Waves I | **EE241 Autumn 2005–2006**

Len Tyler | **Oct. 26, 2005**

Problem Set #5

Due Date: November 4, 2005. Submit in class, or outside Packard 331 before 4:30 PM.

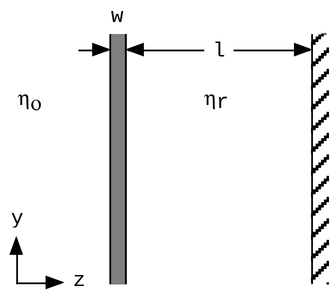
Reading Assignment:

"Reader"	Chapter 6
R W & VD	Sections 6.1-6.3, 6.5-6.8

Problems:

1. Salisbury Screen [30 points]

   Consider a thin screen of thickness w in front of a perfectly conducting plane and separated from it by a lossless spacer of relative impedance η_r and width l, as shown.



   The screen is made of space cloth, which has the property that the resistance of a square is given by $R = 377\ \Omega = \eta_0$. Remember that

$$R = \frac{s}{\sigma a} = \frac{s}{\sigma w s} = \frac{1}{\sigma w}$$

   where

$$\begin{aligned} s &= \text{length of side (m)} \\ a &= \text{area of edge (m}^2\text{)} \\ \sigma &= \text{conductivity of space cloth } (\Omega^{-1}\text{m}^{-1}) \end{aligned}$$

   A plane electromagnetic wave is normally incident upon the screen from the left (free space).

   (a) Determine the boundary conditions for **E** and **H** at the screen and at the conductor, assuming that $w \ll \lambda$ and that $w \ll \delta$ where the skin depth $\delta \equiv \frac{1}{\sqrt{\pi f \mu \sigma}}$. [20 points]

(b) Determine the distance l between the screen and the conductor for which there are no reflections back towards the source. [10 points]

2. Dielectric slabs [15 points]

   An equal number of quarter wave ($\varepsilon_r = 2$) and half-wave ($\varepsilon_r = 4$) dielectric slabs are alternately stacked to make an electromagnetic window for normal incidence between two regions of free space.

   (a) Find the reflection coefficient and standing-wave ratio if there is an odd number of each of the two kinds of slabs. [5 points]

   (b) What value of ε_r for the quarter-wave slabs (keep $\varepsilon_r = 4$ for the half-wave slabs) will transmit the same power as is reflected? [5 points]

   (c) Find the reflection coefficient and standing-wave ratio if there is an even number of each of the two kinds of slabs (with values from part a). [5 points]

3. Normal incidence upon a Dilectric boundary [20 points]

   A linearly polarized wave $\vec{E}_1^+$ is propagating in the dielectric (medium 1) and is normally incident upon another dielectric (medium 2), resulting in a reflected wave $\vec{E}_1^-$ and a transmitted wave $\vec{E}_2^+$. Assume that the boundary is at $z = 0$, and that the dielectric media are non-magnetic ($\mu = \mu = 0$) and lossless ($\sigma = 0$).

   (a) Consider the case when the reflection coefficient $\rho = \frac{|E_1^-|}{|E_1^+|} = \frac{1}{3}$.

      i. What is ϵ_1/ϵ_2? [4 points]

      ii. What is the transmission coefficient, $t = \frac{|E_2^+|}{E_1^+|}$? [2 points]

      iii. Assuming a frequency ω, sketch the total electric fields in bothe media at different times. Please label your sketches. [7 points]

   (b) Consider the case where a third of the incident power is reflected and the rest is transmitted:

      i. What is ϵ_1/ϵ_2? [4 points]

      ii. What are ρ and t? [3 points]

4. Sandwich problem [20 points]

   Two quarter-wave layers of intrinsic impedance η_2 and η_3 are sandwiched between dielectrics of intrinsic impedance η_1 and η_4. Determine the relationship between the intrinsic impedances of all four layers for perfect matching to occur.

5. Impedance Transformation [15 points]

A plane wave travelling in the positive z-direction in a dielectric with intrinsic impedance η_1 is incident on another dielectric of impedance η_2 at $z = 0$. We define the impedance function $\Gamma(z)$ as follows:

$$\Gamma(z) = \frac{E(z)}{H(z)}$$

Express $\Gamma(z)$ as a function of η_1, η_2, k and z, where k is the wavenumber of the incident wave.