Waves I **EE241 Autumn 2005–2006**

Len Tyler **Dec 1, 2002**

Problem Set #8

Due Date: *Wednesday*, December 9, 2005. *Submit in class.*

Problems:

1. Rayleigh Waves [30 points]

   (a) Find the six roots of the Rayleigh equation for a perfectly compressible (μ finite, $\lambda = 0$) semi-infinite isotropic solid with a vacuum interface. Hint: one of the roots is $\sqrt{2}$. [10 points]

   (b) Which of the six roots is the $+x$-directed slow-wave solution? Find the Rayleigh wave speed. [5 points]

   (c) Sketch the particle motion at the surface and find the depth where the particle motion is only in the z direction. [15 points]

2. Mixed Waves in Stratified Media [30 points]

   For a wave in a stratified and isothermal atmosphere, assume a solution having vertical displacement

   $$\zeta = \zeta_0 e^{\nu z} \cos(\omega t - \alpha x - \gamma z)$$

   where $(2\nu)^{-1}$ is the scale height of the atmosphere. Consider the incompressible case ($c \to \infty$):

   (a) Find the expression for ξ. [4 points]

   (b) For a fixed point in space ($x = 0, z = 0$), eliminate time from the expressions for ξ and ζ to obtain the following equation for the particle orbits [10 points]:

   $$\xi^2 + \zeta^2 \left(\frac{N^2}{\omega^2} - 1 \right) + 2\frac{\gamma}{\alpha}\xi\zeta = \frac{\nu^2}{\alpha^2}\zeta_0^2$$

   (c) Describe the shape of the orbit for the cases below. For v through viii, compare the lengths of the vertical and horizontal displacements for the prescribed ellipses. Use the orbit equation above. [16 points]

      i. $\omega < N$

ii. $\omega \to N, \alpha \to \infty$

iii. $\omega \to 0$

iv. $\omega \approx N$

v. $\gamma = 0$

vi. $\frac{N}{\sqrt{2}} < \omega < N$

vii. $\omega = \frac{N}{\sqrt{2}}$

viii. $0 < \omega < \frac{N}{\sqrt{2}}$

3. Equatorial waves [30 points]

When we apply the shallow water equations on the rotating earth in Cartesian geometry, we obtain the following characteristic equation for propagating waves near the equator

$$\frac{\sqrt{gh}}{\beta}\left(\frac{\omega^2}{gh} - k^2 - \frac{k}{\omega}\beta\right) = 2n + 1$$

where h is the equivalent depth of the ocean near the equator (constant),
β is the Coriolis parameter (constant near the equator),
g is the acceleration due to gravity (constant),
k is the longitudinal wavenumber,
ω is the angular frequency, and
n (an integer) corresponds to the meridonial mode number (for different solutions).

(a) Plot the ω vs k diagrams for each of the following modes.

(b) Comment on the direction of propagation (positive k corresponds to eastward propagating waves along the equator) and the nature of dispersion.

(c) What are the corresponding cut-off frequencies for eastward propagating waves (positive k)?

i. Mixed Rossby-gravity wave ($n = 0$)

ii. Kelvin wave ($n = -1$)

iii. Rossby wave ($n = 2$, Low frequency)

iv. Inertio-gravity wave ($n = 2$, High frequency)

Note : You can plot $\omega^* = \omega/(\beta\sqrt{gh})^{\frac{1}{2}}$ vs $k^* = k(\sqrt{gh}/\beta)^{\frac{1}{2}}$ to make your plots indepedent of the constants, g, h and β.

ee241_hw08_2005.tex