CS222 / Phil358 Homework 3

Due: May 26, 2010

1. In the basic linear temporal logic, show that for any given frame, $\mathbb{F} \vDash PF\varphi \to (P\varphi \vee \varphi \vee F\varphi)$, if and only if $\mathbb{F}$ is linear in the future, that is, if $x < y$ and $x < z$, then either $y < z$, $z < y$, or $y = z$.

2. (a) Write a program in the program language of Propositional Dynamic Logic (PDL) that means $\textsf{Do } a \textsf{ until } \varphi$.

   (b) Show that the following two programs are equivalent: $(a \cup b)^*$ and $(a^*b)^*a^*$.

3. Using the "induction axiom" for the Kleene star in PDL, that is, the formula:
$$\varphi \wedge [a^*](\varphi \to [\alpha]\varphi) \to [\alpha^*]\varphi,$$
show that the following rule is admissible in PDL:

   From $\varphi \to [\alpha]\varphi$, infer $\varphi \to [\alpha^*]\varphi$.

4. Show that in the Public Announcement Logic (PAL) that the following holds: $\vDash [\varphi \wedge [\varphi]\psi]\chi \leftrightarrow [\varphi][\psi]\chi$.

5. A formula in PAL is *successful* just in case $\vDash [\varphi]\varphi$.

   (a) Show the successful formulas are not closed under negation. (That is, show there is a successful formula φ such that $\neg\varphi$ is not successful.)

   (b) Show there are successful formulas ψ and χ such that $[\psi]\chi$ is not successful.

6. **Bonus:** (+10) Give an example of two successful formulas ψ and χ such that $\psi \vee \chi$ is not successful.