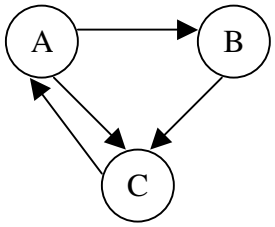
Problem 1

The diagram of the Web and its corresponding stochastic matrix M are as follows:



$$M = \begin{bmatrix} 0 & 0 & 1 \\ 1/2 & 0 & 0 \\ 1/2 & 1 & 0 \end{bmatrix}$$

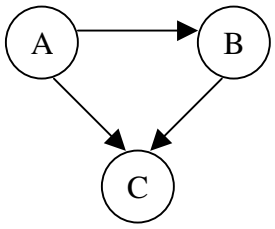
The following is the system of simultaneous equations:

1. A = C
2. B = 1/2 A
3. C = 1/2 A + B
4. A + B + C = 3

Solving the above system, we have A = 6/5, B = 3/5, and C = 6/5.

Problem 2

The diagram of the Web and its corresponding stochastic matrix M are as follows:



$$M = \begin{bmatrix} 0 & 0 & 0 \\ 1/2 & 0 & 0 \\ 1/2 & 1 & 0 \end{bmatrix}$$

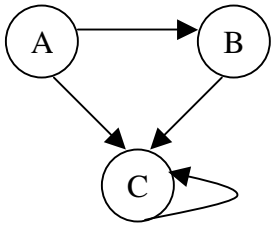
The following is the system of simultaneous equations:

1. $A = 0 + 0.3$
2. $B = 0.7\ (1/2\ A) + 0.3$
3. $C = 0.7\ (1/2\ A + B) + 0.3$

Solving the above system, we have $A = 0.3$, $B = 0.405$, and $C = 0.6885$.

Problem 3

The diagram of the Web and its corresponding stochastic matrix M are as follows:



$$M = \begin{bmatrix} 0 & 0 & 0 \\ 1/2 & 0 & 0 \\ 1/2 & 1 & 1 \end{bmatrix}$$

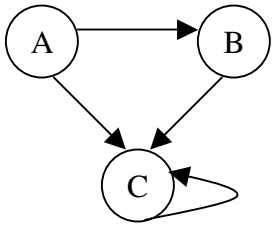
The following is the system of simultaneous equations:

1. $A = 0 + 0.3$
2. $B = 0.7\ (1/2\ A) + 0.3$
3. $C = 0.7\ (1/2\ A + B + C) + 0.3$

Solving the above system, we have $A = 0.3$, $B = 0.405$, and $C = 2.295$.

Problem 4

The diagram of the Web and its corresponding transition matrix A are as follows:



$$A = \begin{bmatrix} 0 & 1 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}, A^T = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix}$$

To calculate the hubbiness of the pages: $h = \lambda\mu AA^T h$, i.e., $\begin{bmatrix} h1 \\ h2 \\ h3 \end{bmatrix} = \lambda\mu \begin{bmatrix} 2 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} h1 \\ h2 \\ h3 \end{bmatrix}$

The following is the system of simultaneous equations:

1. $h_1 = \lambda\mu (2 h_1 + h_2 + h_3)$
2. $h_2 = \lambda\mu (h_1 + h_2 + h_3)$
3. $h_3 = \lambda\mu (h_1 + h_2 + h_3)$

By observation, we have:

$\Rightarrow h_2 = h_3$

$\Rightarrow h_1 / (2 h_1 + h_2 + h_2) = h_2 / (h_1 + h_2 + h_2)$

If we set $h_1 = 1$, then we have $h_2 = h_3 = 1/2^{0.5}$.

To calculate the authorities of the pages: $a = \lambda\mu AA^T a$, i.e., $\begin{bmatrix} a1 \\ a2 \\ a3 \end{bmatrix} = \lambda\mu \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 3 \end{bmatrix} \begin{bmatrix} a1 \\ a2 \\ a3 \end{bmatrix}$

The following is the system of simultaneous equations:

1. $a_1 = 0$
2. $a_2 = \lambda\mu (a_2 + a_3)$
3. $a_3 = \lambda\mu (a_2 + 3 a_3)$

By observation, we have:

$\Rightarrow a_2 / (a_2 + a_3) = a_3 / (a_2 + 3 a_3)$

If we set $a_2 = 1$, then we have $a_3 = 1 + 2^{0.5}$.