CS276A
Information Retrieval

Lecture 7

Recap of the last lecture

- Parametric and field searches
  - Zones in documents
- Scoring documents: zone weighting
  - Index support for scoring
- *tf*×*idf* and vector spaces

This lecture

- Vector space scoring
- Efficiency considerations
  - Nearest neighbors and approximations

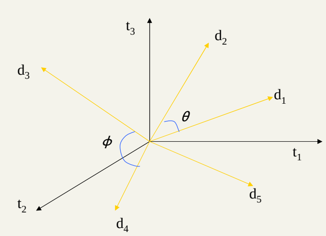
Documents as vectors

- At the end of Lecture 6 we said:
- Each doc *j* can now be viewed as a vector of *wf*×*idf* values, one component for each term
- So we have a vector space
  - terms are axes
  - docs live in this space
  - even with stemming, may have 20,000+ dimensions

Why turn docs into vectors?

- First application: Query-by-example
  - Given a doc D, find others “like” it.
- Now that D is a vector, find vectors (docs) “near” it.

Intuition



Postulate: Documents that are “close together” in the vector space talk about the same things.

The vector space model

Query as vector:
- We regard query as short document
- We return the documents ranked by the closeness of their vectors to the query, also represented as a vector.

Desiderata for proximity

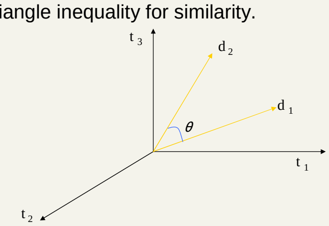
- If d_1 is near d_2, then d_2 is near d_1.
- If d_1 near d_2, and d_2 near d_3, then d_1 is not far from d_3.
- No doc is closer to d than d itself.

First cut

- Distance between d_1 and d_2 is the length of the vector $|d_1 - d_2|$.
  - Euclidean distance
- Why is this not a great idea?
- We still haven't dealt with the issue of length normalization
  - Long documents would be more similar to each other by virtue of length, not topic
- However, we can implicitly normalize by looking at *angles* instead

Cosine similarity

- Distance between vectors d_1 and d_2 *captured* by the cosine of the angle x between them.
- Note – this is *similarity*, not distance
  - No triangle inequality for similarity.



Cosine similarity

- A vector can be *normalized* (given a length of 1) by dividing each of its components by its length – here we use the L_2 norm $\|\mathbf{x}\|_2 = \sqrt{\sum_i x_i^2}$
- This maps vectors onto the unit sphere:
- Then, $\left|\vec{d}_j\right| = \sqrt{\sum_{i=1}^{n} w_{i,j}} = 1$
- Longer documents don't get more weight

Cosine similarity

$$sim(d_j, d_k) = \frac{\vec{d}_j \cdot \vec{d}_k}{\left|\vec{d}_j\right|\left|\vec{d}_k\right|} = \frac{\sum_{i=1}^{n} w_{i,j} w_{i,k}}{\sqrt{\sum_{i=1}^{n} w_{i,j}^2}\sqrt{\sum_{i=1}^{n} w_{i,k}^2}}$$

- Cosine of angle between two vectors
- The denominator involves the lengths of the vectors.

Normalization

Normalized vectors

- For normalized vectors, the cosine is simply the dot product:

$$\cos(\vec{d}_j, \vec{d}_k) = \vec{d}_j \cdot \vec{d}_k$$

Cosine similarity exercises

- *Exercise: Rank the following by decreasing cosine similarity:*
  - Two docs that have only frequent words ***(the, a, an, of)*** in common.
  - Two docs that have no words in common.
  - Two docs that have many rare words in common ***(wingspan, tailfin).***

Exercise

- Euclidean distance between vectors:

$$\left| d_j - d_k \right| = \sqrt{\sum_{i=1}^{n} \left(d_{i,j} - d_{i,k} \right)^2}$$

- Show that, for normalized vectors, Euclidean distance gives the same proximity ordering as the cosine measure

Example

- Docs: Austen's *Sense and Sensibility*, *Pride and Prejudice*; Bronte's *Wuthering Heights*

	SaS	PaP	WH
affection	**115**	**58**	**20**
jealous	**10**	**7**	**11**
gossip	**2**	**0**	**6**

	SaS	PaP	WH
affection	**0.996**	**0.993**	**0.847**
jealous	**0.087**	**0.120**	**0.466**
gossip	**0.017**	**0.000**	**0.254**

- cos(SAS, PAP) = .996 x .993 + .087 x .120 + .017 x 0.0 = 0.999
- cos(SAS, WH) = .996 x .847 + .087 x .466 + .017 x .254 = 0.929

Digression: spamming indices

- This was all invented before the days when people were in the business of spamming web search engines:
  - Indexing a sensible passive document collection vs.
  - An active document collection, where people (and indeed, service companies) are shaping documents in order to maximize scores

Summary: What's the real point of using vector spaces?

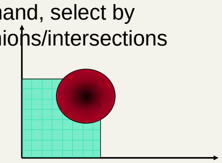
- Key: A user's query can be viewed as a (very) short document.
- Query becomes a vector in the same space as the docs.
- Can measure each doc's proximity to it.
- Natural measure of scores/ranking – no longer Boolean.
  - Queries are expressed as bags of words
- Other similarity measures: see http://www.lans.ece.utexas.edu/~strehl/diss/node52.html for a survey

Interaction: vectors and phrases

- Phrases don't fit naturally into the vector space world:
  - ***"tangerine trees" "marmalade skies"***
  - Positional indexes don't capture tf/idf information for ***"tangerine trees"***
- Biword indexes (lecture 2) treat certain phrases as terms
  - For these, can pre-compute tf/idf.
- A hack: we cannot expect end-user formulating queries to know what phrases are indexed

Vectors and Boolean queries

- Vectors and Boolean queries really don't work together very well
- In the space of terms, vector proximity selects by spheres: e.g., all docs having cosine similarity ≥0.5 to the query
- Boolean queries on the other hand, select by (hyper-)rectangles and their unions/intersections
- Round peg - square hole



Vectors and wild cards

- How about the query ***tan* marm****?
  - Can we view this as a bag of words?
  - Thought: expand each wild-card into the matching set of dictionary terms.
- Danger – unlike the Boolean case, we now have *tf*s and *idf*s to deal with.
- Net – not a good idea.

Vector spaces and other operators

- Vector space queries are apt for no-syntax, bag-of-words queries
  - Clean metaphor for similar-document queries
- Not a good combination with Boolean, wild-card, positional query operators
- But …

Query language vs. scoring

- May allow user a certain query language, say
  - Freetext basic queries
  - Phrase, wildcard etc. in Advanced Queries.
- For scoring (oblivious to user) may use all of the above, e.g. for a freetext query
  - Highest-ranked hits have query as a phrase
  - Next, docs that have all query terms near each other
  - Then, docs that have some query terms, or all of them spread out, with tfxidf weights for scoring

Exercises

- How would you augment the inverted index built in lectures 1–3 to support cosine ranking computations?
- Walk through the steps of serving a query.
- *The math of the vector space model is quite straightforward, but being able to do cosine ranking efficiently at runtime is nontrivial*

Efficient cosine ranking

- Find the k docs in the corpus “nearest” to the query $\Rightarrow$ k largest query-doc cosines.
- Efficient ranking:
  - Computing a single cosine efficiently.
  - Choosing the k largest cosine values efficiently.
    - Can we do this without computing all n cosines?

Efficient cosine ranking

- What we’re doing in effect: solving the k-nearest neighbor problem for a query vector
- In general, do not know how to do this efficiently for high-dimensional spaces
- But it is solvable for short queries, and standard indexes are optimized to do this

Computing a single cosine

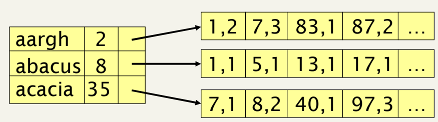
- For every term i, with each doc j, store term frequency tf_{ij}.
  - Some tradeoffs on whether to store term count, term weight, or weighted by idf_i.
- At query time, accumulate component-wise sum

$$sim(\vec{d}_j,\vec{d}_k)=\sum_{i=1}^{m} w_{i,j}\times w_{i,k}$$

- If you’re indexing 5 billion documents (web search) an array of accumulators is infeasible

Ideas?

Encoding document frequencies



- Add $tf_{t,d}$ to postings lists
  - Almost always as frequency – scale at runtime
  - Unary code is very effective here
  - γ code (Lecture 3) is an even better choice
  - Overall, requires little additional space

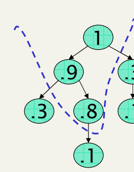
Why?

Computing the k largest cosines: selection vs. sorting

- Typically we want to retrieve the top k docs (in the cosine ranking for the query)
  - not totally order all docs in the corpus
  - can we pick off docs with k highest cosines?

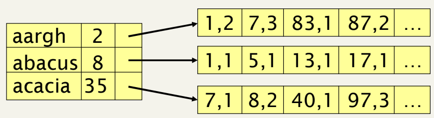
Use heap for selecting top k

- Binary tree in which each node’s value > values of children
- Takes $2n$ operations to construct, then each of k log n “winners” read off in 2log n steps.
- For n=1M, k=100, this is about 10% of the cost of sorting.



Bottleneck

- Still need to first compute cosines from query to each of n docs → several seconds for n = 1M.
- Can select from only non-zero cosines
  - Need union of postings lists accumulators (<<1M): on the query ***aargh abacus*** would only do accumulators 1,5,7,13,17,83,87 (below).



Removing bottlenecks

- Can further limit to documents with non-zero cosines on rare (high idf) words
- Enforce conjunctive search (a la Google): non-zero cosines on *all* words in query
  - Get # accumulators down to {min of postings lists sizes}
- But still potentially expensive
  - Sometimes have to fall back to (expensive) soft-conjunctive search:
  - If no docs match a 4-term query, look for 3-term subsets, etc.

Can we avoid this?

- Yes, but may occasionally get an answer wrong
  - a doc *not* in the top k may creep into the answer.

Best m candidates

- Preprocess: Pre-compute, for each term, its m nearest docs.
  - (Treat each term as a 1-term query.)
  - lots of preprocessing.
  - Result: “preferred list” for each term.
- Search:
  - For a t-term query, take the union of their t preferred lists – call this set S, where $|S| \leq mt$.
  - Compute cosines from the query to only the docs in S, and choose the top k.

Need to pick $m>k$ to work well empirically.

Exercises

- Fill in the details of the calculation:
  - Which docs go into the preferred list for a term?
- Devise a small example where this method gives an incorrect ranking.

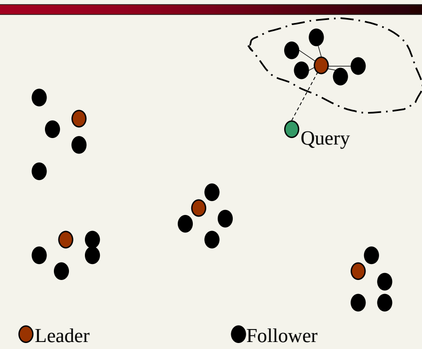
Cluster pruning: preprocessing

- Pick $\sqrt{n}$ *docs* at random: call these *leaders*
- For each other doc, pre-compute nearest leader
  - Docs attached to a leader: its *followers;*
  - Likely: each leader has ~ $\sqrt{n}$ followers.

Cluster pruning: query processing

- Process a query as follows:
  - Given query *Q*, find its nearest *leader L.*
  - Seek *k* nearest docs from among *L*'s followers.

Visualization



Why use random sampling

- Fast
- Leaders reflect data distribution

General variants

- Have each follower attached to *a*=3 (say) nearest leaders.
- From query, find *b*=4 (say) nearest leaders and their followers.
- Can recur on leader/follower construction.

Exercises

- To find the nearest leader in step 1, how many cosine computations do we do?
  - Why did we have $\sqrt{n}$ in the first place?
- What is the effect of the constants *a,b* on the previous slide?
- Devise an example where this is *likely to* fail – i.e., we miss one of the *k* nearest docs.
  - *Likely* under random sampling.

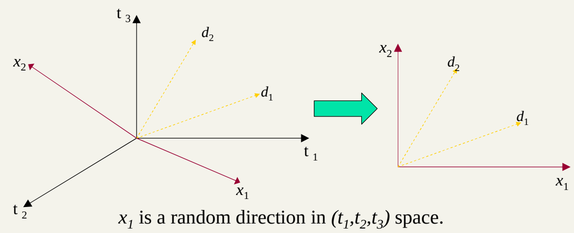
Dimensionality reduction

- What if we could take our vectors and "pack" them into fewer dimensions (say 50,000→100) while preserving distances?
- (Well, almost.)
  - Speeds up cosine computations.
- Two methods:
  - Random projection.
  - "Latent semantic indexing".

Random projection onto $k<<m$ axes

- Choose a random direction x_1 in the vector space.
- For $i = 2$ to k,
  - Choose a random direction x_i that is orthogonal to $x_1, x_2, \ldots x_{i-1}$.
- Project each document vector into the subspace spanned by $\{x_1, x_2, \ldots, x_k\}$.

E.g., from 3 to 2 dimensions



x_1 is a random direction in (t_1,t_2,t_3) space.
x_2 is chosen randomly but orthogonal to x_1.

Dot product of x_1 and x_2 is zero.

Guarantee

- With high probability, relative distances are (approximately) preserved by projection.
- Pointer to precise theorem in Resources.

Computing the random projection

- Projecting n vectors from m dimensions down to k dimensions:
  - Start with $m \times n$ matrix of terms $\times$ docs, A.
  - Find random $k \times m$ orthogonal projection matrix R.
  - Compute matrix product $W = R \times A$.
- j^{th} column of W is the vector corresponding to doc j, but now in $k << m$ dimensions.

Cost of computation

- This takes a total of *kmn* multiplications. Why?
- Expensive – see Resources for ways to do essentially the same thing, quicker.
- *Question: by projecting from 50,000 dimensions down to 100, are we really going to make each cosine computation faster?*

Latent semantic indexing (LSI)

- Another technique for dimension reduction
- Random projection was data-*independent*
- LSI on the other hand is data-*dependent*
  - Eliminate redundant axes
  - Pull together “related” axes – hopefully
    - ***car*** and ***automobile***
- More on LSI when studying clustering, later in this course.

Resources

- MG Ch. 4.4-4.6; MIR 2.5, 2.7.2; FSNLP 15.4
- Random projection theorem – Dasgupta and Gupta. An elementary proof of the Johnson-Lindenstrauss Lemma (1999).
- Faster random projection - A.M. Frieze, R. Kannan, S. Vempala. Fast Monte-Carlo Algorithms for finding low-rank approximations. IEEE Symposium on Foundations of Computer Science, 1998.