CS143 Midterm
Spring 2022

- Please read all instructions (including these) carefully.
- There are 5 questions on the exam, some with multiple parts. You have 90 minutes to work on the exam.
- The exam is open note. You may use laptops, phones and e-readers to read electronic notes, but not for computation or access to the internet for any reason other than to access the class webpage.
- Please write your answers in the space provided on the exam, and clearly mark your solutions. Do not write on the back of exam pages or other pages.
- Solutions will be graded on correctness and clarity. Each problem has a relatively simple and straightforward solution. You may get as few as 0 points for a question if your solution is far more complicated than necessary. Partial solutions will be graded for partial credit.

NAME: ____

In accordance with both the letter and spirit of the Honor Code, I have neither given nor received assistance on this examination.

SIGNATURE: ____

Problem	Max points	Points
1	10	
2	25	
3	15	
4	25	
5	25	
TOTAL	100	

1. Regular Grammars

In class, we discussed how a CFG can be more expressive than a regular expression. However, a subset of CFGs we will call the *regular grammars* (RGs) have exactly the same expressive power as regular expressions.

A regular grammar is a CFG with any number of nonterminals in which every production follows one of three forms:

- $A \to \varepsilon$
- $A \to a$
- $A \to aB$

where a lowercase letter is a single terminal. Note that we allow the case where $B = A$.

For alphabet $\Sigma = \{\texttt{a}, \texttt{b}, \texttt{c}, \texttt{d}\}$, define an RG that is equivalent to the regular expression

$$(\texttt{a}|\texttt{c})(\texttt{dbb}^*)^*$$

Answer: (your answer may use at most 5 non-terminal symbols)

$$\begin{aligned} S &\to \texttt{a}\ A \\ &\mid\ \texttt{c}\ A \\ A &\to \varepsilon \\ &\mid\ \texttt{d}\ B \\ B &\to \texttt{b}\ A \\ &\mid\ \texttt{b}\ B \end{aligned}$$

2. Context-Free Grammars

Given the following two CFGs over the alphabet $\Sigma = \{a, b, c\}$, what is the most restrictive language that can describe them among the following:

$$\text{LR}(0) \subset \text{SLR}(1) \subset \text{Unambiguous CFGs} \subset \text{All CFGs}.$$

For each CFG, explain why it cannot be expressed in the next more restrictive language. If the grammar is LR(0) then no explanation is needed.

(a) $A \to a \mid BaB \mid C$
$B \to b \mid A$
$C \to c \mid BcB$

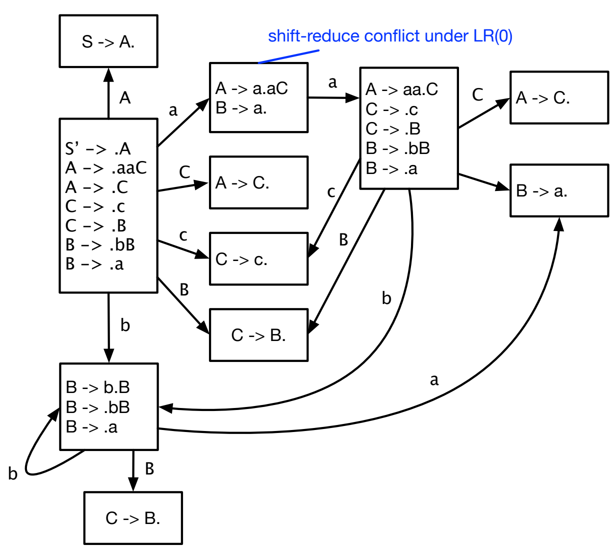
Answer:

All CFGs. The grammar can not be described by an unambiguous CFG, because the grammar is ambiguous. Since C can be replaced by BcB, then A can go to either BaB or BcB. Given a string "$BaBcB$", there are two possible trees: one 'a' lower in the tree and one with 'c' lower in the tree.

(b) $A \to aaC \mid C$
$B \to bB \mid a$
$C \to c \mid B$

Answer:

The CFG is SLR(1). It is not in LR(0) because there is a shift-reduce conflict, which can be resolved in SLR(1) since Follow(B) = {\$} does not contain 'a'.



3. Syntax-Directed Translation

Consider a non-standard binary number system where the value of each binary number b is defined as the *alternating sum* of the decimal numbers that non-zero binary digits represent from right to left. For example, $val(\varepsilon) = 0$, $val(100) = 2^2 = 4$, $val(100010) = 2^1 - 2^5 = -30$, and $val(1100001) = 2^0 - 2^5 + 2^6 = 33$.

Given the following grammar for a non-standard binary numbers b, add semantic actions that computes $val(b)$. You must use the following attributes only: `int val` and `int tmp`. Use Bison syntax: `$i.val` refers to the `val` attribute of the i^{th} symbol of the production and `$$.val` refers to the `val` attribute of the production's result. You should not use any global variables or any attributes other than `val` and `tmp`.

$$
\begin{aligned}
S &\rightarrow T \\
&\mid\ \varepsilon \\
T &\rightarrow 0T \\
&\mid\ 1T \\
&\mid\ 0 \\
&\mid\ 1
\end{aligned}
$$

Answer:

```
S -> T {

        $$.val = $1.val;

}

S -> ε  {

        $$.val = 0;

}
```

```
T -> 0T {

        $$.tmp = $2.tmp * 2;
        $$.val = $2.val;

}

T -> 1T {

        $$.tmp = $2.tmp * (-2);
        $$.val = $2.val + $$.tmp;

}

T -> 0  {

        $$.tmp = -1;
        $$.val = 0;

}

T -> 1  {

        $$.tmp = 1;
        $$.val = 1;

}
```

4. First and Follow Sets

We have lost our CFG, but luckily we have the First sets and all of the First/Follow relationships. Construct a grammar that is consistent with the following information:

Each nonterminal has exactly two productions.

$$\begin{aligned}
\text{First(A)} &= \{a, b\} \\
\text{First(B)} &= \{a, b\} \\
\text{First(D)} &= \{d, \varepsilon\}
\end{aligned}$$

$$\begin{aligned}
\text{First(B)} - \{\varepsilon\} &\subseteq \text{Follow(a)} \\
\text{d} &\in \text{Follow(B)} \\
\text{Follow(A)} &\subseteq \text{Follow(d)} \\
\text{First(D)} - \{\varepsilon\} &\subseteq \text{Follow(B)} \\
\text{Follow(A)} &\subseteq \text{Follow(D)} \\
\text{Follow(A)} &\subseteq \text{Follow(B)} \\
\text{First(D)} - \{\varepsilon\} &\subseteq \text{Follow(b)} \\
\text{Follow(B)} &\subseteq \text{Follow(D)} \\
\text{Follow(B)} &\subseteq \text{Follow(b)} \\
\text{Follow(B)} &\subseteq \text{Follow(a)} \\
\text{Follow(D)} &\subseteq \text{Follow(d)}
\end{aligned}$$

Answer:

$$\begin{aligned}
&A \to aBd \mid BD \\
&B \to bD \mid a \\
&D \to d \mid \varepsilon
\end{aligned}$$

Note: some variations here are possible. E.g., instead of $A \to aBd$, you could have $A \to aB$.

5. Bottom-Up Parsing

Each of the following two subproblems describe a deterministic (i.e., DFA) LR(0) parsing automaton. Show your grammar and fill in the parsing automaton with transitions and each state labeled with its set of LR(0) items. You do *not* need to analyze the automaton to determine whether the grammar is LR(0) or SLR(1). The grammar and the automaton constitute a complete answer to each subproblem.

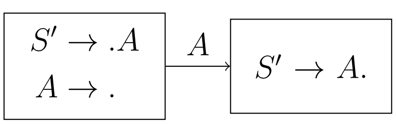
Assume that the first step of the automaton construction is to add a new production $S' \to S$ to the grammar, as described in class. This production should be included in your grammars, in your automatons, and in your counts.

Give the simplest possible grammar (fewest productions *and* fewest terminals) that result in a parsing automaton satisfying the description.

(a) An automaton (and corresponding CFG) with two states and one transition from the start state to the second state.

Answer:

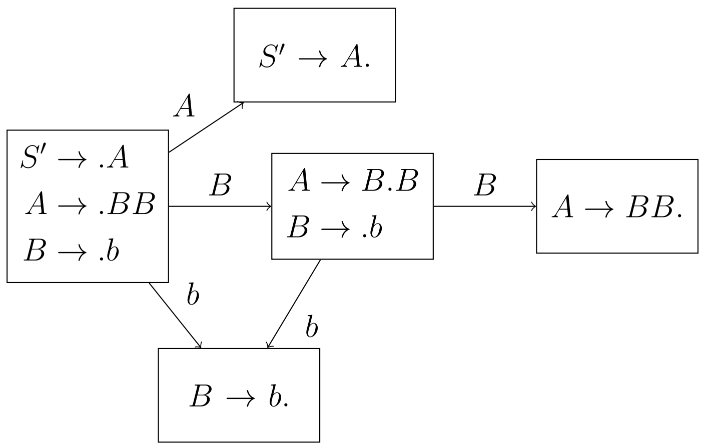
$$A \to \varepsilon$$



(b) An automaton (and corresponding CFG) with a minimal number of states, without loops, where one state has two incoming transitions.

Answer:

$$A \to BB$$
$$B \to b$$



(Note: We accepted any solution with at most 7 states as close enough to minimal.)