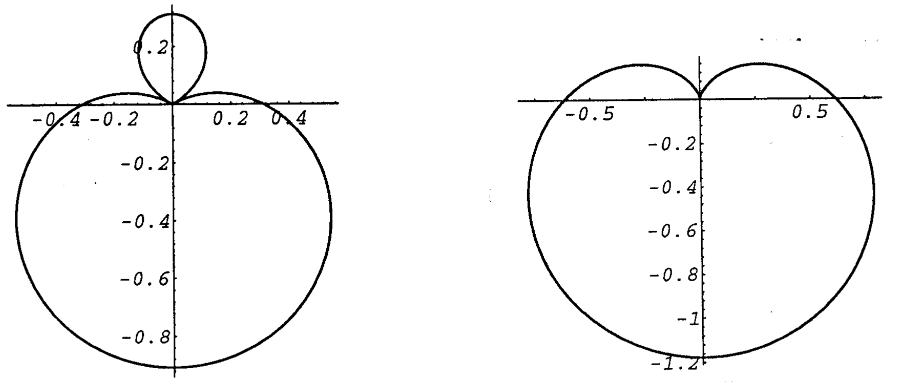



Again sketching the relative currents as viewed by an observer in the $\theta = 0, \pi$ directions




In this case a beam is formed in the $\theta = \pi$ direction.

The figures below provide two example array factor patterns for a dipole + capacitive passive element.




$\gamma = +18°$ $\qquad\qquad$ $\gamma = +36°$

$kd = 36°$



Thus we see that a "short," or capacitive passive element forms a beam on the same side as the passive element location. For this reason such elements are refered to as <u>directors</u>.

In the above, the function plotted is

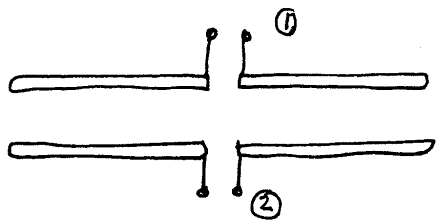
$$f(\theta; d, \gamma) = 1 - \exp\{j[\gamma - \tfrac{2\pi}{\lambda} d \cos\theta]\}$$

corresponding to

$$f(\theta) = 1 + \frac{I_2}{I_1} e^{-jkd\cos\theta} \; ; \quad \frac{I_2}{I_1} = -e^{+j\gamma}$$

# How to analyze the Yagi-Uda Array?

Consider the two element array as a two-port network!




$$V_1 = Z_{11} I_1 + Z_{12} I_2$$

$$V_2 = Z_{21} I_1 + Z_{22} I_2$$

V, I relationship at the terminals can be completely specified by the $Z_{ij}$'s.



For one element driven and the other coupled parasitically, $\underline{V_2 = 0}$.

$$V_1 = Z_{11} I_1 + Z_{12} I_2$$

$$0 = Z_{21} I_1 + Z_{22} I_2$$

The second equation $\Rightarrow$ $\dfrac{I_2}{I_1} = -\dfrac{Z_{21}}{Z_{22}}$

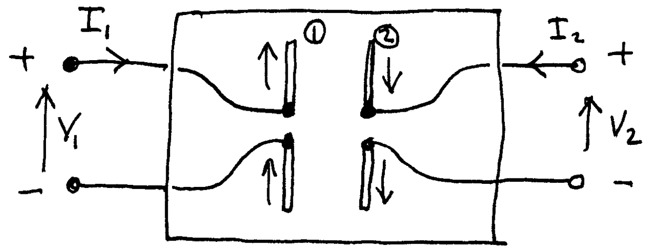
$$f(\theta) = I_1 + I_2 e^{-jkd\cos\theta} = I_1\left(1 + \frac{I_2}{I_1} e^{-jkd\cos\theta}\right)$$

Evidently, array performance is controlled by

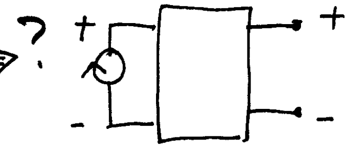
$\dfrac{Z_{21}}{Z_{22}}$, $d$, $\lambda$

19-21

It is important to be clear on the sign conventions. Drawing our antenna as part of a two-port network.




$$V_2\Big|_{I_2=0} = Z_{21} I_1 \; ; \quad Z_{21} > 0 \Rightarrow ?$$




What is the direction of current flow in dipole # 2, and hence the direction of $I_2$ relative to the circuit convention?

19-22

Below we give curves showing the mutual impedance between two closely spaced $\lambda/2$ dipoles (from Kraus), and a table of values for dipoles of slightly different lengths (from Elliott). You need not take these curves on faith, as you now are able to compute these by the method of moments.

From Kraus, 2nd Edition



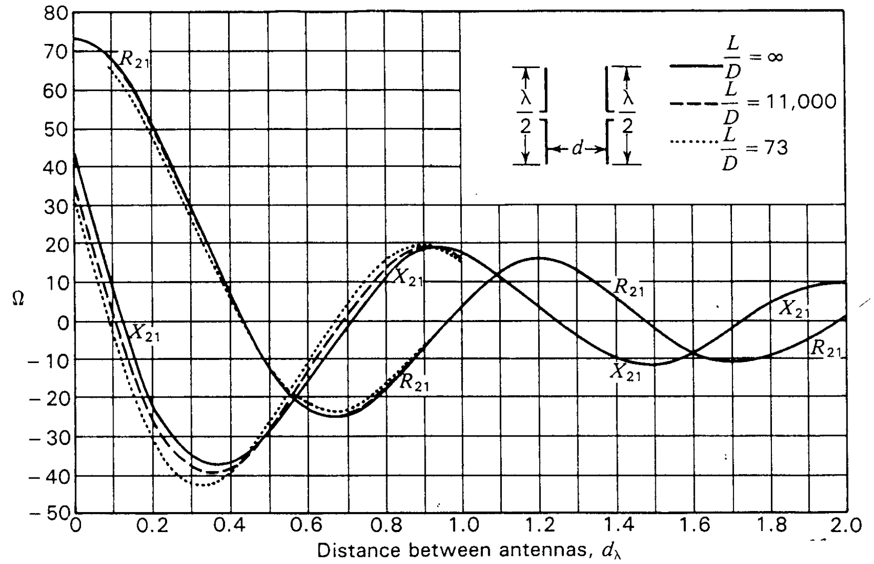



**Figure 10-12** Curves of mutual resistance ($R_{21}$) and reactance ($X_{21}$) of two parallel side-by-side linear $\lambda/2$ antennas as a function of distance between them. Solid curves are for infinitesimally thin antennas as calculated from Carter's formulas. Dashed and dotted curves between 0 and 1.0$\lambda$ spacing are from Tai's data for antennas with $L/D$ ratios of 11 000 and 73 respectively.

19-24

Mutual impedance between two closely spaced dipoles.

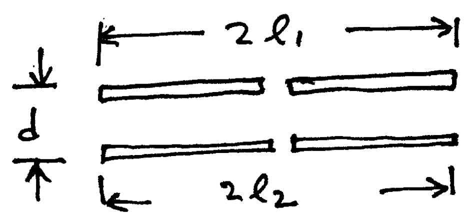



TABLE 8.4 Mutual impedance versus spacing between two parallel dipoles: $2l_1/\lambda = 0.475$

$a/\lambda = 0.0032$

| | $Z_{12}$ ohms | | |
|---|---|---|---|
| $d/\lambda$ | $2l_2/\lambda = 0.450$ | $2l_2/\lambda = 0.475$ | $2l_2/\lambda = 0.500$ |
| 0.10 | 53.94 \|1.52° | 58.19 \|3.22° | 62.78 \|4.98° |
| 0.15 | 49.08 \|−9.38° | 52.73 \|−8.45° | 56.62 \|−7.50° |
| 0.20 | 44.42 \|−21.93° | 47.67 \|−21.42° | 51.12 \|−20.90° |
| 0.25 | 40.23 \|−35.53° | 43.18 \|−35.28° | 46.30 \|−35.02° |
| 0.30 | 36.55 \|−49.91° | 39.26 \|−49.82° | 42.11 \|−49.73° |
| 0.35 | 33.35 \|−64.87° | 35.84 \|−64.89° | 38.47 \|−64.91° |
| 0.40 | 30.57 \|−80.32° | 32.86 \|−80.40° | 35.31 \|−80.48° |
| 0.45 | 28.14 \|−96.14° | 30.28 \|−96.26° | 32.56 \|−96.38° |
| 0.50 | 26.04 \|−112.25° | 29.02 \|−112.40° | 30.16 \|−112.56° |

Elliott, p. 369

/...

Observe that in terms of the induced open circuit voltage it is the spacing, d, of the dipoles that is important. This validates (at some level) our previous handwaving regarding the direct effects.

From Elliott's table the steps are $\Delta d = 0.05$ or 1/20 cycle $\Rightarrow 18^\circ$. The angle $\angle Z_{12}$ varies more slowly than this. The variation in $\angle Z_{12}$ is more like $10^\circ$/step when d small, then $\sim 15^\circ$/step for $d/\lambda > 0.15$. Also note the insensitivity in $|Z_{12}|$, and that the $\Gamma = 0$ catastrophe is avoided!



What about $Z_{22}$? This is different, as we already know.

TABLE 8.5 Self-impedance of a cylindrical dipole versus length (King-Middleton corrected second-order approximation; ($a_2/\lambda = 0.0032$)

| | $2l_2/\lambda$ | $Z_{22}$ ohms | |
|---|---|---|---|
| $-\delta$ | 0.450 | $60.56 - j29.58 = 67.40\angle{-26.03^\circ}$ | capacitive |
| resonant | 0.475 | $72.06 + j\,4.04 = 72.17\angle{3.21^\circ}$ | |
| $+\delta$ | 0.500 | $83.60 + j41.34 = 93.26\angle{26.31^\circ}$ | inductive |

Elliott, p. 370

Thinking back over our preliminary calculations, we saw that we could form a beam either on the side opposite the passive element ( $2\ell+\delta$, inductive, current lags, <u>reflector</u>), or on the same side ($2\ell-\delta$, capacitive, current leads, <u>director</u>). How do we relate this to the Table 8.6 from Elliott?

First, consider that there are two ways in which we can "form" beams. 1) We can arrange for the radiation from our two elements to add constructively in the desired direction, or

/...

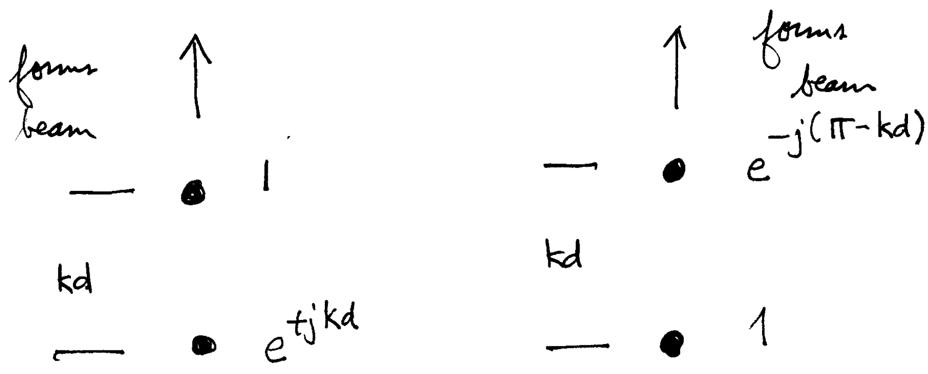


2) we can arrange for destructive interference in the direction opposite that of the desired beam.

In the first instance we need the relative phase between the two elements to equal kd, with the lag in the direction of the intended beam.

In the second instance we need the relative phase between the two elements to equal $\pi - kd$, with the lag opposite the direction of the beam.

Sketching this,




not necessarily a null here

null or strong minimum forms here

<u>Case 1</u>

<u>Case 2</u>

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19-30

Examining Table 8.6, Elliott, ... we see

1. That Case 2 is satisfied for the <u>capacitive</u> element for <u>$d/\lambda \simeq 10°$</u>. There is no entry that will satisfy Case 1 for a <u>capacitive</u> element. $(\ell_2 < \ell_1)$

2. That Case 1 is satisfied for the inductive element for <u>$d/\lambda \simeq 0.3$</u>. There is no entry that will satisfy Case 2 for an <u>inductive</u> element. $(\ell_2 > \ell_1)$

Rather than just searching the Table 8.6, as we have just done, it is possible (straight-forward in fact) to calculate the effectiveness of the two element arrays corresponding to the "director" and "reflector" columns for all spacings. Elliott presents such a calculation in his Fig 8.14, below. You can see that even a simple two element array with element separation $\sim 0.15\lambda$ gives marked performance improvement over a simple dipole!



Combining these results we find $-\frac{Z_{21}}{Z_{22}}$

TABLE 8.6 Relative current versus spacing for two parallel dipoles, one driven, one parasitic: $2l_1/\lambda = 0.475$

| $d/\lambda^{\circ}$ | $d/\lambda$ | $I_2/I_1 = -Z_{21}/Z_{22}$ <br> $2l_2/\lambda = 0.450$ | $2l_2/\lambda = 0.475$ | $2l_2/\lambda = 0.500$ |
|---|---|---|---|---|
| 36 | 0.10 | $0.800\,\underline{\vert -152.45^\circ}$ | $0.806\,\underline{\vert 180.01^\circ}$ | $0.673\,\underline{\vert 158.67^\circ}$ |
| 54 | 0.15 | $0.728\,\underline{\vert -163.35^\circ}$ | $0.731\,\underline{\vert 168.34^\circ}$ | $0.607\,\underline{\vert 146.19^\circ}$ |
| 72 | 0.20 | $0.659\,\underline{\vert -175.90^\circ}$ | $0.661\,\underline{\vert 155.37^\circ}$ | $0.548\,\underline{\vert 132.79^\circ}$ |
| 90 | 0.25 | $0.597\,\underline{\vert 170.50^\circ}$ | $0.598\,\underline{\vert 141.51^\circ}$ | $0.496\,\underline{\vert 118.67^\circ}$ |
| 108 | 0.30 | $0.542\,\underline{\vert 156.12^\circ}$ | $0.544\,\underline{\vert 126.97^\circ}$ | $0.452\,\underline{\vert 103.96^\circ}$ |
| 126 | 0.35 | $0.495\,\underline{\vert 141.16^\circ}$ | $0.497\,\underline{\vert 111.90^\circ}$ | $0.413\,\underline{\vert 88.78^\circ}$ |
| 144 | 0.40 | $0.454\,\underline{\vert 125.71^\circ}$ | $0.455\,\underline{\vert 96.39^\circ}$ | $0.379\,\underline{\vert 73.21^\circ}$ |
| 162 | 0.45 | $0.418\,\underline{\vert 109.89^\circ}$ | $0.420\,\underline{\vert 80.53^\circ}$ | $0.349\,\underline{\vert 57.31^\circ}$ |
| 180 | 0.50 | $0.386\,\underline{\vert 93.78^\circ}$ | $0.388\,\underline{\vert 64.39^\circ}$ | $0.323\,\underline{\vert 41.13^\circ}$ |

$l_2 \rightarrow$ capacitive   resonant   inductive

Elliott, p. 370

Result of calculating gain for each entry in Table 8.6



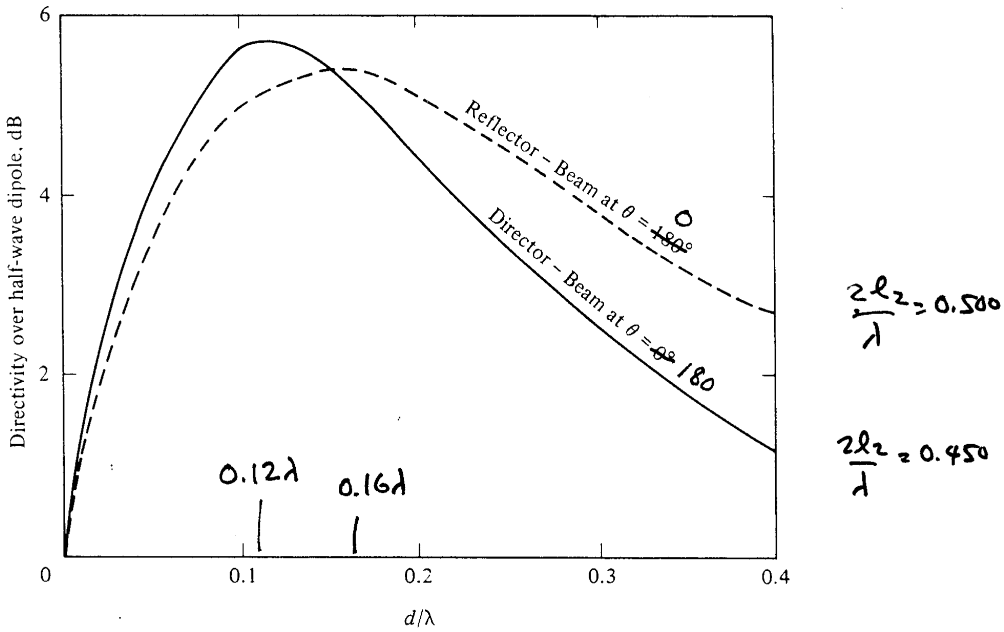



**Fig. 8.14** Peak Directivity versus Element Spacing for an Array of Two Parallel Dipoles; One Element Driven, the Other Parasitic

Elliott, p. 371



So. —

Shorted dipole placed at an appropriate distance from a driven dipole can act as a

<u>director</u> (parasite is shorter*)

or a <u>reflector</u> ( " " longer)

thereby generating an "end-fire" array beam.

*than resonant length

What is input $Z$?

Assume $Z_{12}$ is insensitive to small changes in $2\ell_1/\lambda$. This is reasonable $(Z_{12} = Z_{21})$

then knowing that $Z_{IN} = \frac{V_1}{I_1} = Z_{11} + \frac{I_2}{I_1} Z_{12}$

$$Z_{IN} = Z_{11} - \frac{Z_{12}^2}{Z_{22}}$$

For optimum director $-\frac{Z_{12}^2}{Z_{22}} = -38 - j14$

/...



To resonate the system lengthen $\ell_1$, adjust $\mathrm{Im}(Z_{11}) \simeq +j14$. This occurs for $\frac{2\ell_1}{\lambda} = 0.482$ $(a/\lambda = 0.0032)$,

where $Z_{11} = 76 + j14$

$$Z_{IN} = \underbrace{76 + j14}_{Z_{11}} \underbrace{-38 - j14} = 38\,\Omega$$

Driven element must be slightly longer than in absence of a director. Note that

/...

the input $Z$ is lower than before, which is again consistent with our ideas of the behavior of a dipole in the presence of a conductor.

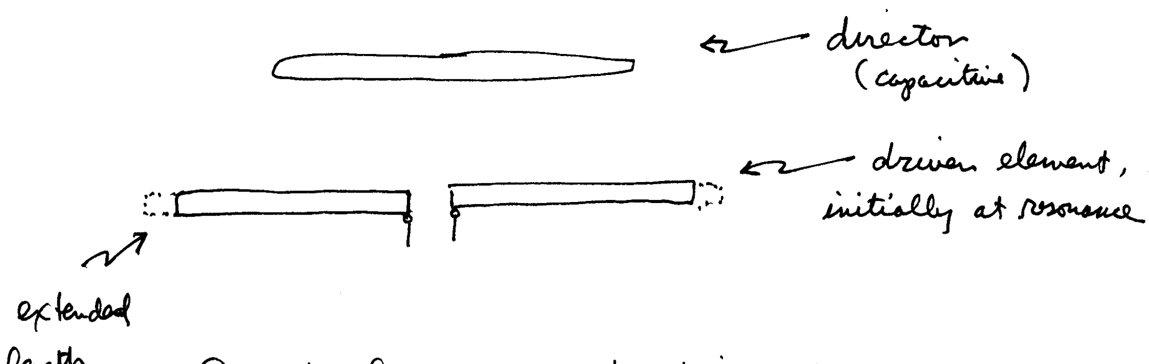
Repeating above for an optimum <u>reflector</u>

$$-\frac{Z_{12}^2}{Z_{22}} = -23+j24 \qquad (\text{inductive})$$

$$2l_1/\lambda = 0.454\ ,\quad Z_{11} = 62.5 - j24\ ,\quad Z_{IN} = 40\ \Omega$$



## <u>Example of the Director + Driven Element</u>




Adding length to the driven element increases the overall inductance, cancelling the capacitance introduced by the director.

(In the case of a <u>reflector</u>, the driven element must be made shorter than resonance.)

Example: $Z_1$ when $d/\lambda = 0.1$, with one driven element plus director

Table 8.6, Elliott, $L/\lambda = 0.450$

$d/\lambda = 0.1 \Rightarrow \quad -Z_{12}/Z_{22} = 0.800 \angle -152.45^\circ$

Table 8.4, Elliott, $L/\lambda = 0.450$

$d/\lambda = 0.1 \Rightarrow \quad Z_{12} = 53.94 \angle 1.52^\circ$

$$-Z_{12}^2/Z_{22} = 43.15 \angle -151^\circ$$

$$= -37.7 - j21 \quad \Omega$$

/...



$$Z_1 = Z_{11} - Z_{12}^2/Z_{22}$$

Table 8.5, Elliott, $L/\lambda = 0.475$

$$Z_{22} = Z_{11} = 72.06 + j4.04 \quad \left\{ \begin{matrix} \text{approx.} \\ \text{resonance} \end{matrix} \right\}$$

$$Z_1 = 72.06 + j4.04 - 37.7 - j21$$

$$\simeq 34 - j17 \quad \Omega$$

/...

By interpolation from Table 8.5, Elliott

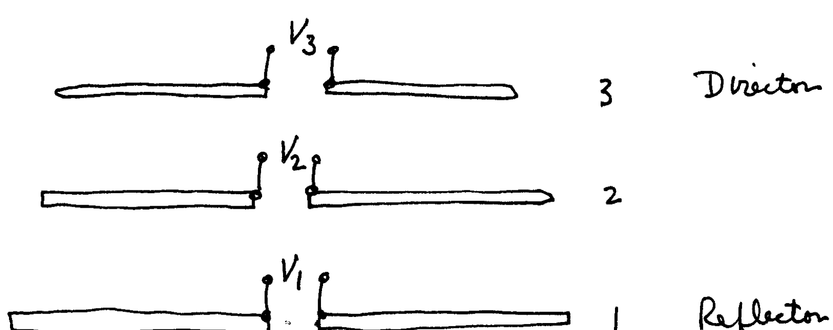
$$L/\lambda = 0.49 \Rightarrow Z_{11} \simeq 78 + j21$$

Use of an element of this length will yield

$$Z_1 \simeq 78 + j21 - 37.7 - j21$$

$$\simeq 40\ \Omega \quad (+j0)$$

19-42




$$V_1 = Z_{11} I_1 + Z_{12} I_2 + Z_{13} I_3 \quad \text{etc.}$$

$$Z_{12} = \left.\frac{V_1}{I_2}\right|_{I_1 = I_3 = 0} \quad \text{typ.}$$

Adding more elements ...

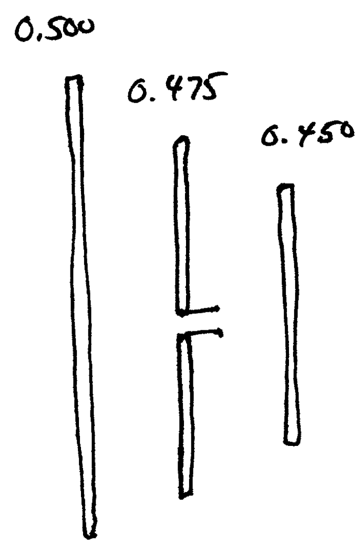
$$\begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} \\ & Z & \\ \\ \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix}$$

$$\frac{I_1}{I_2} = \frac{Z_{13} Z_{23} - Z_{12} Z_{33}}{Z_{11} Z_{33} - Z_{13}^2}, \text{ etc}$$

$$\frac{I_3}{I_2} = \frac{Z_{13} Z_{12} - Z_{23} Z_{11}}{Z_{11} Z_{33} - Z_{13}^2}$$

/...

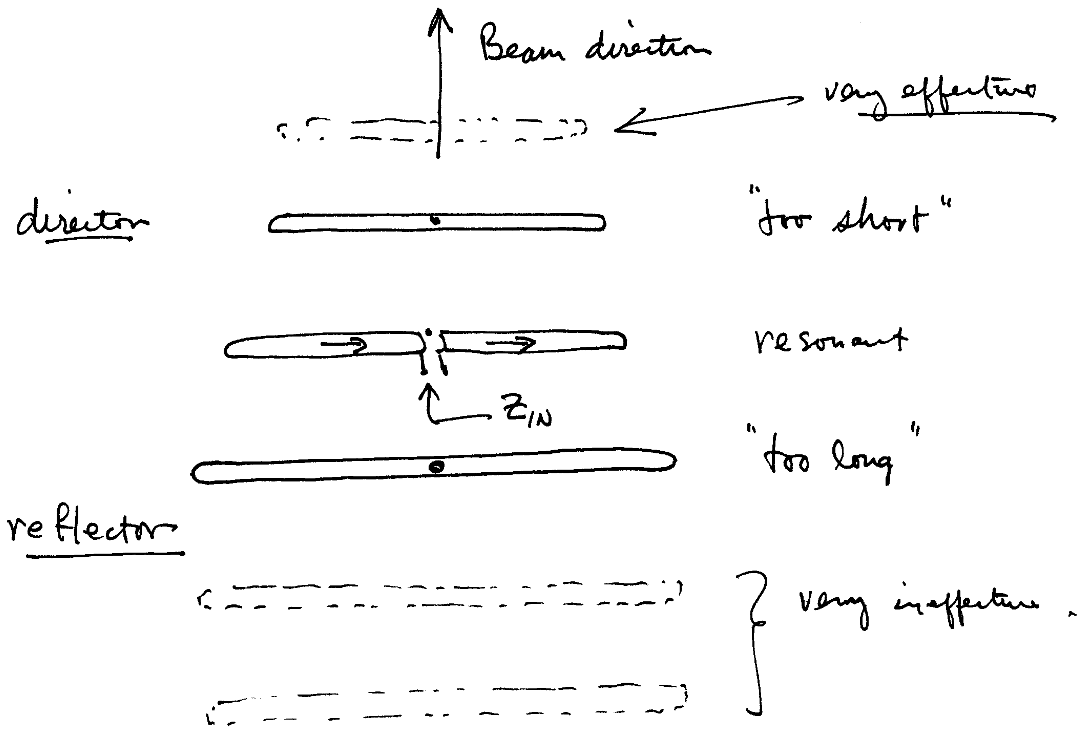





$Z_{IN} = 26 \angle 10^\circ$

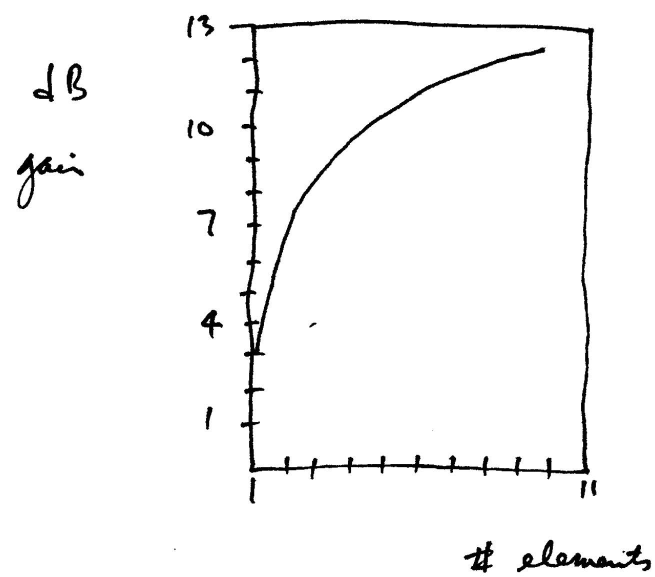
$$\frac{d_1}{\lambda} = \frac{d_2}{\lambda} = 0.20$$

Commonly use  driver, $Z_{IN} \to 4 Z_{IN}\Big|_{\text{single dipole}}$






from S & T 5-32, p 225



Generally adding directors brings $\approx 1$ dB per element up to 10 - 13 dB. Adding reflectors doesn't help v. much after the first. <u>Why should this be so?</u>

The Yagi-Uda and similar antennas can be readily designed through use of MoM codes