CME306 / CS205B Homework 9

1. Write out the *symmetric* matrix equation for the standard second order central difference approximation to the equation

$$\nabla \cdot \left(\frac{1}{\rho} \nabla p \right) = \nabla \cdot \vec{u} \tag{1}$$

with the following boundary conditions:

$$\begin{cases} p(0, y) = 1 \\ p(1, y) = 1 \\ p_y(x, y) = 0 \quad \text{for } y \in \{0, 1\} \end{cases} \tag{2}$$

You should assume a MAC grid (ie. that velocities live on cell faces, and that pressure and density live in the cell centers), and you may *not* assume a constant density. Write the equations for the following three cells:

(a) an internal cell (something sufficiently far from the boundary, ie. p_{ij})

(b) a cell that lies along the x-axis boundary (ie. p_{i1}), and

(c) a cell that lies along the y-axis boundary (ie. p_{1j}).

2. Physically, when we have an incompressible flow with all Neumann boundary conditions, what does the compatibility condition require? Is something similar required for compressible flow?