CS276A

Information Retrieval

Lecture 3

Recap: lecture 2

- Stemming, tokenization etc.
- Faster postings merges
- Phrase queries

This lecture

- Index compression
- Space estimation

Corpus size for estimates

- Consider *n* = 1M documents, each with about *L*=1K terms.
- Avg 6 bytes/term incl spaces/punctuation
  - 6GB of data.
- Say there are *m* = 500K *distinct* terms among these.

Don’t build the matrix

- 500K x 1M matrix has half-a-trillion 0’s and 1’s.
- But it has no more than one billion 1’s.
  - matrix is extremely sparse.
- So we devised the inverted index
  - Devised query processing for it
- Where do we pay in storage?

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Terms

Term	N docs	Tot Freq
ambitious	1	1
be	1	1
brutus	2	2
capitol	1	1
caesar	2	3
did	1	1
enact	1	1
hath	1	1
I	1	2
i'	1	1
it	1	1
julius	1	1
killed	1	2
let	1	1
me	1	1
noble	1	1
so	1	1
the	2	2
told	1	1
you	1	1
was	2	2
with	1	1

Doc #	Freq
2	1
2	1
1	1
2	1
1	1
1	1
2	2
1	1
1	1
2	1
1	2
1	1
2	1
1	1
1	2
2	1
1	1
2	1
2	1
1	1
2	1
2	1
2	1
1	1
2	1
2	1

Pointers

Storage analysis

- First will consider space for postings pointers
- Basic Boolean index only
  - Devise compression schemes
- Then will do the same for dictionary
- No analysis for positional indexes, etc.

Pointers: two conflicting forces

- A term like ***Calpurnia*** occurs in maybe one doc out of a million - would like to store this pointer using $\log_2$ 1M ~ 20 bits.
- A term like ***the*** occurs in virtually every doc, so 20 bits/pointer is too expensive.
  - Prefer 0/1 vector in this case.

Postings file entry

- Store list of docs containing a term in increasing order of doc id.
  - ***Brutus***: 33,47,154,159,202 …
- Consequence: suffices to store *gaps*.
  - 33,14,107,5,43 …
- Hope: most gaps encoded with far fewer than 20 bits.

Variable encoding

- For ***Calpurnia***, will use ~20 bits/gap entry.
- For ***the***, will use ~1 bit/gap entry.
- If the average gap for a term is G, want to use ~$\log_2 G$ bits/gap entry.
- Key challenge: encode every integer (gap) with ~ as few bits as needed for that integer.

γ codes for gap encoding (Elias)

Length	Offset

- Represent a gap G as the pair <*length,offset*>
- *length* is in unary and uses $\lfloor \log_2 G \rfloor$ +1 bits to specify the length of the binary encoding of
- *offset* = $G - 2^{\lfloor \log_2 G \rfloor}$ in binary.

Recall that the unary encoding of x is a sequence of x 1's followed by a 0.

γ codes for gap encoding

- e.g., 9 represented as <1110,001>.
- 2 is represented as <10,1>.
- Exercise: does zero have a γ code?
- Encoding G takes 2 $\lfloor \log_2 G \rfloor$ +1 bits.
  - γ codes are always of odd length.

Exercise

- Given the following sequence of γ–coded gaps, reconstruct the postings sequence:

1110001110101011111101101111011

From these γ–decode and reconstruct gaps, then full postings.

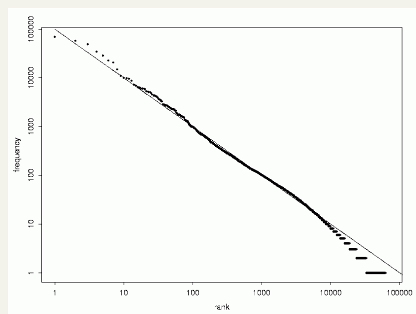
What we've just done

- Encoded each gap as tightly as possible, to within a factor of 2.
- For better tuning (and a simple analysis) - need a handle on the <u>distribution</u> of gap values.

Zipf's law

- The *k*th most frequent term has frequency proportional to *1/k*.
- Use this for a crude analysis of the space used by our postings file pointers.
  - Not yet ready for analysis of dictionary space.

Zipf's law log-log plot



Rough analysis based on Zipf

- The *i* th most frequent term has frequency proportional to 1/*i*
- Let this frequency be *c/i.*
- Then $\sum_{i=1}^{500,000} c/i = 1.$
- The *k* th <u>Harmonic number</u> is $H_k = \sum_{i=1}^{k} 1/i.$
- Thus *c* = 1/H_m , which is ~ 1/ln *m* = 1/ln(500k) ~ 1/13.
- So the *i* th most frequent term has frequency roughly 1/13*i.*

Postings analysis contd.

- Expected number of occurrences of the *i* th most frequent term in a doc of length *L* is:

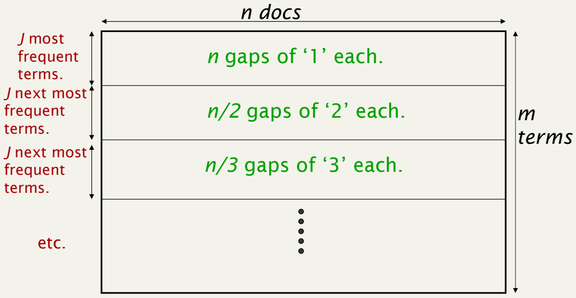
Lc/i ~ *L*/13*i* ~ 76/*i* for *L*=1000.

Let *J* = *Lc* ~ 76.

Then the *J* most frequent terms are likely to occur in every document.

Now imagine the term-document incidence matrix with rows sorted in decreasing order of term frequency:

Rows by decreasing frequency



J-row blocks

- In the *i* th of these *J*-row blocks, we have *J* rows each with *n/i* gaps of *i* each.
- Encoding a gap of *i* takes us $2\log_2 i + 1$ bits.
- So such a row uses space ~ $(2n \log_2 i)/i$ bits.
- For the entire block, $(2n\, J \log_2 i)/i$ bits, which in our case is ~ $1.5 \times 10^8 (\log_2 i)/i$ bits.
- Sum this over *i* from 1 upto *m/J* = 500K/76~ 6500. (Since there are *m/J* blocks.)

Exercise

- Work out the above sum and show it adds up to about 53 x 150 Mbits, which is about 1GByte.
- So we've taken 6GB of text and produced from it a 1GB index that can handle Boolean queries!

Make sure you understand all the approximations in our probabilistic calculation.

Caveats

- This is not the entire space for our index:
  - does not account for dictionary storage – next up;
  - as we get further, we'll store even more stuff in the index.
- Assumes Zipf's law applies to occurrence of terms in docs.
- All gaps for a term taken to be the same.
- Does not talk about query processing.

More practical caveat

- γ codes are neat but in reality, machines have word boundaries – 16, 32 bits etc
  - Compressing and manipulating at individual bit-granularity is overkill in practice
  - Slows down architecture
- In practice, simpler word-aligned compression (see Scholer reference) better

Word-aligned compression

- Simple example: fix a word-width (say 16 bits)
- Dedicate one bit to be a *continuation bit c.*
- If the gap fits within 15 bits, binary-encode it in the 15 available bits and set *c*=0.
- Else set *c*=1 and use additional words until you have enough bits for encoding the gap.

Exercise

- How would you adapt the space analysis for γ–coded indexes to the scheme using continuation bits?

Exercise (harder)

- How would you adapt the analysis for the case of positional indexes?
- Intermediate step: forget compression. Adapt the analysis to estimate the number of positional postings entries.

Dictionary and postings files

Term	Doc #	Freq
ambitious	2	1
be	2	1
brutus	1	1
brutus	2	1
capitol	1	1
caesar	1	1
caesar	2	2
did	1	1
enact	1	1
hath	2	1
I	1	2
i'	1	1
it	2	1
julius	1	1
killed	1	2
let	2	1
me	1	1
noble	2	1
so	2	1
the	1	1
the	2	1
told	2	1
you	2	1
was	1	1
was	2	1
with	2	1

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I	1	2
i'	1	1
it	1	1
julius	1	1
killed	1	2
let	1	1
me	1	1
noble	1	1
so	1	1
the	2	2
told	1	1
you	1	1
was	2	2
with	1	1

Usually in memory

Doc #	Freq
2	1
2	1
1	1
2	1
1	1
1	1
2	2
1	1
1	1
2	1
1	2
1	1
2	1
1	1
1	2
2	1
1	1
2	1
2	1
1	1
2	1
2	1
2	1
1	1
2	1
2	1

Gap-encoded, on disk

Inverted index storage

- Have estimated pointer storage
- Next up: Dictionary storage
  - Dictionary in main memory, postings on disk
    - This is common, especially for something like a search engine where high throughput is essential, but can also store most of it on disk with small, in-memory index
- Tradeoffs between compression and query processing speed
  - Cascaded family of techniques

How big is the lexicon V?

- Grows (but more slowly) with corpus size
- Empirically okay model:

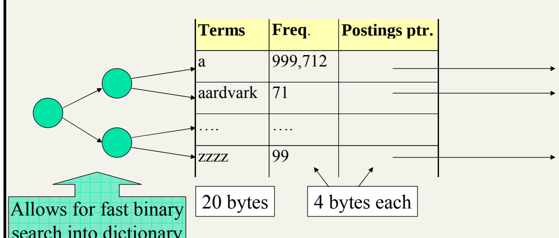
$$m = kN^b$$

Exercise: Can one derive this from Zipf's Law?

- where $b \approx 0.5$, $k \approx 30–100$; N = # tokens
- For instance TREC disks 1 and 2 (2 Gb; 750,000 newswire articles): ~ 500,000 terms
- V is decreased by case-folding, stemming
- Indexing all numbers could make it extremely large (so usually don't*)
- Spelling errors contribute a fair bit of size

Dictionary storage - first cut

- Array of fixed-width entries
  - 500,000 terms; 28 bytes/term = 14MB.



Exercises

- Is binary search really a good idea?
- What are the alternatives?

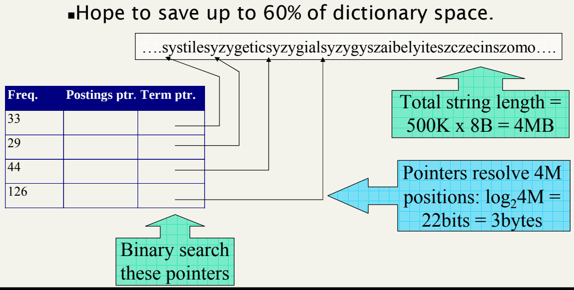
Fixed-width terms are wasteful

- Most of the bytes in the **Term** column are wasted – we allot 20 bytes for 1 letter terms.
  - And still can't handle *supercalifragilisticexpialidocious.*
- Written English averages ~4.5 characters.
  - Exercise: Why is/isn't this the number to use for estimating the dictionary size?
  - Short words dominate token counts.
- Average word in English: ~8 characters.

Explain this.

Compressing the term list

- Store dictionary as a (long) string of characters:
  - Pointer to next word shows end of current word
  - Hope to save up to 60% of dictionary space.



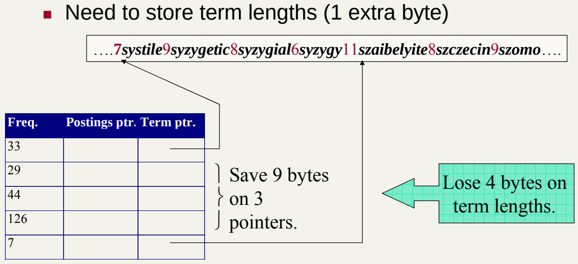
Total space for compressed list

- 4 bytes per term for Freq.
- 4 bytes per term for pointer to Postings.
- 3 bytes per term pointer
- Avg. 8 bytes per term in term string
- 500K terms ⇒ 9.5MB

Now avg. 11 bytes/term, not 20.

Blocking

- Store pointers to every *k*th on term string.
  - Example below: *k*=4.
- Need to store term lengths (1 extra byte)



Net

- Where we used 3 bytes/pointer without blocking
  - 3 x 4 = 12 bytes for *k*=4 pointers,

now we use 3+4=7 bytes for 4 pointers.

Shaved another ~0.5MB; can save more with larger *k*.

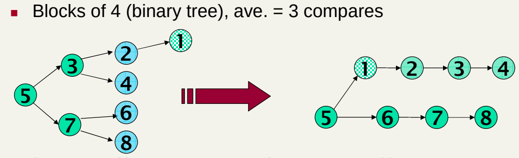
Why not go with larger *k*?

Exercise

- Estimate the space usage (and savings compared to 9.5MB) with blocking, for block sizes of *k = 4, 8* and *16.*

Impact on search

- Binary search down to 4-term block;
- Then linear search through terms in block.
- 8 documents: binary tree ave. = 2.6 compares
- Blocks of 4 (binary tree), ave. = 3 compares



$= (1+2\cdot2+4\cdot3+4)/8$ $=(1+2\cdot2+2\cdot3+2\cdot4+5)/8$

Exercise

- Estimate the impact on search performance (and slowdown compared to *k*=1) with blocking, for block sizes of *k = 4, 8* and *16.*

Total space

- By increasing *k*, we could cut the pointer space in the dictionary, at the expense of search time; space 9.5MB → ~8MB
- Net – postings take up most of the space
  - Generally kept on disk
  - Dictionary compressed in memory

Some complicating factors

- Accented characters
  - Do we want to support accent-sensitive as well as accent-insensitive characters?
  - E.g., query ***resume*** expands to ***resume*** as well as ***résumé***
  - But the query ***résumé*** should be executed as only ***résumé***
  - Alternative, search application specifies
- If we store the accented as well as plain terms in the dictionary string, how can we support both query versions?

Index size

- Stemming/case folding cut
  - number of terms by ~40%
  - number of pointers by 10-20%
  - total space by ~30%
- Stop words
  - Rule of 30: ~30 words account for ~30% of all term occurrences in written text
  - Eliminating 150 commonest terms from indexing will cut almost 25% of space

Extreme compression (see *MG*)

- <u>Front-coding</u>:
  - Sorted words commonly have long common prefix – store differences only
  - (for last *k-1* in a block of *k*)

8***automata***8***automate***9***automatic***10***automation***

→8{***automat***}***a***1◊***e***2◊***ic***3◊***ion***

Encodes ***automat***

Extra length beyond ***automat.***

Begins to resemble general string compression.

Extreme compression

- Using (perfect) hashing to store terms "within" their pointers
  - not great for vocabularies that change.
- Large dictionary: partition into pages
  - use B-tree on first terms of pages
  - pay a disk seek to grab each page
  - if we're paying 1 disk seek anyway to get the postings, "only" another seek/query term.

Compression: Two alternatives

- <u>Lossless compression</u>: all information is preserved, but we try to encode it compactly
  - What IR people mostly do
- <u>Lossy compression</u>: discard some information
  - Using a stopword list can be viewed this way
  - Techniques such as Latent Semantic Indexing (later) can be viewed as lossy compression
  - One could prune from postings entries unlikely to turn up in the top *k* list for query on word
    - Especially applicable to web search with huge numbers of documents but short queries (e.g., Carmel et al. *SIGIR 2002)*

Top *k* lists

- Don't store all postings entries for each term
  - Only the "best ones"
  - Which ones are the best ones?
- More on this subject later, when we get into ranking

Resources

- MG 3.3, 3.4.
- F. Scholer, H.E. Williams and J. Zobel. Compression of Inverted Indexes For Fast Query Evaluation. Proc. ACM-SIGIR 2002.