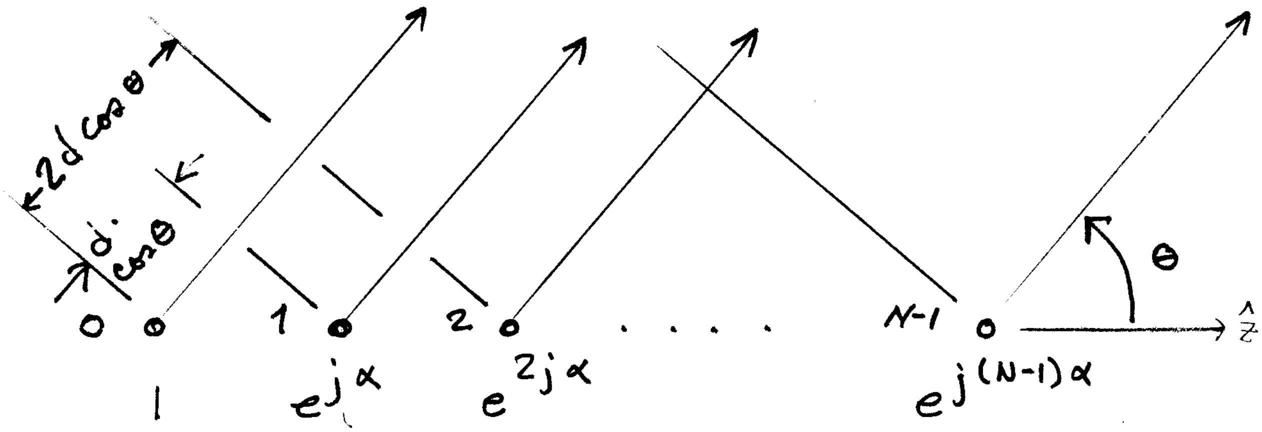
Uniform Array of N-elements.



Each element radiates to a distant ~~source~~ receiver as $\dfrac{A_o(\theta)}{r_i} e^{-jkr_i}$

Phase advance per element is $\alpha + dk\cos\theta$

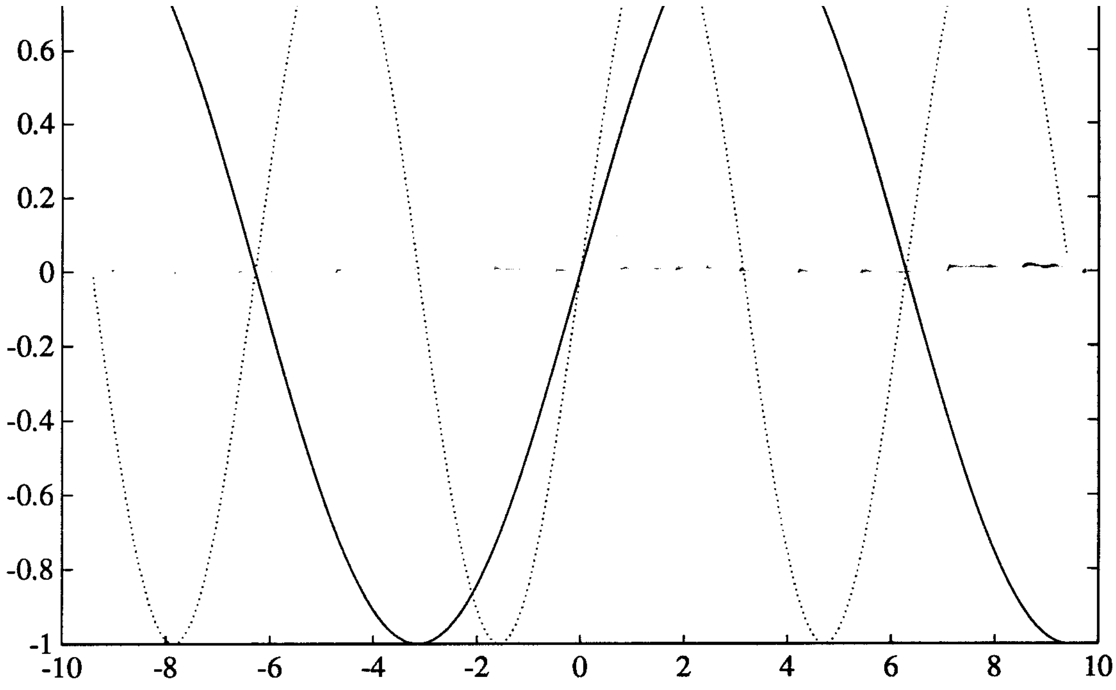
/...

/...

$$E_{Total} = \frac{A_o(\theta)}{r_o} e^{-jkr_o} \, e^{j\left(\frac{N-1}{2}\right)\psi} \, \frac{\sin(N\psi/2)}{\sin(\psi/2)},$$

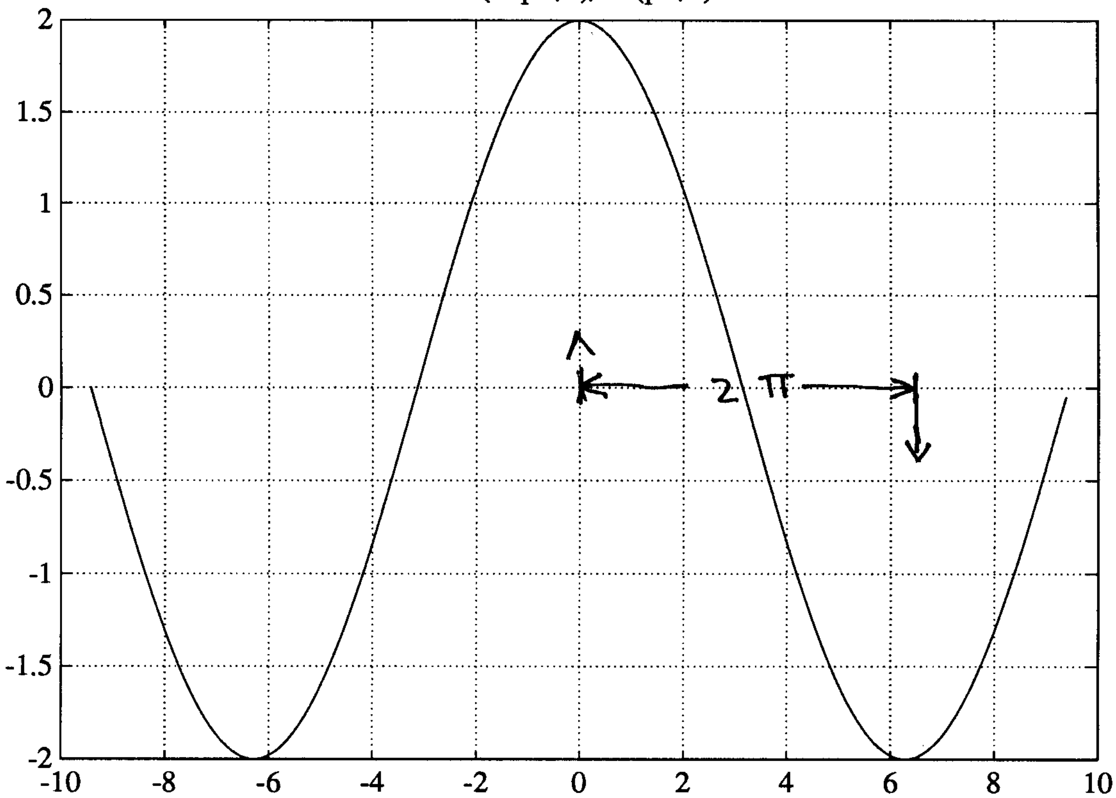
where phase is referenced to the center of the array.

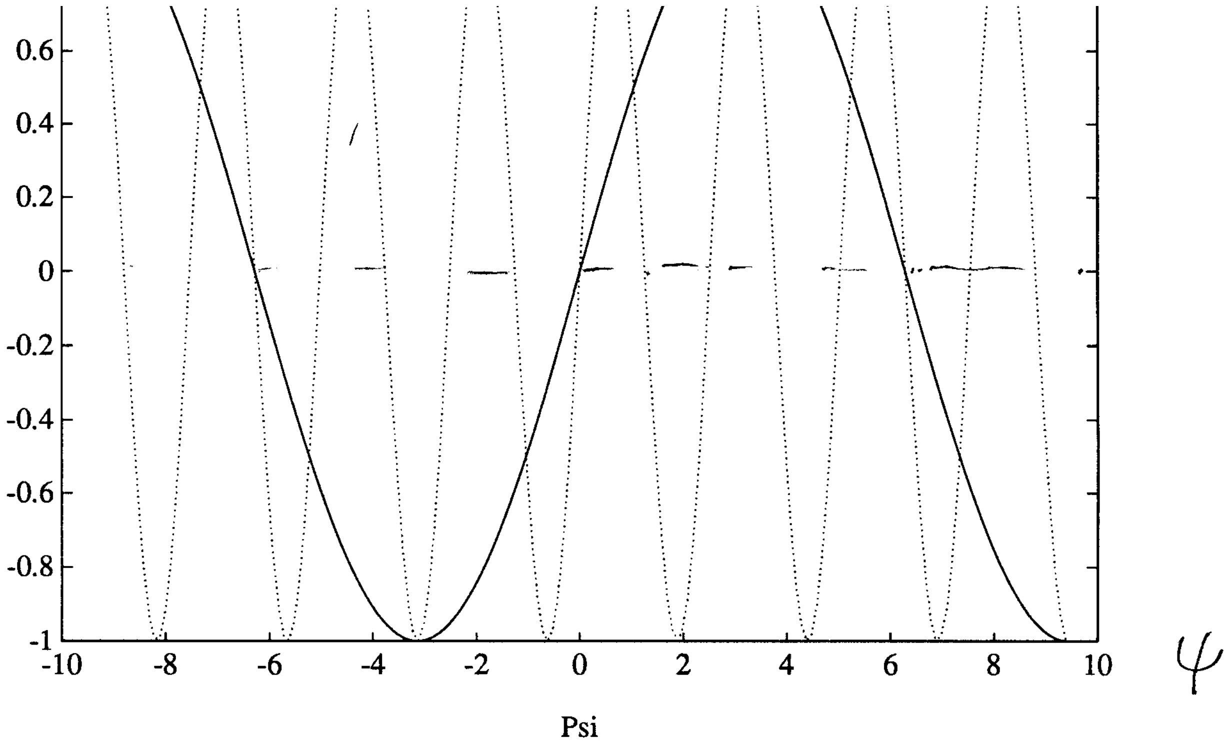
$$E_{total} = \underbrace{\frac{A_o(\theta)\, e^{-jkr_o}}{r_o}}_{\text{element factor}} \; \underbrace{F(\theta)}_{\text{array factor}}$$



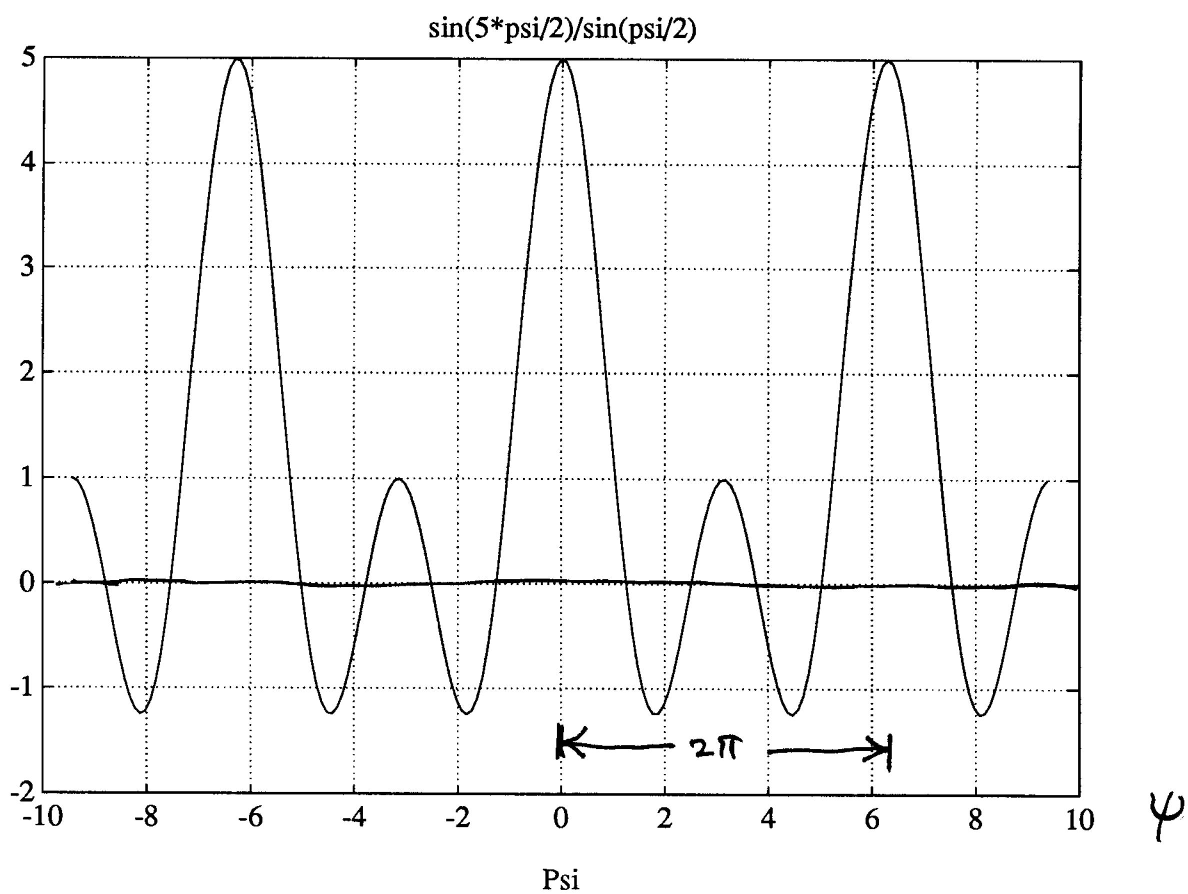
18-4

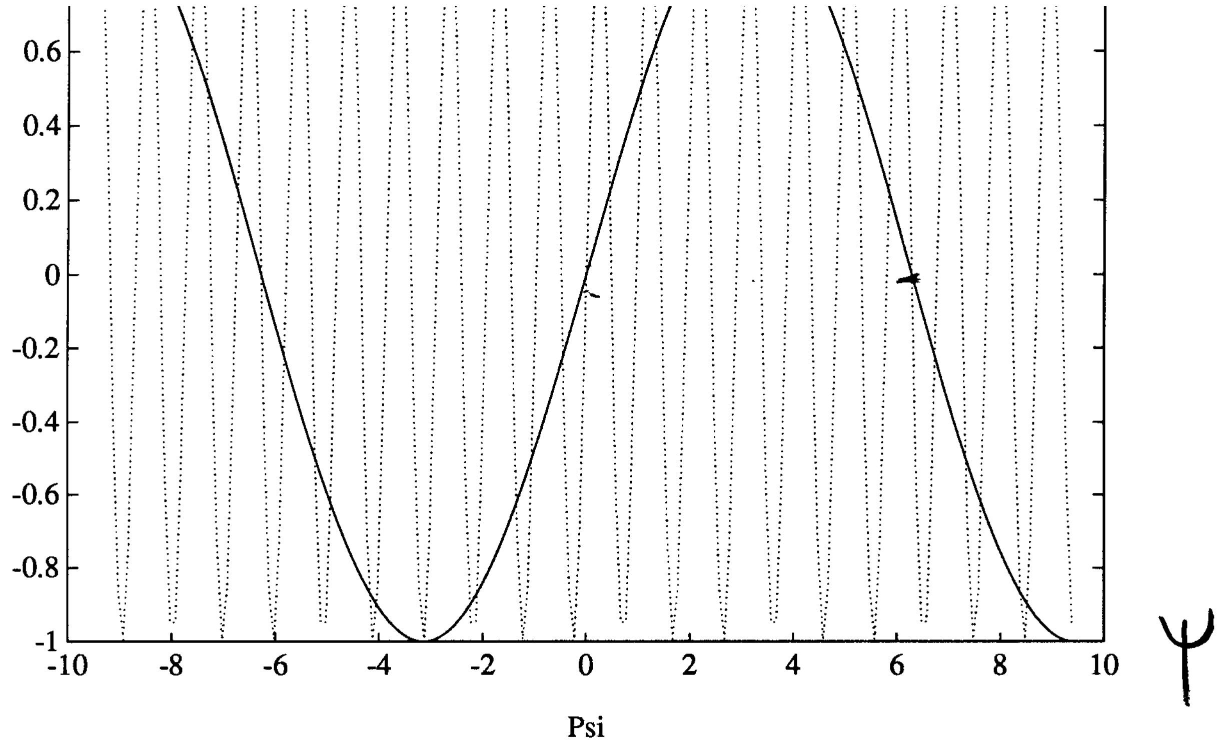
sin(2*psi/2)/sin(psi/2)



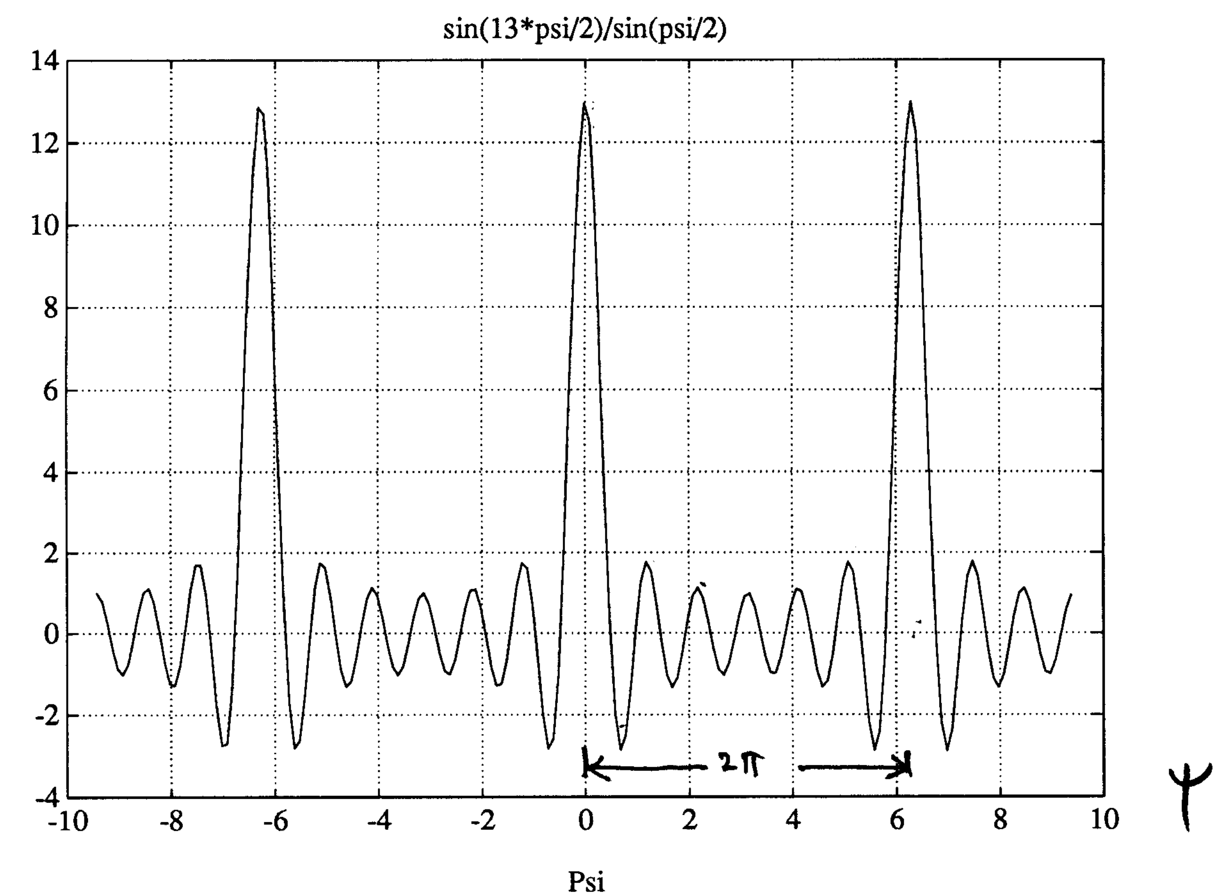

0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
-10
-8
-6
-4
-2
0
2
4
6
8
10
ψ
Psi

18-6


sin(5*psi/2)/sin(psi/2)
5
4
3
2
1
0
-1
-2
-10
-8
-6
-4
-2
0
2
4
6
8
10
2π
ψ
Psi


0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
-10
-8
-6
-4
-2
0
2
4
6
8
10
Psi
ψ

18-8


sin(13*psi/2)/sin(psi/2)
14
12
10
8
6
4
2
0
-2
-4
-10
-8
-6
-4
-2
0
2
4
6
8
10
2π
Psi
ψ

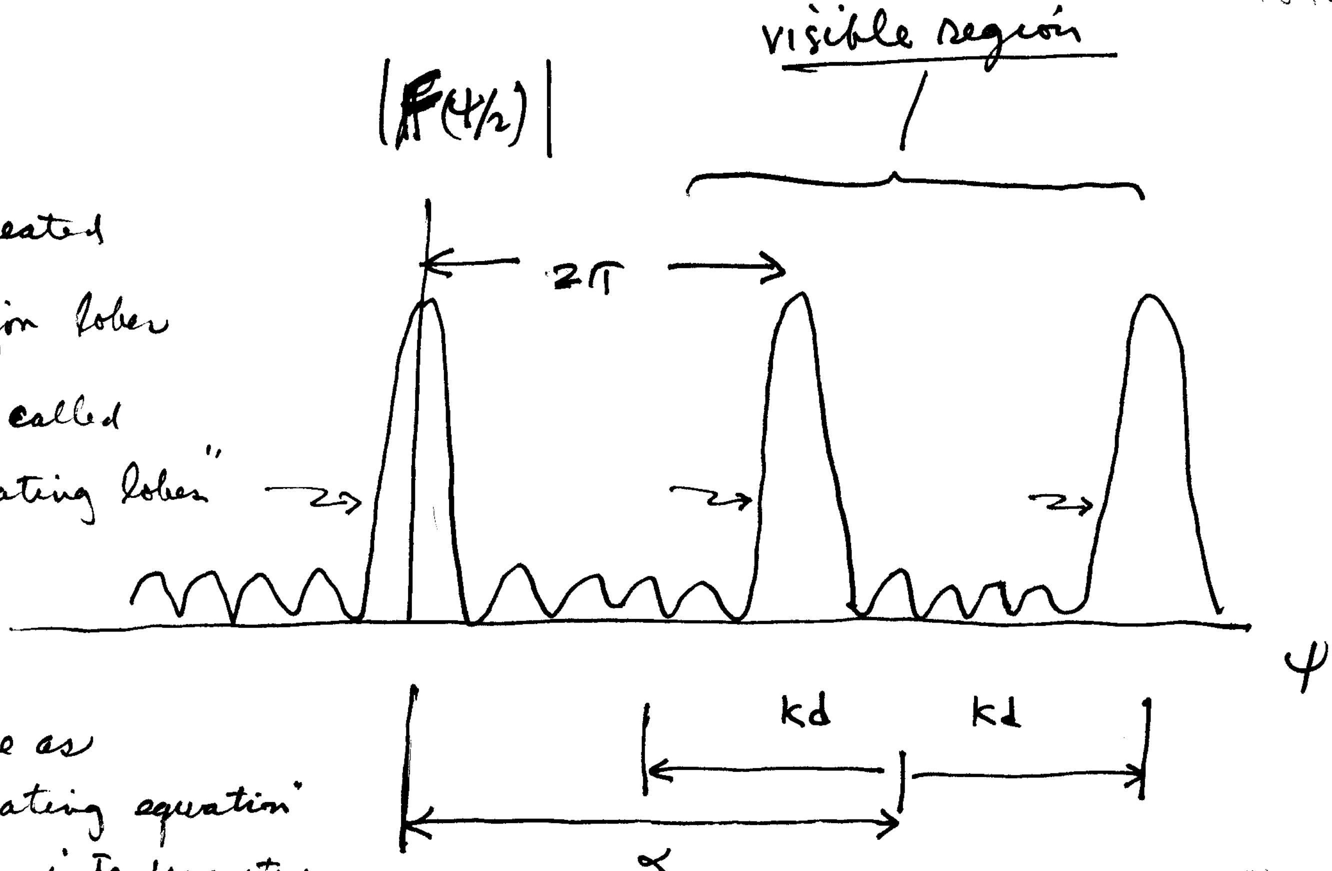
$\alpha - dk < \psi < \alpha + dk$, called <u>visible</u> region.

$\Delta\psi = 2dk = \frac{4\pi d}{\lambda}$, which is independent of α!

$N, \Delta\psi$ ~~determines~~ number of lobes in the array pattern.

There are always $N-1$ lobes per π variation in $\frac{\psi}{2}$ (see examples, lobes measured from "zero" to next "zero")

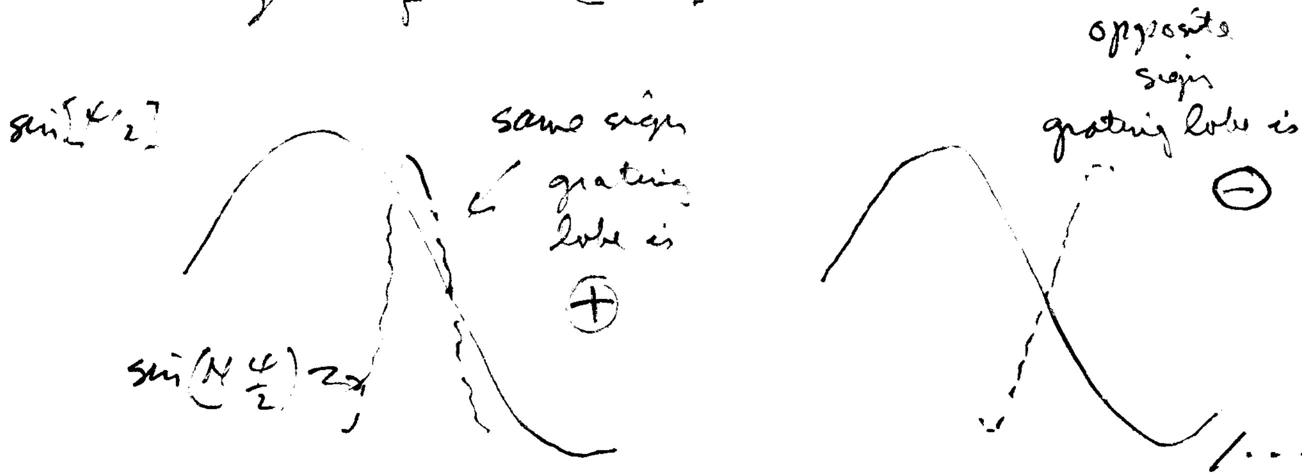
Repeated major lobes are called "grating lobes"



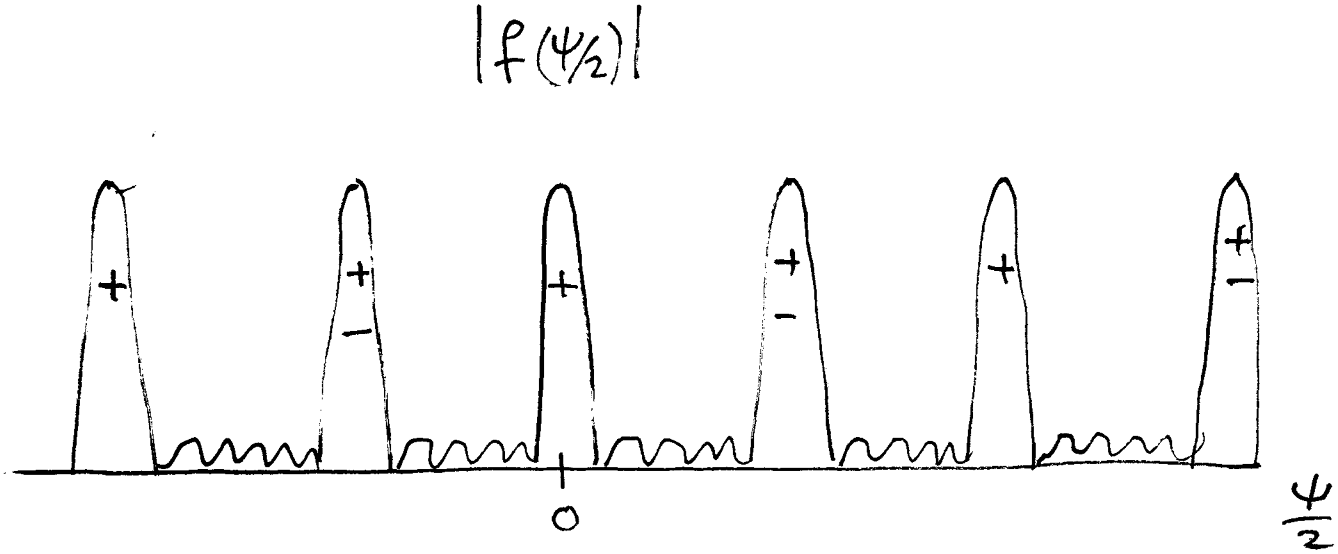
same as "grating equation" from interferometry.

Grating lobes occur at $\frac{\psi}{2} = \pm n\pi$

Signs of grating lobes depend on relative slopes of $\sin[N(\psi/2)]$ and $\sin[\psi/2]$ at zeros of $\sin[\psi/2]$

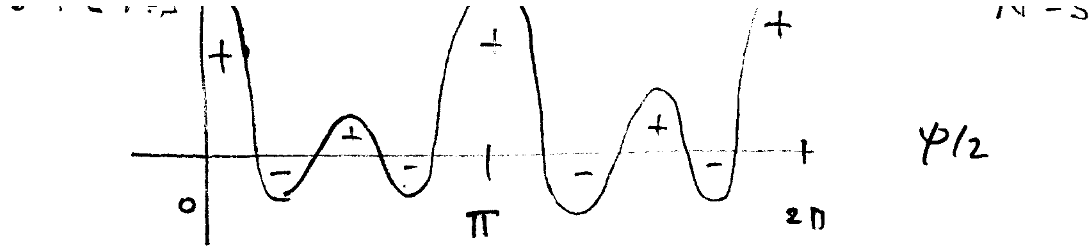


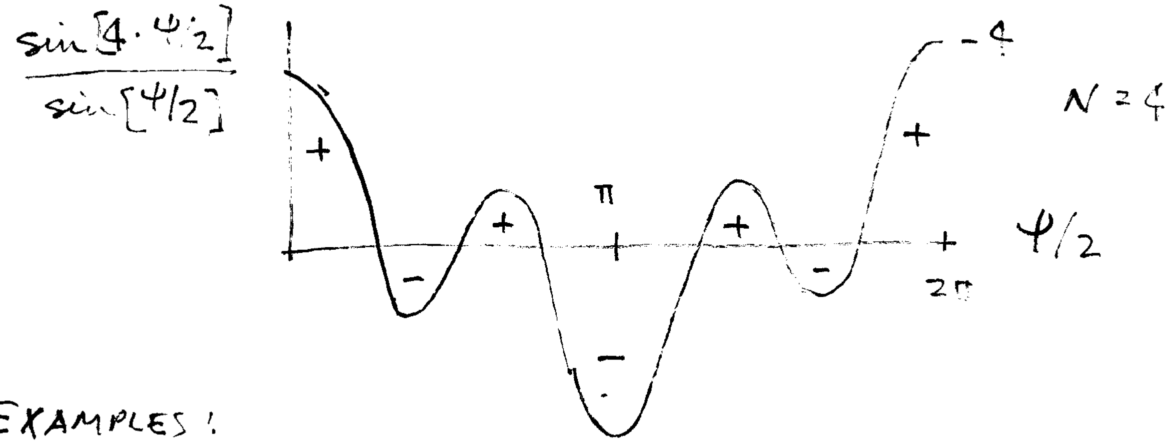
18-12



Grating lobes at $\frac{\psi}{2} = n \cdot 2\pi$ are always "+"

Grating lobes at $\frac{\psi}{2} = (2n-1)\cdot\pi$ are "+" if N is odd, and are "−" if N is even.





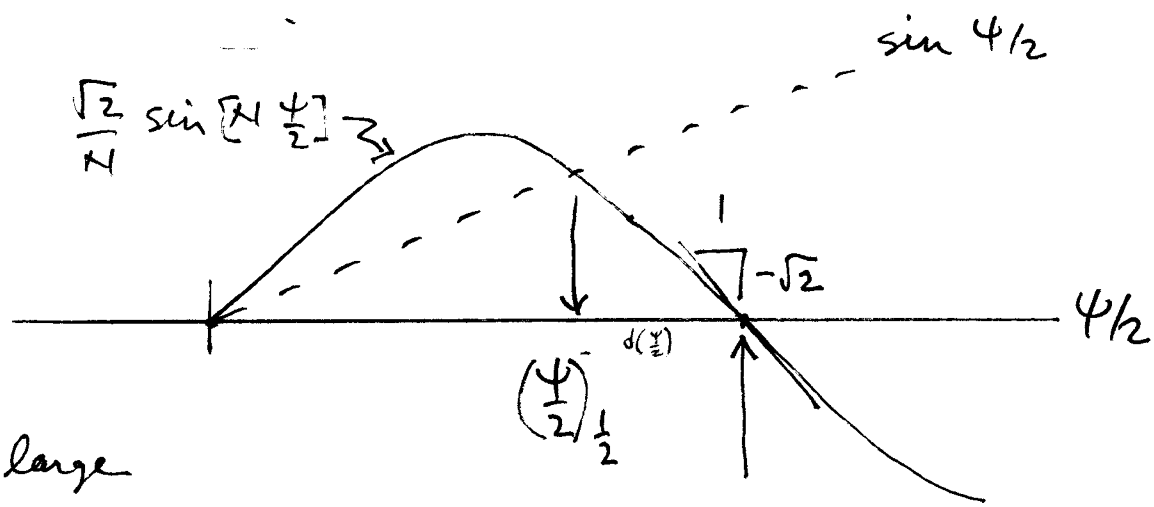
EXAMPLES:
ODD OR EVEN "N"

18-14

Half Power Width:

We need to solve $$\frac{1}{N}\,\frac{\sin[N\psi/2]}{\sin\psi/2} = \frac{1}{\sqrt{2}}$$

or $$\frac{\sqrt{2}}{N}\sin[N\psi/2] = \sin[\psi/2]$$



for N large

$$\left(\frac{\psi}{2}\right)_{\frac{1}{2}}\left[1+\sqrt{2}\right] \simeq \frac{\pi}{N} \qquad \left|\frac{\psi}{2}\right|_{\frac{1}{2}} = \frac{\pi}{N}\frac{1}{(1+\sqrt{2})} \quad \text{for } \tfrac{1}{2} \text{ power}$$

$$\frac{\psi}{2} = \pm\frac{\pi}{N} \quad \text{for first Nulls – exactly.}$$

So $\frac{1}{2}$ power points are at about $0.4 \cdot \Delta\theta_{Null}$.

Broadside $\frac{\psi}{2} = \frac{kd}{2}\cos\theta \simeq \frac{2\pi}{2}\cdot\frac{d}{\lambda}\cos\theta = \frac{\pi d}{\lambda}\cos\theta$

$$\Delta\theta_{Null} \simeq\ ? \qquad \frac{\pi d}{\lambda}\cos\left(\Delta\theta_{null}-\frac{\pi}{2}\right) = \frac{\pi}{N}\ ; \quad \Delta\theta_{Null} = \frac{\lambda}{Nd} \quad (\text{HW})$$

$$\Delta\theta_{Null_{FW}} = \frac{2\lambda}{Nd}\ ; \qquad \Delta\theta_{\frac{1}{2}\text{ power}, FW} \simeq 0.8\frac{\lambda}{Nd} \quad (\underline{\underline{\text{Broadside}}})$$

<u>Some points regarding uniform arrays</u> . . .

Pattern "structure" depends only on N (!)

$$f_N(\psi) = \frac{\sin[N\psi/2]}{\sin[\psi/2]}, \quad \underline{\underline{\text{it is independent of } \alpha}}\ (!)$$

As N increases!

Main lobe becomes more narrow

Number of side lobes / minor lobes increases

$f_1(\psi) \rightarrow \pm\sin[N\psi/2]$ in vicinity of grating

Extent of the active pattern - i.e., the visible region is determined by

$\Delta\psi = 2kd = 4\pi \frac{d}{\lambda}$

Maximum element spacing for which there is only a single grating lobe is $d = \lambda/2$ *

Width of the grating lobes is twice that of the minor lobes.

* Note that this permits two maxima for $\alpha = \pi$, however.

/...

18-18

Points (3)

Actual pattern is determined by combination of N, α, d, λ

broadside

Beam width to first null of grating lobe is

$\theta_{FN} = \cos^{-1}\left[\pm \frac{\lambda}{Nd}\right]^*$, for $\alpha = 0$ (broadside)

total BWFN $= \left| \cos^{-1}\left(-\frac{\lambda}{Nd}\right) - \cos^{-1}\left(\frac{\lambda}{Nd}\right)\right|$

Endfire beamwidth:

$$\text{total BWFN} \approx 2\sqrt{\frac{2\lambda}{Nd}}, \quad Nd \gg \lambda$$

Half Power BW:

$$\theta_{HP} \simeq 0.886 \frac{\lambda}{Nd} \csc\theta_0 \quad \text{near broadside}$$

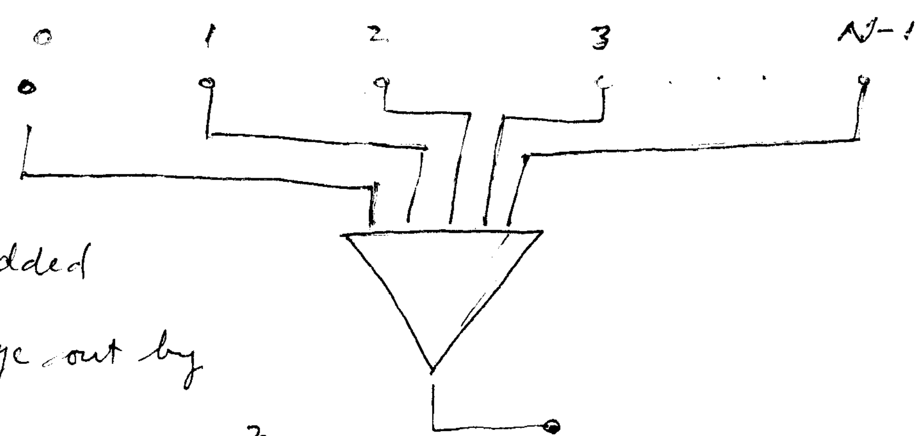
$$\theta_{HP} \simeq 2\sqrt{0.886\frac{\lambda}{Nd}} \quad \text{endfire}$$

18-20

/... Points (5)

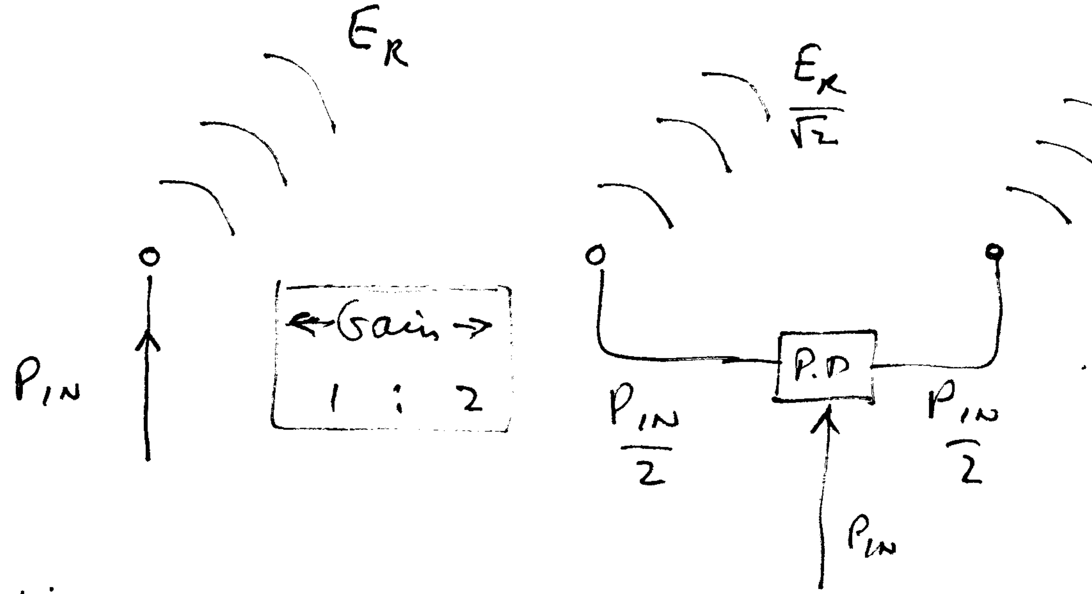
What is the gain of an N-element array?

You might say N^2 —



N-elements added increase voltage out by N, so power goes as N^2.

Consider transmitting case. (Either way, antenna remains the same)



Receiving, received power is also doubled.

At distant point

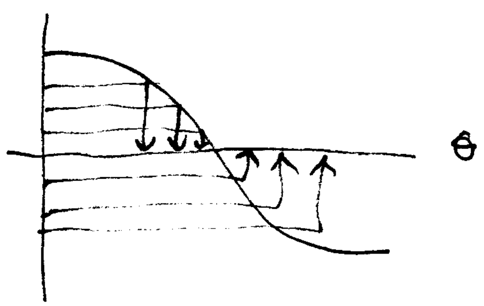
total field E_R

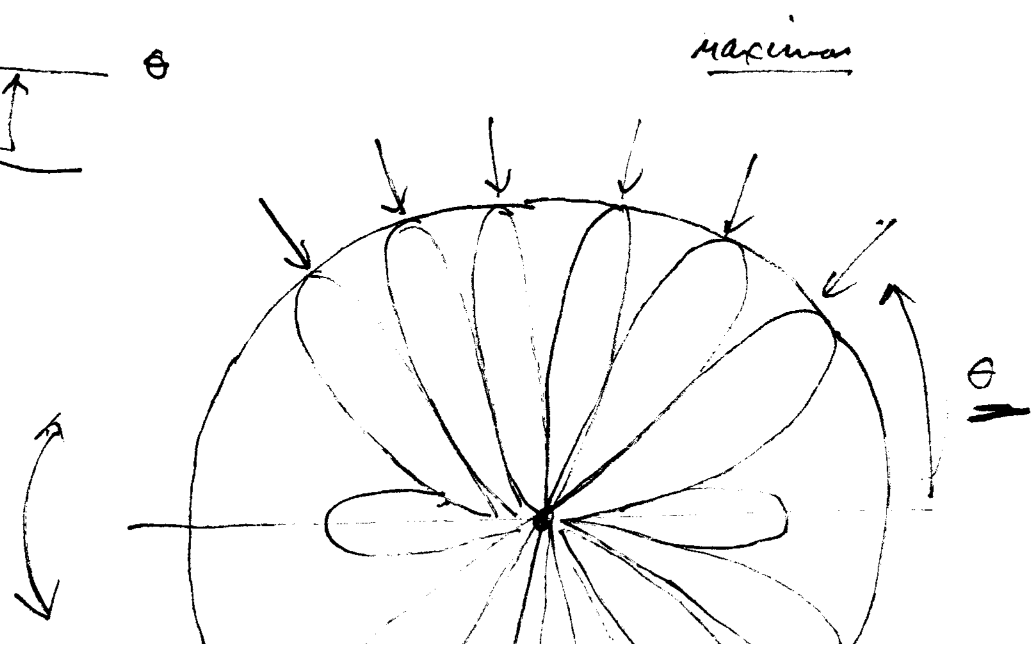
$$P_R = \frac{1}{2}\frac{|E_R|^2}{\eta}$$

At distant point total field $\frac{2}{\sqrt{2}}E_R$, $P_R' = \frac{|E_R|^2}{\eta}$

18-22

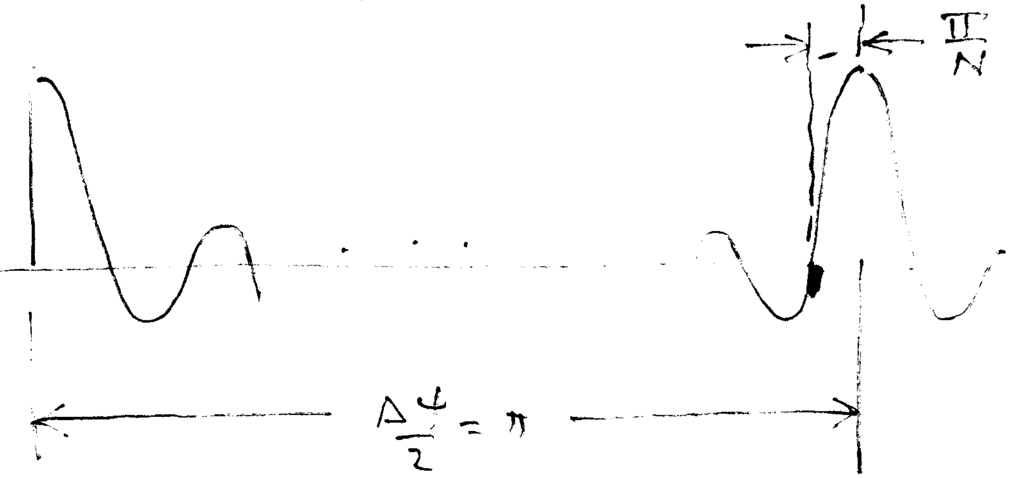
Typical Array Pattern – How to manipulate





Consider lobe spacing for endfire condition

$\frac{\psi}{2} = \frac{\pi}{N}$ for first lobe, on side



$\frac{\Delta\psi}{2} = \frac{2\pi d}{\lambda}$ (see above) ; $\frac{\Delta\psi}{2} = \pi - \frac{\pi}{N} = \pi\left[1 - \frac{1}{N}\right]$ ensures only one peak

(max - beyond array) $\Rightarrow$ $\frac{d}{\lambda} = \frac{1}{2}\left(1 - \frac{1}{N}\right)$ for spacing.

18-24

<u>How to create a pencil beam (loosely speaking)?</u>

(This is in contrast with creating only a single main lobe.)

<u>Answer</u>: Phase up the array for a maximum at one end, say $\Theta = 0$. This can be accomplished by choosing $\alpha = \pm kd$. Use $\alpha = -kd$, and apply the result just obtained for creating only a single main peak.

spacing $\rightarrow$ | d | d |

. . . o o o . . . $\Leftarrow\!\!\!\#$

phase shift $\rightarrow$ | kd | kd |

$$\alpha = -kd$$

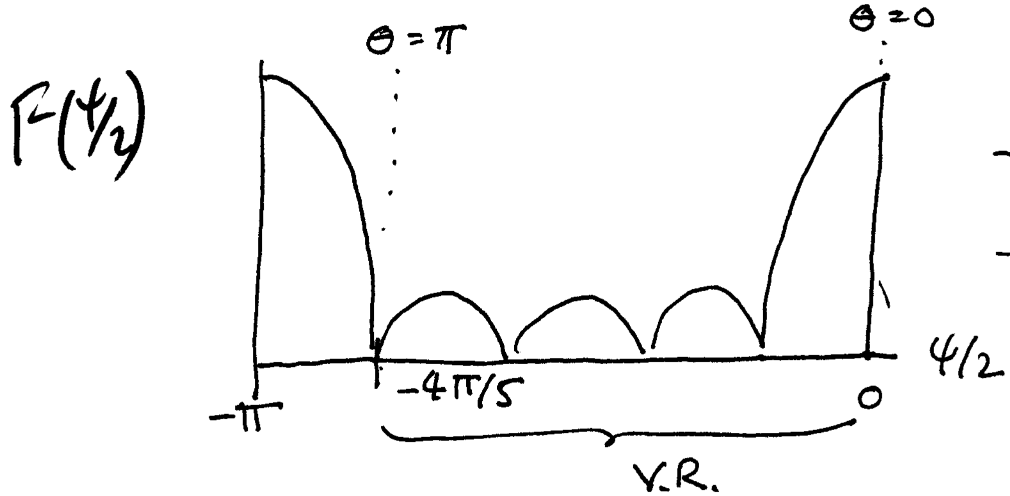
$$\psi = \alpha + kd\cos\theta \rightarrow kd(\cos\theta - 1)$$

$$kd = \pi\left(1 - \frac{1}{N}\right)\Big|_{N=5} = \frac{4\pi}{5}$$

Example $N = 5$

Visible region

$$-\frac{4\pi}{5} \le \frac{\psi}{2} \le 0$$



18-26

Pencil Beam (3)

$$F(\theta) = \sum_{n=0}^{N-1} e^{+j\psi n} = \sum_{n=0}^{N-1} e^{-j(kd - kd\cos\theta)n}$$

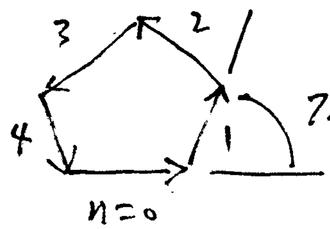
$$F(0) = N \longrightarrow 5$$

$$F(\pi) = \sum_{n=0}^{N-1} e^{-j2kdn} = \sum_{n=0}^{N-1} e^{-j2\pi\left(1-\frac{1}{N}\right)n}$$

$$= \sum_{n=0}^{N-1} e^{-j2\pi n} \cdot e^{+j\frac{2\pi n}{N}} = 0$$

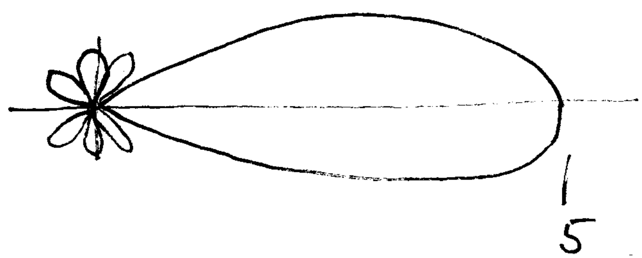
$F(0) = 5$

$\theta = \pi$



$F(\pi) = 0$

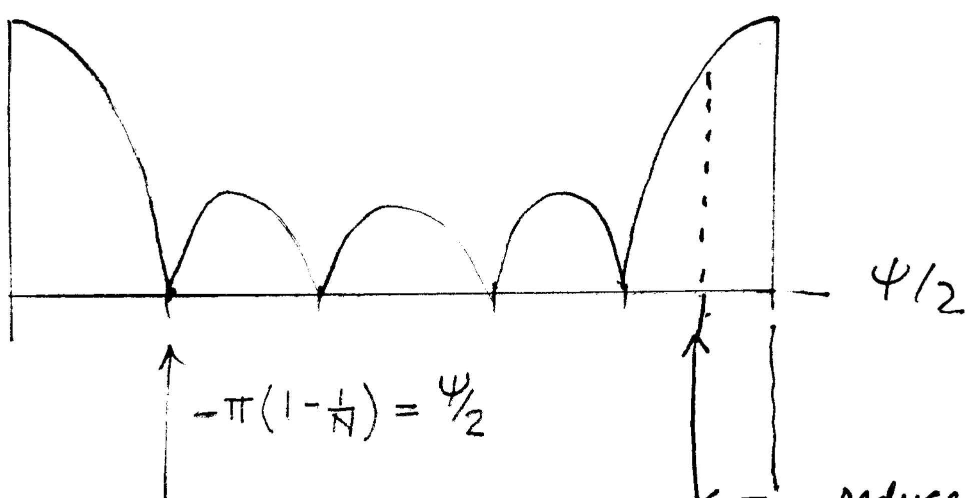
Array Pattern $\quad N = 5 \quad kd = \pi(1 - \frac{1}{5})$



18-28

Can $N = 5$ pencil beam be improved?

Yes - consider pattern structure $F(\psi/2)$



① pin the null here

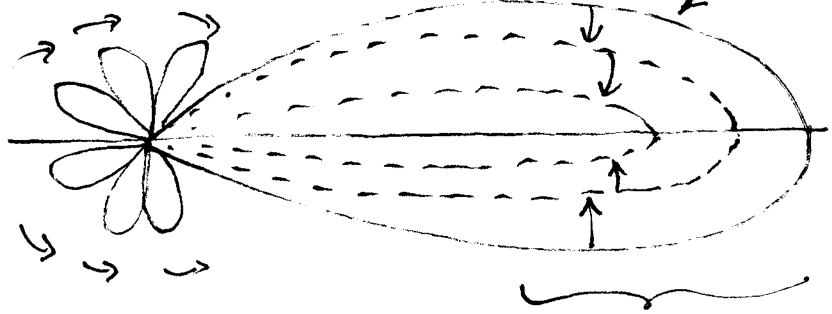
reduce range $\frac{\Delta\psi}{2}$ to move peak to left; for

Hansen - Woodyard

but reduces width of main beam @ $\theta = 0$

small, move to lower θ's

5 element array



$\theta = 0$

squeezing effect of reducing $\frac{\Delta\psi}{2}$

Net effect is to increase gain!

Example of a practical super gain/directivity array!

18-30

How is range of $\frac{\Delta\psi}{2}$ reduced?

$\Delta\psi = 2kd$; so let $kd' \to kd - \Delta$

Then, in order to maintain $\boxed{\alpha' - kd' = -2\pi\left(1 - \frac{1}{N}\right)}$

$\alpha' \to \alpha - \Delta$

So array "looks like"

$\leftarrow kd - \Delta \rightarrow$ $\quad e^{-j(kd+\Delta)\cdot 2}$

o o o o o . . . .

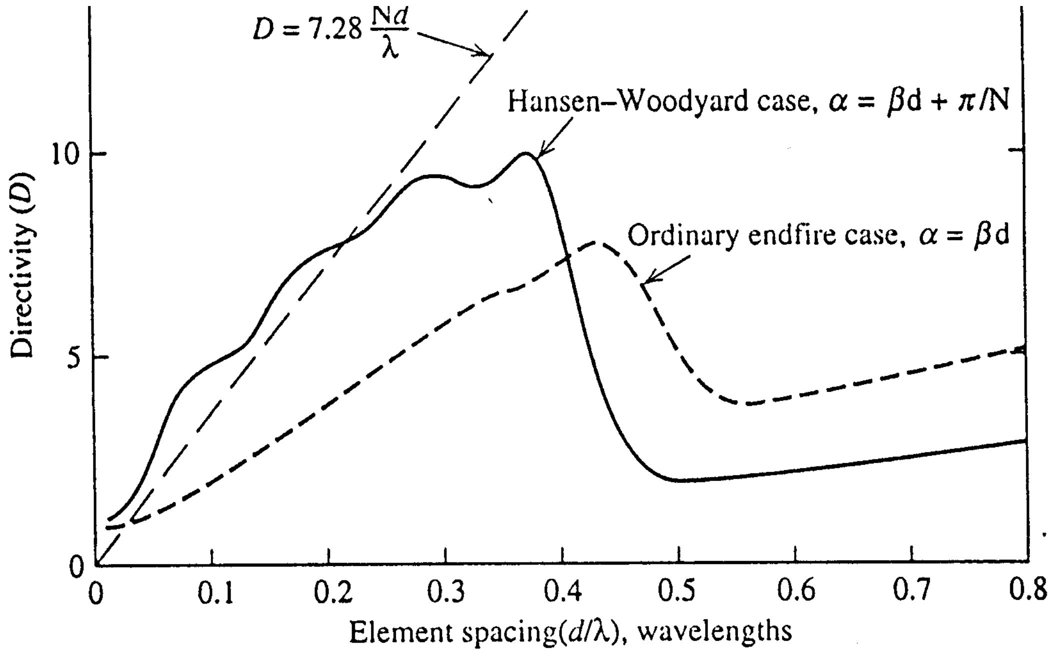


Figure 3-15 Comparison of directivities for two five-element equally spaced, uniformly excited endfire arrays: ordinary endfire (dotted curve) and Hansen–Woodyard endfire (solid curve). Also shown is the directivity approximation (3-82) for the Hansen–Woodyard case.