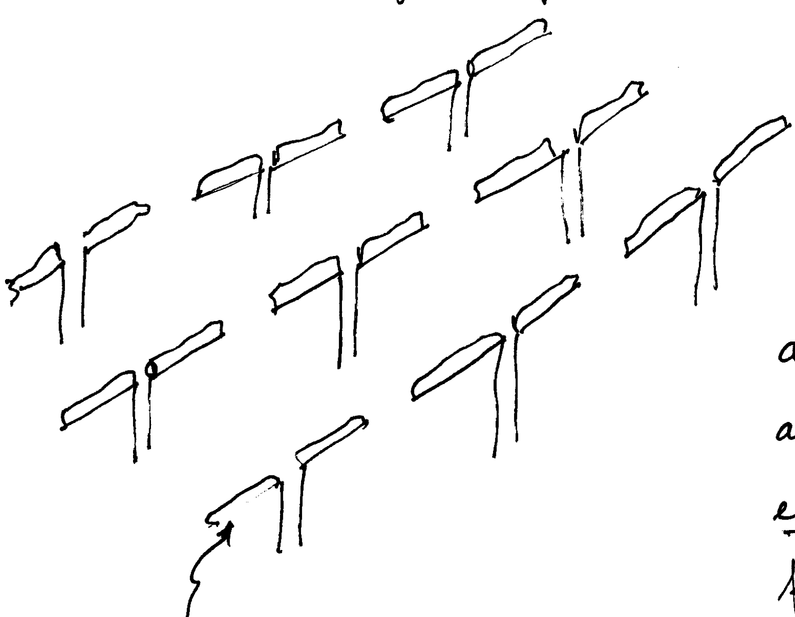
Array Theory – Reading

S & T Sections 3.1 – 3.4

Kraus Sections 4-1, 2, 3, 4, 5, 6, 7, 9, 10, 14

Arrays – Multiple radiating and receiving elements connected together for enhanced performance.



Cu pipe

Jicamarca (Peru)
El Campo (Texas)
arrays covered several acres with dipole elements constructed from plumbing materials

Used for research – Sun & ionosphere

VLA

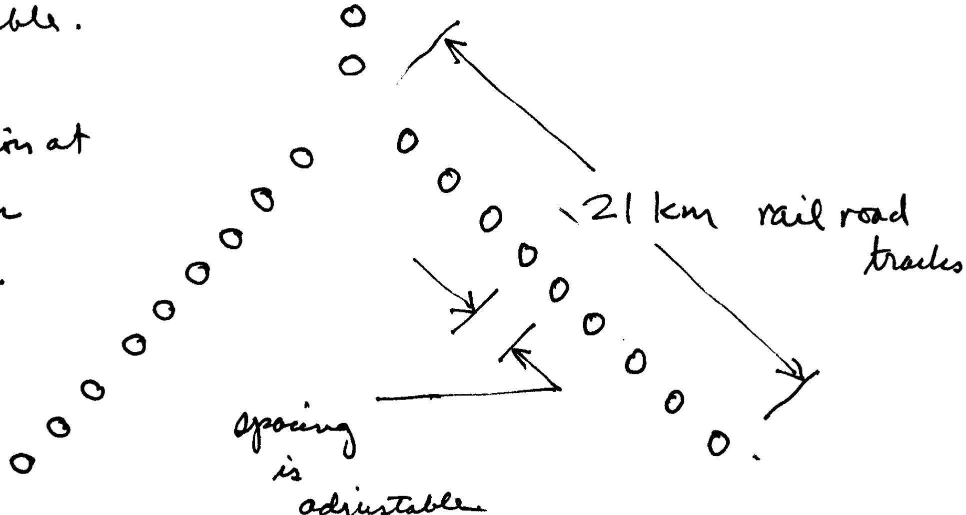
Very Large Array

Socorro, New Mexico

$\lambda \sim$ cm

"Elements" are 27, 25 m dia antennas, each steerable.

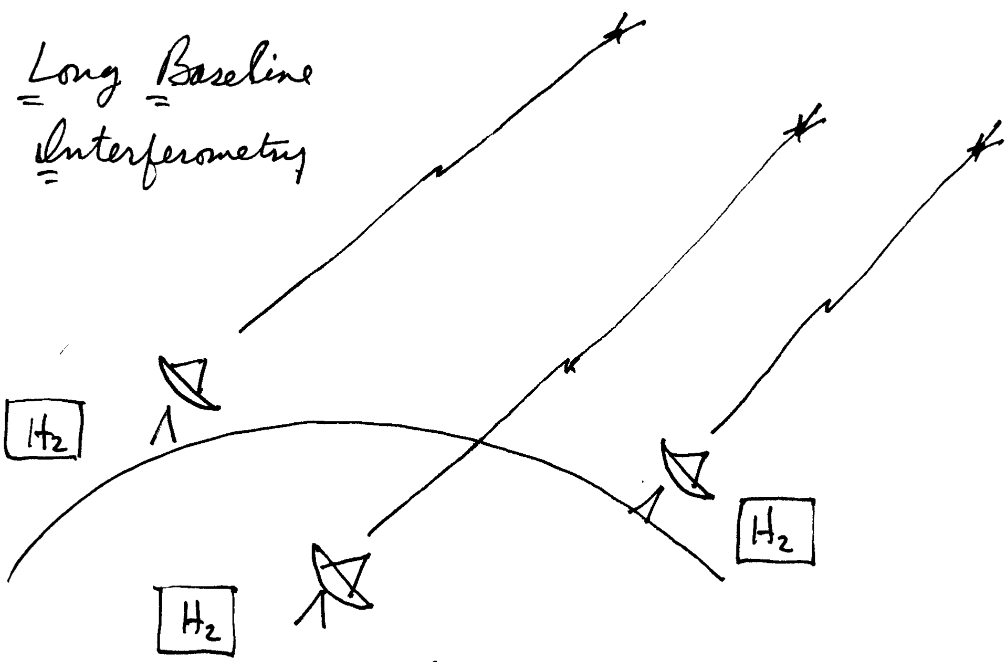
resolution at $\lambda = 6$ cm $\sim \frac{1}{2}$ sec



Maximum element spacing is $2 \times 21 \cdot \sin 60° \cong 36$ km

VLBI

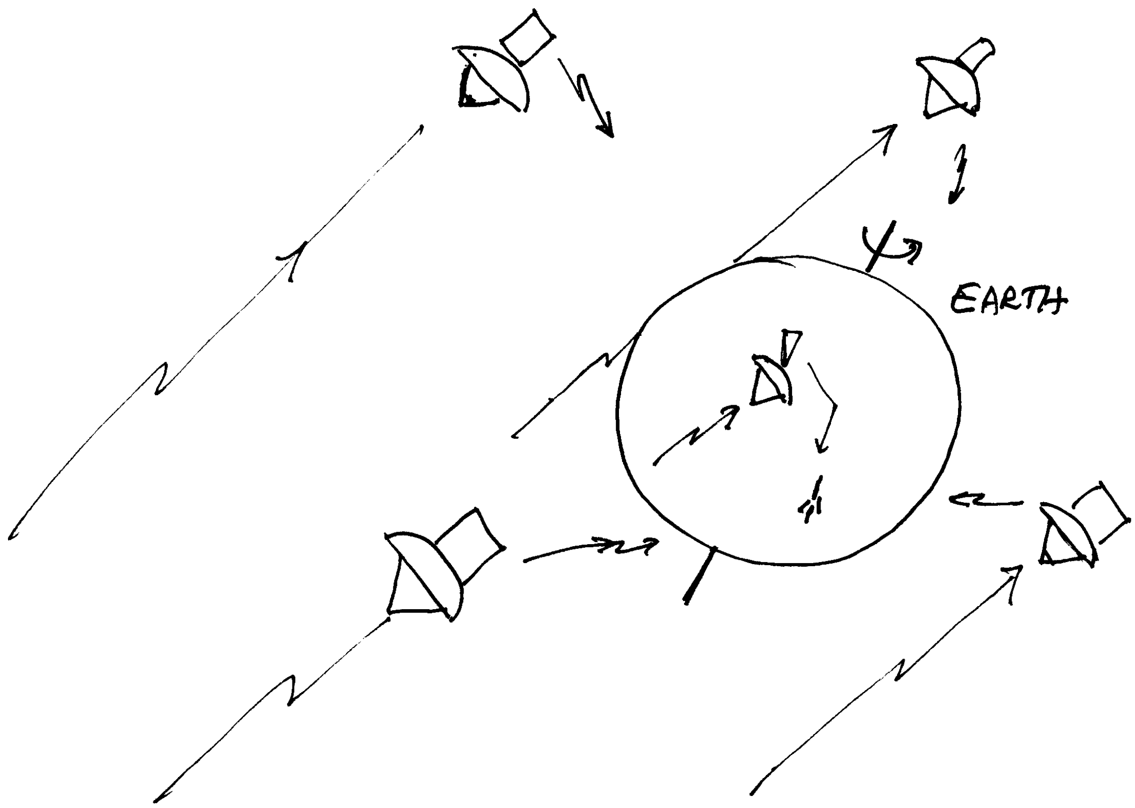
Very Long Baseline Interferometry



Contenental Scale Spacing

Non-real time operation

OVLBI Orbiting VLBI

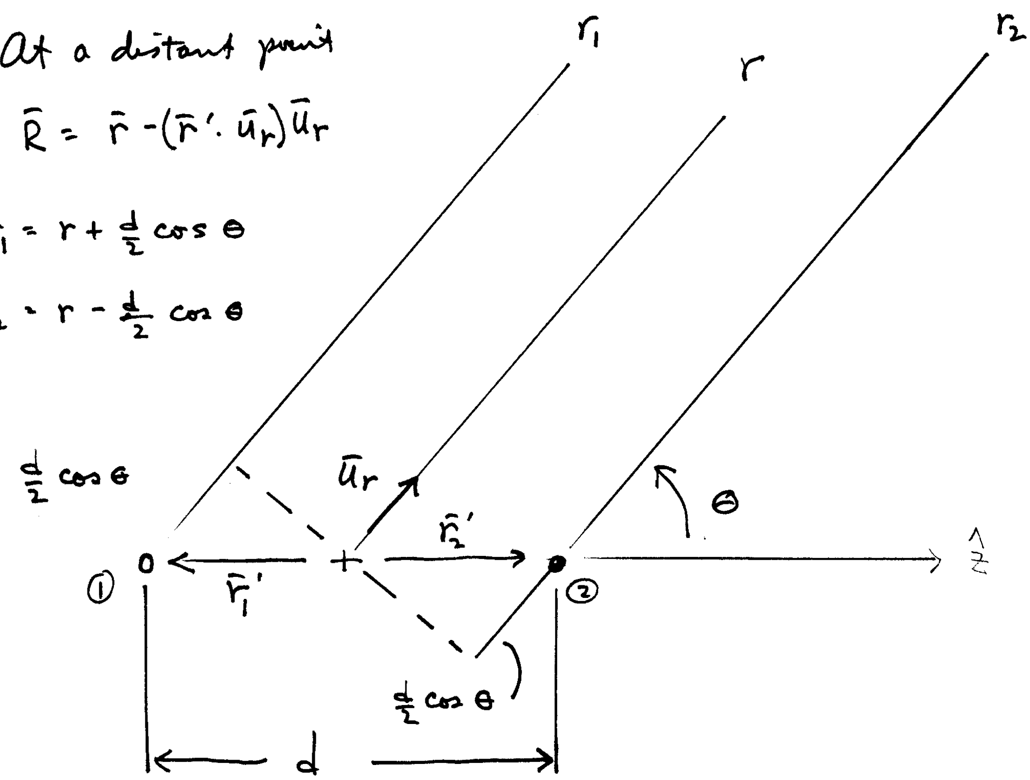


At a distant point

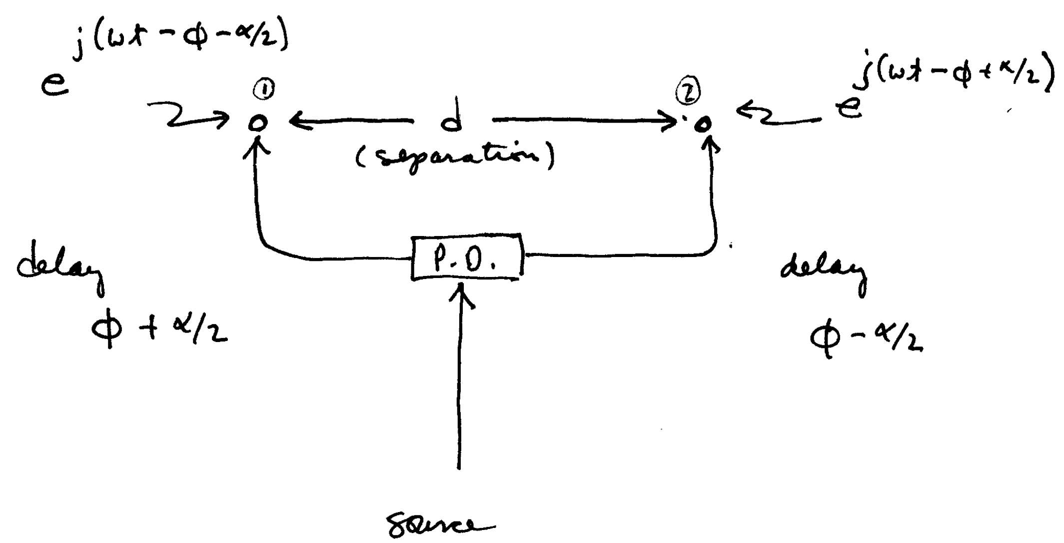
$\bar{R} = \bar{r} - (\bar{r}' \cdot \bar{u}_r)\bar{u}_r$

$r_1 = r + \frac{d}{2}\cos\theta$

$r_2 = r - \frac{d}{2}\cos\theta$



Two Element Array



At some distant point in θ direction

$$E_{Tot} = 2 \cdot \frac{A_o(\theta)}{r} e^{-jkr} \left[\frac{\underbrace{e^{+j(\frac{\alpha}{2}+\frac{dk}{2}\cos\theta)}}_{\substack{\text{less delay}\\ \text{side 2}}} + \underbrace{e^{-j(\frac{\alpha}{2}+\frac{dk}{2}\cos\theta)}}_{\substack{\text{more delay}\\ \text{side 1}}}}{2} \right]$$

Phase is referenced to center of baseline

$$= 2 \frac{A_o(\theta)}{r} e^{-jkr} \cos\left(\frac{\alpha}{2} + \frac{dk}{2}\cos\theta\right)$$

$$= 2 \frac{A_o(\theta)}{r} e^{-jkr} \cos(\psi/2) \;;\; \psi = \alpha + dk\cos\theta$$

/...

$$E_{Tot} = \underbrace{\frac{A_0(\theta)e^{-jkr}}{r}}\cdot\underbrace{2\cos(\psi/2)}$$

radiation from an individual element (one of two, in this case) | composite array factor giving effect of combining elements.

[Here elements are identical, but this need not be the case. A more general calculation would take into account the possible individual differences.]

17.10

$|E_{Tot}|$ - what is behavior?

maxima at $\frac{\alpha}{2} + \frac{dk}{2}\cos\theta = \pm n\pi = \frac{\psi}{2}$

minima $= 0$ at $\frac{\alpha}{2} + \frac{dk}{2}\cos\theta = \pm n\pi + \frac{\pi}{2} = \frac{\psi}{2}$

$dk\cos\theta_n = -\alpha \pm 2n\pi$ are maxima

$dk\cos\theta_n = -\alpha \pm 2n\pi + \pi$ are minima

$\underbrace{\frac{d}{\lambda}\cos\theta_n = -\frac{\alpha}{2\pi} \pm n, \text{max}}_{m}$; $\frac{d}{\lambda}\cos\theta_n = -\frac{\alpha}{2\pi} \pm n + \frac{1}{2}$, min

/...

17.11

Referring to figure above –

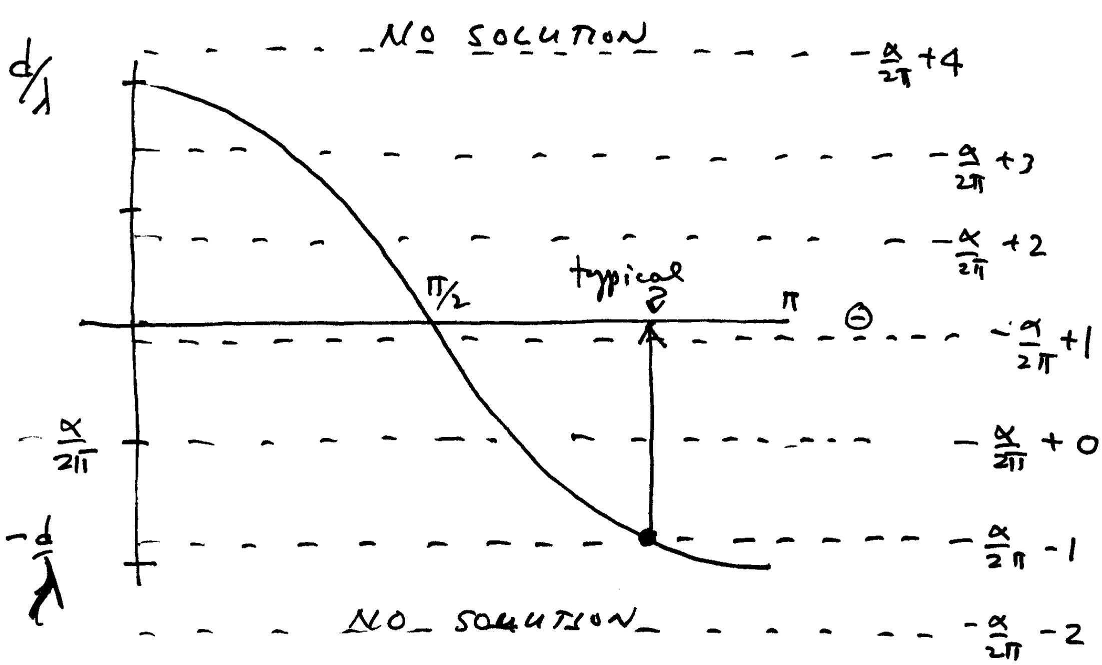
– Orientation of the lobes (locations of maxima) is set by α

– Spacing of lobes is set by d/λ

$$\begin{cases} \text{If} & \alpha = 0, \text{ always a maximum at } \theta = \frac{\pi}{2}, \ \forall d \\ \text{and} & d = n\lambda, \text{ always a maximum at } \theta = 0, \pi \\ \text{and} & d = n\lambda + \frac{\lambda}{2}, \text{ always a minimum at } \theta = 0, \pi \end{cases}$$

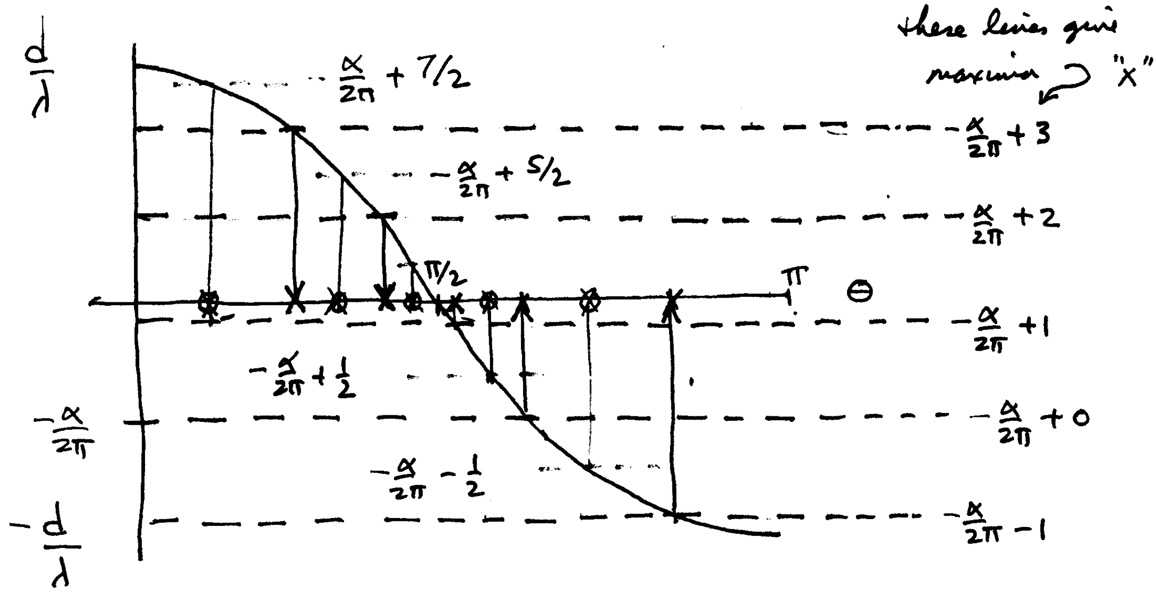
For $\alpha = \pi$, always a minimum at $\theta = \pi/2$

17.12



Plotting up these results . . .

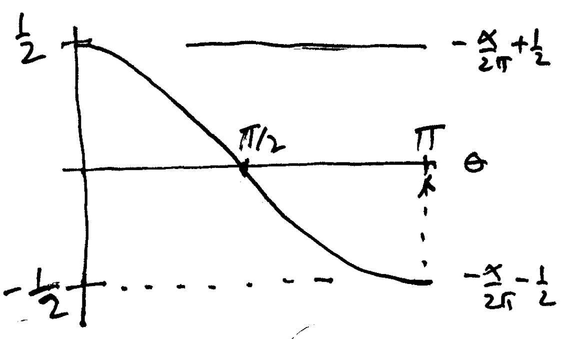
Intersections of $\frac{d}{\lambda}\cos\theta$ w/ these lines give maxima "x"

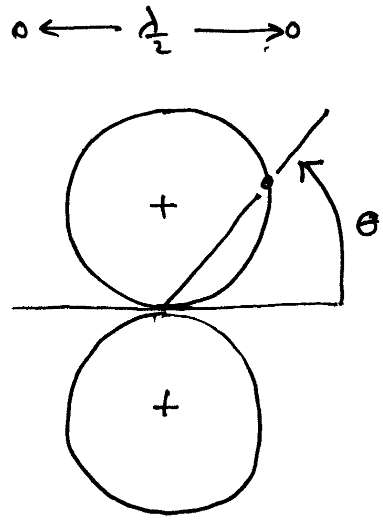


Number of maxima is about $\left[2\frac{d}{\lambda} + 1\right]$

/ . . .

Examples. 1. $\alpha = 0$





Max at $\theta = \frac{\pi}{2}$

Min at $\theta = 0, \pi$

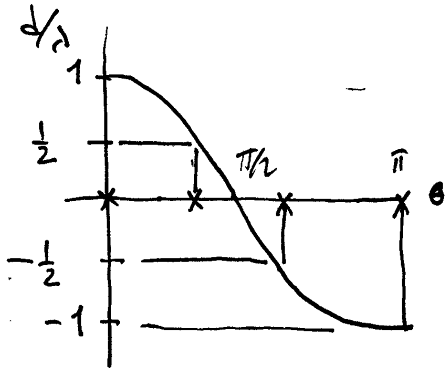
Pattern is figure of rotation – doughnut w/o hole

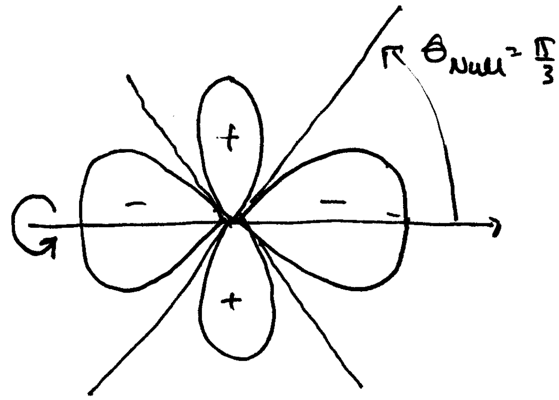
Field pattern is a circle in every plane containing axis.

2 . $\alpha = 0$

$0 \longleftarrow \lambda \longrightarrow 0$

Maxima at $\theta = 0, \frac{\pi}{2}, \pi$



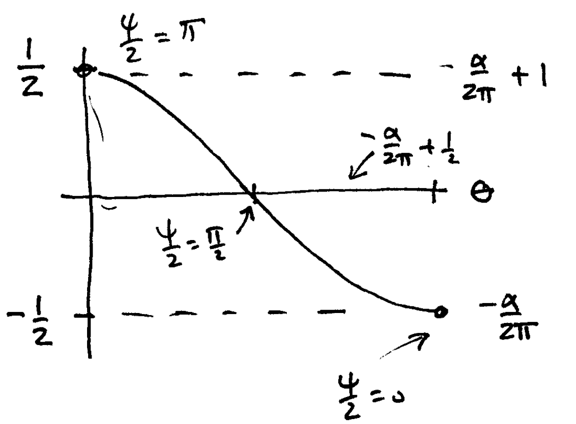


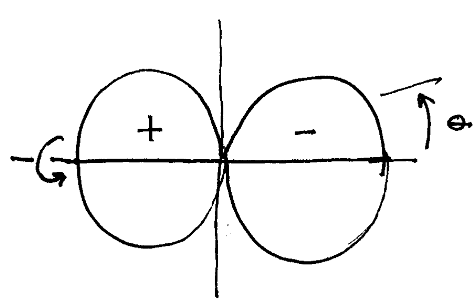
Curve is $\cos\left(\pi \frac{d}{\lambda} \cos\theta\right)\Big|_{d=\lambda}$

$= 2\cos(\pi \cos\theta)$

Example 3, $\alpha = \pi$

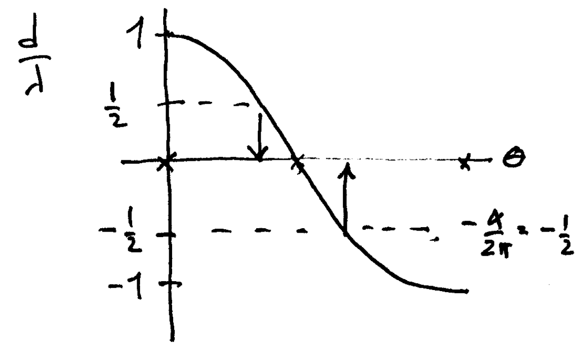
$0 \longleftarrow \frac{\lambda}{2} \longrightarrow 0$

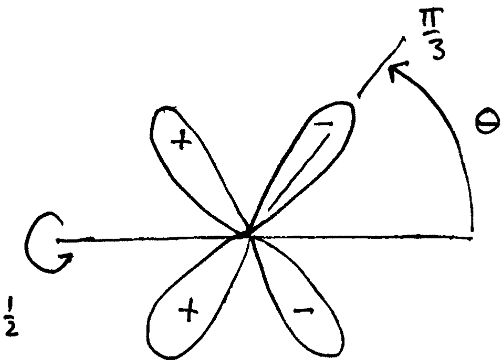




Example 4. $\alpha = \pi$

$o \longleftarrow \lambda \longrightarrow o$

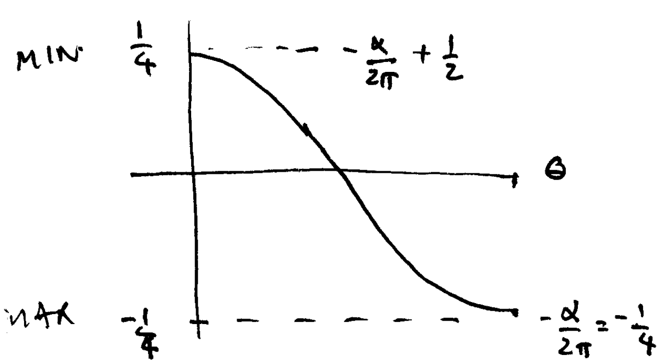


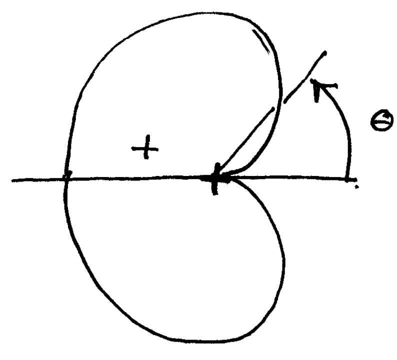


Example 5. $\alpha = \frac{\pi}{2}$, $d = \lambda/4$

$o \longleftarrow \frac{\lambda}{4} \longrightarrow o$

$e^{-j\pi/4}$ $\qquad$ $e^{+j\pi/4}$





Cardiod

$$2\cos\left[\frac{\pi}{4}(1+\cos\theta)\right]$$

Note that $\dfrac{d}{\lambda} = -\dfrac{\alpha}{2\pi} + \dfrac{1}{2}$ puts null at $\theta = 0$;

there may or may not be a max. (for arbitrary d)

$$\alpha = 2\pi\left(\frac{1}{2} - \frac{d}{\lambda}\right)$$

Example $\dfrac{d}{\lambda} = \dfrac{1}{10}$, $\alpha = 2\pi\left(\dfrac{4}{10}\right) = \dfrac{4\pi}{5}$

① ②

$o \xleftarrow{\quad} \dfrac{\lambda}{10} \xrightarrow{\quad} o$

$e^{-j\pi\frac{2}{5}}$ $e^{+j\pi\frac{2}{5}}$

$$e^{j\frac{2\pi}{5}} + e^{-j\frac{2\pi}{5} - j\frac{2\pi}{10}} = 0$$