Matrices in Julia

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Matrices

- Matrices in Julia are represented by 2D arrays
- \[
\begin{bmatrix}
2 & -4 & 8.2 \\
-5.5 & 3.5 & 63
\end{bmatrix}
\]
  creates the \(2 \times 3\) matrix \(A\)

\[
A = \begin{bmatrix}
2 & -4 & 8.2 \\
-5.5 & 3.5 & 63
\end{bmatrix}
\]

- Spaces separate entries in a row; semicolons separate rows
- \(\text{size}(A)\) returns the size of \(A\) as a pair, i.e.,
  \[
  A_{\text{rows}}, A_{\text{cols}} = \text{size}(A) # \text{ or}
  \]
  \[
  # A_{\text{rows}} \text{ is size}(A)[1], A_{\text{cols}} \text{ is size}(A)[2]
  \]

- Row vectors are \(1 \times n\) matrices, e.g., \([4 \ 8.7 \ -9]\)
Indexing and slicing

- $A_{ij}$ is found with $A[i,j]$
- can use ranges: $A[1:2,1:3]$ is $2 \times 3$ submatrix or slice $A_{1:2,1:3}$
- : selects all elements along that dimension
  - $A[:,3]$ is third column
  - $A[2,:]$ is second row
- $A[:,]$ stacks the columns of $A$ as a vector (column-major order)
- $A\,'[:,]$ stacks the rows of $A$ as a vector (row-major order)
Block matrices

- block matrix

\[
X = \begin{bmatrix}
A & B \\
C & D \\
\end{bmatrix}
\]

(with A, B, C, and D matrices) is formed with

\[
X = \begin{bmatrix}
A & B \\
C & D \\
\end{bmatrix}
\]

- usual rules governing dimensions of A, B, C, and D apply
Useful matrices in Julia

- $0_{m \times n}$ is `zeros(m,n)`
- $m \times n$ matrix with all entries 1 is `ones(m,n)`
- $I_{n \times n}$ is `eye(n)`
- `diag(x)` is `diagm(x)` (where $x$ is a vector)
Transpose and matrix addition

▶ $A^T$ is written $A'$ (single quote mark)
▶ $+/-$ are used for matrix addition/subtraction (matrices must have the same size)
▶ for example,

\[
\begin{bmatrix}
4.0 & 7 \\
-10.6 & 89.8
\end{bmatrix} + \begin{bmatrix}
19 & -34.7 \\
20 & 1
\end{bmatrix}
\]

is written

$[4.0 \ 7; -10.6 \ 89.8] + [19 \ -34.7; \ 20 \ 1]$
Matrix-scalar operations

- matrix-scalar operations (+, -, *, \") apply elementwise
- scalar-matrix multiplication:
  \[ 10 \times \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \]
gives
  \[ 10 \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 10 & 20 \\ 30 & 40 \end{bmatrix} \]
  (scalar can also appear on right of matrix)

- matrix-scalar addition:
  \[ \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} + 10 \]
gives
  \[ \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} + \begin{bmatrix} 10 & 10 \\ 10 & 10 \end{bmatrix} = \begin{bmatrix} 11 & 12 \\ 13 & 14 \end{bmatrix} \]
  (which is not standard mathematical notation)
Matrix-vector multiplication

• * operator is used for matrix-vector multiplication
• for example,

\[
\begin{bmatrix}
1 & 2 \\
3 & 4
\end{bmatrix}
\begin{bmatrix}
5 \\
6
\end{bmatrix}
\]

is written

\[\begin{bmatrix}
1 & 2; & 3 & 4
\end{bmatrix}
\begin{bmatrix}
5, & 6
\end{bmatrix}\]
Matrix multiplication

* is also used for matrix-matrix multiplication:

\[
\begin{bmatrix}
2 & 4 & 3 \\
3 & 1 & 5
\end{bmatrix}
\begin{bmatrix}
3 & 10 \\
4 & 2 \\
1 & 7
\end{bmatrix}
\]

is written

\[
\begin{bmatrix}
2 & 4 & 3 \\
3 & 1 & 5
\end{bmatrix}
\begin{bmatrix}
3 & 10 \\
4 & 2 \\
1 & 7
\end{bmatrix}
\]

\[A^k\] is \[A^k\] (for square matrix \[A\])
Other functions

- sum of entries of a matrix: \( \sum(A) \)
- average of entries of a matrix: \( \text{mean}(A) \)
- \( \max(A,B) \) and \( \min(A,B) \) finds the element-wise \( \max \) and \( \min \) respectively
  - the arguments must have the same size unless one is a scalar
- \( \text{norm}(A) \) is not what you might think
  - to find \( \left( \sum_{i,j} A_{ij}^2 \right)^{1/2} \) use \( \text{norm}(A[:]) \) or \( \text{vecnorm}(A) \)
Computing regression model RMS error

the math:

▶ \( X \) is an \( n \times N \) matrix whose \( N \) columns are feature \( n \)-vectors
▶ \( y \) is the \( N \)-vector of associated outcomes
▶ regression model is \( \hat{y} = X^T \beta + v \) (\( \beta \) is \( n \)-vector, \( v \) is scalar)
▶ RMS error is \( \text{rms}(\hat{y} - y) \)

in Julia:

\[
y_{\text{hat}} = X' \ast \text{beta} + v \\
\text{rms\_error} = \text{norm}(y_{\text{hat}} - y) / \text{sqrt}(\text{length}(y))
\]