

EE102 Homework 7 and 8 Solutions

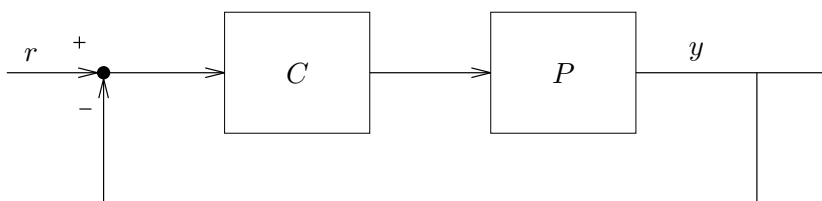
69. *Stability analysis of a PI controller.* Suppose that PI control,

$$C(s) = k_p + \frac{k_i}{s},$$

is used with the plant

$$P(s) = \frac{1}{s^2 + 2s + 2},$$

in the standard feedback control configuration:



Find the conditions on k_p and k_i for which the closed-loop transfer function T from r to y is stable. You can assume that $k_p > 0$ and $k_i > 0$.

Express your conditions in the simplest form possible.

Solution:

The transfer function T is

$$T = \frac{PC}{1 + PC} = \frac{k_p s + k_i}{s(s^2 + 2s + 2) + k_p s + k_i}.$$

We need to check when the denominator,

$$s^3 + 2s^2 + (k_p + 2)s + k_i,$$

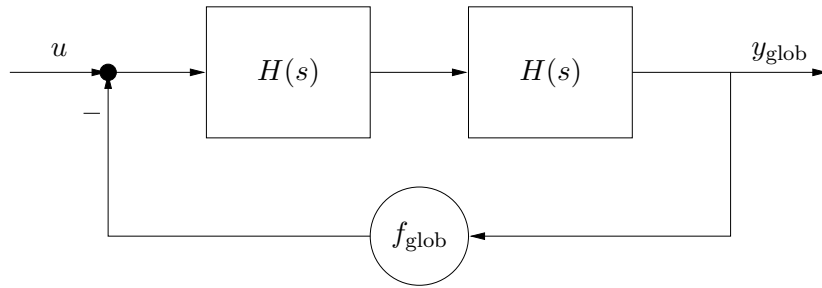
is Hurwitz. Using the result from lecture 13, we find the conditions are

$$2 > 0, \quad k_p + 2 > 0, \quad k_i > 0, \quad (2)(k_p + 2) > k_i.$$

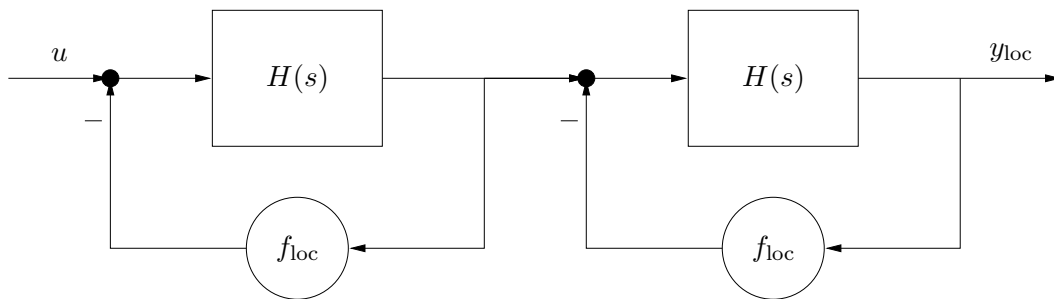
We already assume that k_p and k_i are positive, so the condition is just $k_i < 2(k_p + 2)$.

70. *Local versus global feedback: dynamic analysis.* We consider two amplifiers, each with a transfer function $H(s) = 100/(1 + s)$. We are going to use these amplifiers, together with feedback, to design a system with DC gain 40dB. (Roughly speaking, then, we have 40dB of ‘extra’ gain.)

Global feedback. In this arrangement we connect the two amplifiers in cascade, and then use feedback around the cascade connection, as shown below.



Local feedback. In this arrangement we use feedback around each amplifier, and then put the two closed-loop systems in cascade, as shown below. (To simplify things, we'll assume the two feedback gains are the same.)



In both arrangements, the feedback consists of a positive gain (independent of s).

- Find the value of f_{glob} that makes the closed-loop DC gain from u to y_{glob} , in the global feedback arrangement, equal to 40dB.
- Find the value of f_{loc} that makes the closed-loop DC gain from u to y_{loc} , in the local feedback arrangement, equal to 40dB.

We'll define the *settling time* of a system as the time it takes for the unit step response to settle to within about 10% of its final (asymptotic) value. If the system is unstable, we'll say the settling time is ∞ .

- Using the value for f_{glob} found in part 3a, find the settling time T_{glob} of the global feedback system.
- Using the value for f_{loc} found in part 3b, find the settling time T_{loc} of the local feedback system.
- Give a *one sentence intuitive explanation* of the results of 3c and 3d.
- You can estimate the settling times, provided you explain how you obtain the estimate. An accuracy of $\pm 30\%$ is fine.

Solution:

Let's first consider global feedback. This is the same as putting feedback around a single amplifier with transfer function $(100/(1+s))^2$. The (open-loop) DC gain is 10000, and we need a closed loop gain of 40dB, *i.e.*, 100. We can find f_{glob} from

$$100 = \frac{10000}{1 + 10000f_{\text{glob}}},$$

which gives $f_{\text{glob}} = 0.0099$ (*i.e.*, very nearly 0.01).

Now let's find the resulting transfer function from u to y_{glob} :

$$T_{\text{glob}}(s) = \frac{10000/(s+1)^2}{1 + 0.0099 \cdot 10000/(s+1)^2} = \frac{10^4}{s^2 + 2s + 100}.$$

(We can check that the DC gain is indeed 100, as required.) The poles are $-1 \pm j\sqrt{99}$.

The decay rate is given by the real part, so we expect convergence in about $T_{\text{glob}} \approx 2\text{sec}$ or so (two time constants, $e^{-2} \approx 0.13$). Note also that the dynamics are highly oscillatory, which is probably very undesirable.

Now let's consider local feedback. Each feedback amplifier must have a closed-loop gain of 10, so we solve

$$10 = \frac{100}{1 + 100f_{\text{loc}}},$$

to get $f_{\text{loc}} = 0.09$. The resulting transfer function from u to y_{glob} is

$$T_{\text{loc}}(s) = \left(\frac{100/(s+1)}{1 + 0.09 \cdot 100/(s+1)} \right)^2 = \frac{100^2}{(s+10)^2}.$$

(Yes, the DC gain is 100.) The poles are real and repeated, at -10 . The step response will therefore have a constant term, and terms of the form e^{-10t} and te^{-10t} . A good approximation of the 10% settling time is about two time constants, which is around 0.2 seconds. Notice that this settling time is far smaller than the one for global feedback. Moreover, instead of a very oscillatory response, we get a nice, critically damped response. So evidently the local feedback arrangement is far better, in terms of dynamic response, than the global feedback arrangement. But remember from a homework exercise that in terms of reduction of sensitivity to amplifier gain, the global arrangement is better.

Here's one sentence that explains it: Delay around a feedback loop, along with too much loop gain, can yield oscillatory or even unstable response; the global feedback arrangement has more delay and more loop gain, so we were asking for trouble.

94. *Feedback amplifier design.* In this problem, you'll use the standard feedback amplifier configuration, described by the equations

$$y = Ae, \quad e = u - Fy.$$

The raw amplifier, denoted A , is linear and time invariant, with transfer function given by

$$A(s) = \frac{A_{\text{dc}}}{1 + sT},$$

and the feedback F is a simple gain (*i.e.*, a constant). The constant A_{dc} (which is positive) is the DC gain of the raw amplifier, and T (which is also positive) is the 63% rise time of the raw amplifier. The DC gain of the amplifier varies over a range (say, with temperature, manufacturing variations, etc.). For simplicity, we'll assume that the rise time T does not vary, and that the feedback F is implemented using high precision components, and so does not vary.

You can choose among four different raw amplifiers, with characteristics:

- *Raw amplifier 1*: DC gain is $40 \pm 3\text{dB}$; rise time is $0.5\mu\text{sec}$.
- *Raw amplifier 2*: DC gain is $65 \pm 5\text{dB}$; rise time is $2\mu\text{sec}$.
- *Raw amplifier 3*: DC gain is $80 \pm 10\text{dB}$; rise time is $15\mu\text{sec}$.

Choose *one* of the raw amplifiers and an appropriate feedback gain F according to the following design rules:

- The (nominal) closed-loop DC gain is 30dB.
- The variation in closed-loop DC gain is no more than $\pm 5\%$.
- The closed-loop 63% rise time is as small as possible.

For calculating the closed-loop rise time you can use the nominal gain of the raw amplifier (*i.e.*, 50dB for raw amplifier 1, etc.), and ignore the gain variation.

Solution.

First some quick analysis to figure out the effect of the feedback on the rise-time. The closed-loop transfer function is

$$G(s) = \frac{A(s)}{1 + A(s)F} = \frac{A_{\text{dc}}/(1 + sT)}{1 + A_{\text{dc}}F/(1 + sT)} = \frac{A_{\text{dc}}}{1 + A_{\text{dc}}F} \frac{1}{1 + s\tilde{T}},$$

where $\tilde{T} = T/(1 + A_{\text{dc}}F)$. In other words, the closed-loop system is also first order, with the obvious DC gain, and a time constant (or rise-time) that is a factor $S(0)$ faster than the open-loop amplifier.

Now let's consider each choice. For each one we solve for $S(0)$ using the formula $G(0) = A_{\text{dc}}S(0)$.

Raw amplifier 1. Here $S(0)$ is 10dB = 3.16. So the closed-loop rise-time is $0.5\mu\text{sec}/3.16 = 0.156\mu\text{sec}$. The variation in DC gain is given by (approximately) $3\text{dB}/3.16 = 0.95\text{dB}$, which is around 11%, so we don't meet the DC gain variation requirement.

Raw amplifier 2. Here $S(0)$ is 35dB = 56.2. The closed-loop rise-time is $2\mu\text{sec}/56.2 = 0.0356\mu\text{sec}$. The variation in DC gain is given by (approximately) $5\text{dB}/56.2 = 0.089\text{dB}$, which is around 1%, so we meet the DC gain variation requirement, with a healthy margin.

Raw amplifier 3. Here $S(0)$ is 50dB = 316. The closed-loop rise-time is $15\mu\text{sec}/316 = 0.0475\mu\text{sec}$. The variation in DC gain is given by (approximately) $10\text{dB}/316 = 0.032\text{dB}$, which is around 0.365%, so we meet the DC gain variation requirement, with a healthy margin.

Conclusion: Raw amplifier 2 is best. The appropriate F is given by

$$\frac{1778}{1 + 1778F} = 31.6$$

(using 65dB = 1778, 30dB = 31.6). We can solve this to get $F = 0.0311$.